SCIENTIFIC BOOKS

THE THEORY OF FUNCTIONS

Théorie Générale des Fonctionelles. By V. VOLTERRA and J. PÉRÈS. (Collection Borel) Paris, Gauthier-Villars, 1936. xii + 358 pp.

THE volume before us is the first in a series of three, the three together to give a résumé of something over fifty years of the work of Volterra. Although the complementary studies of others are given extensively and a systematic treatment is constructed, thus portraying the whole development of the subject, the exposition would nevertheless demand the attention of mathematicians of this generation, even if it limited itself to the inventions and discoveries of the author himself. Moreover, the author has the good fortune of having as collaborator Professor J. Pérès, who has made many contributions to the subject of functionals and integral equations. This volume and the other two in prospect, as well as the recent treatise, "Opérations infinitésimales linéaires," by Volterra and Hostinsky, form an amplification and modernization of the two volumes on functions of curves and integral equations published in the same Borel series on the theory of functions, some twenty-five years ago.

It is not necessary to detail the entire history of Volterra's contributions to this subject. But it is interesting to recall one or two steps. In the account of the meeting of the Lincei of June 15, 1884, appeared a short note "Sopra un problema di elettrostatica," which deals with the equation,

$$\varphi(x) = \int_o^a f(\alpha)\, F(\alpha, x)\, d\alpha, \quad o < x < a,$$

by means of variational methods. The significance of its result is best expressed in terms of the physical application: Given a conductor which is in the form of a symmetrical portion of a surface of revolution, subject to the action of insulators containing arbitrary charges of electricity provided that these are symmetrical about the axis of revolution, the equilibrium distribution on the conductor can be constructed in terms of the corresponding distribution of unit mass on the conductor when there are no inducing masses present. In 1887 we find these variational principles further developed as the sketch of a general theory of functionals; and also the study of linear differential equations in terms of the infinitesimal analysis of substitutions. In 1894 we have the algebraic theory of integral equations "of Volterra type." A man is indeed fortunate who sows in his youth the seeds of such a harvest.

The first half of the volume is devoted to the general theory of functionals. In this, we have the "passage from the discontinuous to the continuous," the relation to abstract metric spaces with the forms of the linear operator which depend on the various definitions of distance, a development of the calculus of functionals, and its relation to the calculus of variations. The author acknowledges the assistance of Professor Tonelli in this last section. These chapters are not to be taken as a substitute for treatises on metric spaces or calculus of variations, of which there already exist notable instances, but to introduce the reader to the general points of view which envisage them. Rather, there are emphasized the special developments of the theory of functionals itself on the basis of the special properties of one definition of distance, and for the sake of their applications.

The second part of the book deals with integral equations and comprises, as well as the classical theory, various developments in the direction of singular types, equations with multiple integrals, systems of equations and non-linear equations. It is not intended to be as nearly complete a survey as the detailed review of Hellinger in the Encyklopädie, but it is entirely readable and is supplemented by an extensive bibliography. The student will get from it the feeling of pressure outward, rather than the impression of in-breeding that one receives from so many sketches of the topic. One wonders when the subject of the numerical calculation and theoretical location of characteristic values will obtain the systematic development which may be hoped for from a subject as vast as the theory of functions of a complex variable (see for instance Section 35 of Hellinger's above-mentioned report).

There is some development of the "functions of composition," in which the symbolic product fg stands for the expression

$$\int_x^y f(x, \xi)\, g\,(\xi, y)\, d\xi$$

or the corresponding integral with constant limits, the systematic treatment of this subject, with its numerous applications, being left to another volume. Of particular interest are the powers, positive, negative and zero. The power f^0 plays somewhat the role of an indicator of an operation to be performed; that is, f^0 itself may have no concrete interpretation, whereas $f^0 g = g$. Whether or not there may be some advantage in separating the roles of unity by introducing a notation analogous to that of quaternions, as the reviewer did at one time, the questions at issue remain fundamentally the same; and the difficulties are in no sense to be obviated by a method of notation.

Altogether the reviewer has no hesitation in recom-

mending the volume to the diligent attention of the student, and looks forward himself to the publication of the later volumes. In particular the reader will find that the treatment serves to give content to those more abstract studies of metric spaces, which indeed have been created in large part out of the fecundity of this topic of functionals.

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Termite City. By ALFRED E. EMERSON and ELEANOR FISH. With a foreword by William Beebe. Illustrations by Keith Ward. Pp. 127, 37 ills. Rand, McNally and Company, New York, 1937. $1.50.

THE inexhaustible store of most interesting natural history pertaining to termites has been tapped in this book by Professor Emerson out of his long accumulated riches of information about this greatly diversified group of social insects. He sets forth in non-technical language the organization of the termite colony; the functions of the several castes; the varied structures found in termite houses; the periodic swarming of the alates; the care of king, queen, eggs, soldiers and larvae; the guest insects found in termite colonies; the destructive activities of cellulose-eating species; and the past history and evolution of the group. A fully annotated glossary adds technical information helpful to the non-specialist reader.

Termites are rather closely related to cockroaches and are thus among the oldest insects known, being found in the carboniferous strata and fully preserved in amber. Although rather simple and generalized in basic structural pattern they are highly evolved in social organization and in structures and instincts related to this mode of life, as shown in the limitation of reproduction to a single pair in the colony, with enormous increase in size and rate of egg-laying in the queen and provision of second and even third form supplementary kings and queens capable of coming quickly into functional activity in the event of the death of either member of the primary pair, or in case of the breaking up of the primary colony by isolation of outlying extensions or transportation of a fragment of the colony to a new site. Other specializations are found in the soldier caste, which also may have second and third forms in the same species. Some of these develop powerful mandibles, while others are nasute types with glands exuding a sticky fluid for entangling enemies or a poison gas for destroying them. As in the case of the military forms among other social animals the soldiers sacrifice themselves for the good of the group, are unproductive and are faithfully fed by the workers; the latter in the more highly evolved termite families form a slave caste deprived of flight and the reproductive function.

More specialized still are the complicated instincts of each caste by which the social organization is conserved. As noted by Darwin in his discussion of the difficulties attendant upon his theory of natural selection, these extraordinary instincts are exhibited by individuals who leave no offspring, and their parents never exhibit these same instinctive behaviors. This may be a highly specialized form of recessiveness of characters linked with the capacity for normal sexual activity.

Professor Emerson brings out of his closet of termite skeletons some of the disjunct members which haunt a student of these perplexing activities in the termite colony. Most of the work of the colony is done by individuals which are blind. Their antennae and mouth parts have a variety of sense organs adapted by structure as tactile and chemical receptors, but auditory organs are lacking and visual ones occur only in a rudimentary state even in the alate phase. How then is coordinated activity established and maintained? How do the blind soldiers find their way to guard the portal of exit, space themselves in order according to rank about the queen, or along the line of march? How do workers without blue prints or supervising architect build earthen towers with a ventilating system, oriented to the points of the compass, or erect rain gutters on the trunk of a tree to shed water from the nest starting these structures at the top and bottom of the series and meeting in good form at the middle? Verily instinct covers a multitude of unsolved problems in the termite world.

Termites also give rise to another set of problems among termitophiles which live in their colonies, which as beetles of strange pattern and stranger behavior, with abdomens reflexed over their back with segmental spigots flowing with milk and honey for their hosts. The fact that termitophile insects come to smell like their hosts may explain their toleration by their otherwise cautious hosts, but their structural changes might perplex even an enthusiastic biochemist.

Termites destroy man's utilized cellulose, preempt his fertile lands in the tropics by massive earthen structures, often sheltering fungus gardens. These are so durable that the recovery of the land for agriculture is unprofitable, though the termite mortar is good road metal. Termites also reduce forest waste, rapidly returning fallen trees to the soil, thus building up humus. They also facilitate erosion and help in building up alluvium and sedimentary deposits. The volumetric aspect of their relation to nature and man is based upon their social activities. The termite colony, given favorable conditions within reach of their exploratory habits, is practically immortal. It is fascism without a Hitler imposing its Kultur upon every individual and ever demanding room for more colonies.

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