corresponding mechanical and acoustical systems. Because of this familiarity, it has become the custom to describe various acoustical phenomena in the language of the electrical engineer by reference to corresponding electrical circuits. In presenting the principles of wave mechanics, Schroedinger and subsequent expositors of the subject made use of acoustical analogies, showing particularly the similarity in the problems of finding the allowed energy levels in atomic physics and the normal modes of vibration of mechanical and acoustical systems. Because of the extraordinary interest in atomic physics in recent years, the former class of problems has become more familiar to the theoretical physicist than the latter. In this book, therefore, some of the mathematical techniques used in wave mechanics and particularly the idioms which there have become familiar are applied to problems in acoustics.

The book deals most particularly with those parts of acoustics to which the mathematical methods used in wave mechanics are especially applicable, that is, those in which normal modes of vibration are determined from a differential wave equation and the boundary conditions and in which the resulting motion under a given force is determined by expansion of the force in a series of terms characterizing the normal modes of motion. The extent to which this method of treatment dominates the discussion throughout the book is indicated by the fact that nothing or little is said about velocity potential, the dynamical equations of Lagrange or the principle of reciprocity. The author devotes considerable space at the beginning of the book to the vibrating string, as this is admirably adapted for priming the student in the mathematical methods used in most of the succeeding chapters, which deal with bars, membranes, plates, radiation, propagation and scattering of sound, speech and hearing, and the acoustics of rooms.

The engineer may not find this book entirely convenient if he is looking for information of the kind which is customarily given in engineering handbooks. Some of the notation, which is often at variance with that adopted by the American Standards Association, may be unfamiliar to him, *e.g.*, ν is used instead of f for frequency and a clockwise instead of the conventional counter-clockwise rotation of the positive time vector is adopted. On the other hand, the student who wishes to become thoroughly grounded in certain methods of the theoretical physicist as applied to acoustical problems and at the same time to obtain a comprehensive picture of the physical relationships involved, will here find an excellent introduction. Although the discussion throughout the book is based on well-established principles, the presentation is refreshingly original as well as clear. A set of well-chosen, illustrative problems is given at the end of each chapter.

It has long been customary to consider the problems in the acoustics of rooms from the standpoint of wave propagation and the mean free path of the wave. This method has been fruitful in dealing with practical problems. More recently a number of investigators have studied the subject analytically by the more conventional methods of mathematical physics whereby the normal modes of vibration together with their periods and rates of decay are determined from the field equation and the boundary conditions. This is the method of treatment adopted in this book. Although it may not as yet have been of great direct help in the acoustical design of rooms because of the difficulty of obtaining even an approximate solution of the equations in practical cases, it is extremely valuable in that it affords a clear physical picture of the nature of the acoustical phenomena in rooms. The discussion by Professor Morse is, I think, more illuminating than anything presented heretofore from this point of view.

The particular forms of the apparatus and instruments chosen as examples for the application of the mathematical methods are not of a kind that have come into commercial use, nor perhaps of a kind that any one should want to build for purposes of study. They apparently have been idealized to illustrate more effectively the points the author wishes to emphasize. This procedure is in line with the whole plan of the book, which is to discuss and bring out those principles that are physically fundamental to the science rather than to give a description of things which may be here to-day and gone to-morrow.

E. C. WENTE

SPECIAL ARTICLES

FUNDAMENTAL THEOREMS OF TRIHORNOMETRY

(1) A *horn angle* is the figure formed by two curves having a common tangent at a common point. Only the case of first order contact is considered here. The unique conformal invariant of a horn angle I have shown to be

$$M_{12}=\frac{(\gamma_2-\gamma_1)^2}{\frac{d\gamma_2}{ds_2}-\frac{d\gamma_1}{ds_1}}$$

where γ represents curvature and s arc length. This combination of the two curvatures and the two rates of curvature is therefore called the *natural measure* of the horn angle. It is a real abstract number.

Since only first order contact is allowed, the measure M_{12} can not be zero, but can be infinity. If M is infinite, the horn angle can be reduced conformally to the circular (or Euclidean) case—a circle and a tangent line. If M is finite, the horn angle can be reduced to the parabolic (or Appolonian) case—a parabola and a line tangent to it at a point that is not its vertex.

For convenience, the curvatures are represented by x and the rates of curvature by y. Then, $M_{12} = (x_2 - x_1)^2/(y_2 - y_1)$. An *auxiliary plane* in which x and y are represented as cartesian coordinates is introduced. The horn angle is represented by two points (x_1, y_1), (x_2, y_2), and M_{12} is their distance in the new metric. Obviously $M_{21} = -M_{12}$.

(2) A *trihorn* is formed when three curves have a common tangent at their common point (but do not have higher contact at this point). A trihorn has six measures: M_{12}, M_{21}, M_{23}, M_{32}, M_{31}, M_{13}. These are evidently connected by the following relations:

$$M_{21} = -M_{12},\ M_{32} = -M_{23},\ M_{13} = -M_{31}.$$

We select M_{12}, M_{23}, M_{31} as the three principal measures of the three horns of the trihorn.

Besides, a trihorn has six new angles: α_{12}, α_{21}, α_{23}, α_{32}, α_{31}, α_{13} which are conformally invariant. Each α-angle measures the angle between two horns and may be called a dihorn angle. The angle α_{12} is defined in the auxiliary plane as follows:

$$\alpha_{12} = \alpha_{3,\,12} = \alpha_{31,\,32} = \frac{(y_2 - y_3)/(x_2 - x_3)}{(y_1 - y_3)/(x_1 - x_3)}.$$

Therefore,

$$\alpha_{21} = \frac{1}{\alpha_{12}},\ \alpha_{32} = \frac{1}{\alpha_{23}},\ \alpha_{13} = \frac{1}{\alpha_{31}}.$$

Select α_{12}, α_{23}, α_{31} as the three principal angles of the trihorn. In the auxiliary plane, the trihorn is represented by a (rectilinear) triangle; the M's are the *sides* of the triangle, and the α's are the *angles*.

We now state the following theorems on *trihornometry;* some are analogous to theorems in ordinary trigonometry, and some are strikingly different.

I. A necessary condition that three numbers be the measures of the sides of a trihorn is

$$M_{12}\, M_{23}\, M_{31}\ (M_{12} + M_{23} + M_{31}) \leqq 0$$

If M_{12} and M_{23} are positive, so also is M_{13}, and we have $M_{12} + M_{23} \geqq M_{13}$. In this sense the sum of the parts is usually greater than the whole, but may be equal to the whole. In the exceptional case of equality we call the trihorn *wide-open.*

II. The absolute values of the three measures are equal if and only if all are infinite. If two of the measures are infinite, the third measure is infinite; the trihorn is then said to be completely circular or euclidean.

Except for these peculiar cases, and the case where the M's are equal in absolute value, the condition

$$M_{12}\, M_{23}\, M_{31}\ (M_{12} + M_{23} + M_{31}) \leqq 0$$

is sufficient for three numbers to be the measures of a trihorn. If $M_{12} + M_{23} + M_{31} = 0$ the trihorn is *wide-open.* The three points in the auxiliary plane are then collinear.

III. The α angles of a trihorn obey the relation $\alpha_{12}\,\alpha_{23}\,\alpha_{31} = 1$. This a fundamental *equality,* instead of *inequality,* as for the sides.

IV. If one and only one side of a trihorn is infinite, one of the adjacent angles of the trihorn is zero, another is infinite, and the third angle (opposite the infinite side) is finite and non-zero. The third angle has two possible values if the sides are given. (Such a trihorn is partially circular.)

V. If one of the angles of a trihorn is infinite (then, it follows, one of the sides is infinite), another angle is zero and the third is finite and non-zero.

VI. If the three measures of a trihorn are infinite (completely circular) the angles are all indeterminate.

VII. If one of the angles of a trihorn is indeterminate, then all the angles are indeterminate and the three measures are infinite.

VIII. If a trihorn is wide-open and not circular, the three angles are unity.

IX. If one of the angles is unity, all the angles are unity and the trihorn is wide-open.

X. Except for unit, zero, infinite and indeterminate values, the equality $\alpha_{12}\,\alpha_{23}\,\alpha_{31} = 1$ is a sufficient condition for three numbers to be the angles of a trihorn.

XI. If the three sides of a trihorn are given, two values of the angles are determined; that is two distinct (non-congruent) triangles exist. The trihorns are conformally distinct.

XII. If the three angles of a trihorn are given, the ratios of the three sides are uniquely determined. If any three parts, except the three angles, are given, all the parts are determined (with one or two solutions). (The detailed formulas, all algebraic, will be given elsewhere).

XIII. Neither equilateral nor equiangular triangles exist.

XIV. If two sides of a triangle are equal, the opposite angles are never equal.

XV. A triangle can have two right angles, but not three.

XVI. In any isosceles triangle, the sum of the base angles is unity.

XVII. A *right angle* has the value $\alpha = \frac{1}{2}$, and an *anti-right angle,* the value $\alpha = 2$. The minimum distance is given by the former value. The perpendicular and the anti-perpendicular distances between two parallel lines are in the ratio of $-8:1$.

XVIII. The medians of a triangle (mid-points correspond to the bisectors of the horns of the trihorn) are concurrent; but the altitudes are not concurrent.

XIX. The congruence group in the new metric is $X = mx + h$, $Y = m^2y + k$.

(4) In conclusion, I remark again that only horn angles of first order contact are allowed in the preceding discussion. If higher contacts are permitted, the theory becomes much more complicated (since each order requires a new theory of measure), and the complete conformal geometry of horn angles becomes *non-Archimedean*. The fact that angles of contact of different orders are not metrically comparable was first noted by Newton in the Principia.

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UPWARD TRANSPORT OF MINERALS THROUGH THE PHLOEM OF STEMS

THE general opinion among botanists is that the mineral elements absorbed by the roots are transported upwards in the xylem with the water. Curtis[1] as a result of his numerous investigations has, however, suggested that at least some of the soil solutes ascend in the phloem. Mason and Phillis[2] dispose of the matter of solute conduction by the statement: "To sum up, ringing experiments have shown that soil solutes ascend the stem in the wood, but they have not demonstrated that they (soil solutes) may not also ascend in the phloem. It must be admitted, however, that the evidence available renders it very unlikely that they normally do so."

With a new tool available, namely, strongly radioactive elements, we have undertaken to clarify this issue. The installation of the new cyclotron equipment in the physics laboratories at the University of Michigan has made available fair quantities of strongly radioactive material. Phosphorus was chosen because it is an important element in every plant and because its half life is fifteen days, a period long enough to allow one to conduct several experiments with the same preparation. Red phosphorus was activated and then made into KH_2PO_4. This salt was made up into a 0.5 per cent. aqueous solution with a pH of 5 to 6. Rooted cuttings of geranium, *Sedum praealtum* and *Bryophyllum calycinum* have been used as plant material. These plants were chosen because the bark separates readily from the wood. The presence of the radioactive phosphorus in the plant was detected by means of an electroscope.

[1] O. F. Curtis, "The Translocation of Solutes in Plants." McGraw-Hill, 1935.

[2] T. G. Mason and E. Phillis, "The Migration of Solutes." *Bot. Rev.*, 3: 47–71, 1937.

As more details will later be published elsewhere, only a few experiments will be cited here. With *Bryophyllum* several experiments were performed in which part of the bark with several leaves was completely separated from the wood except at the base, and at this point the remainder of the plant was cut off. This left the plant with only a few leaves attached to the bark, which was connected with the roots through a portion of the unmutilated stem. In one of these experiments the piece of bark was 22 cm long and had four leaves attached to it.

The roots of this plant were kept in the active phosphorus solution for about 40 hours. At the end of this period the leaves were still quite turgid. The bark was cut up into pieces 2 cm long and the activity determined. Table I gives the results for this experiment. The electroscope discharged itself in 40 minutes, so that any time less than 40 minutes denotes active phosphorus in the plant material.

TABLE I
TIME REQUIRED TO DISCHARGE THE ELECTROSCOPE WITH 2 CM-LONG PIECES OF BRYOPHYLLUM BARK

Distance above solution, cm	Time in minutes
3	13.0
7	16.0
11	19.25
15	22.5
17	25.0
19	30.0
23	38.0

This shows that even in the last section, which was 23 cm above the solution, there was some radioactive phosphorus present. In this experiment the leaves were not tested, but in others where tests were made, active phosphorus was found to be present in them also.

In another group of experiments with *Bryophyllum* a complete section of the xylem, about 2 cm long, was removed from the stem, leaving the leaves connected with the roots only through the bark, which was left complete. In one of these experiments a well-rooted plant remained in an active phosphorus solution for 17 hours. Table II gives the results.

TABLE II
TIME REQUIRED TO DISCHARGE THE ELECTROSCOPE WITH 2 CM-LONG PIECES OF BRYOPHYLLUM STEM (SECTION OF WOOD REMOVED)

Distance above solution, cm	Bark	Xylem underneath bark
6	8.0 min.	20.0 min.
8	14.0 "	Wood removed
10	25.0 "	35.0 min.
13	32.8 "	39.5 "

There is evidence here that phosphorus was conducted through the phloem of the *Bryophyllum* stem