sey. A visit will be made to a modern oil refinery and to oil leases where oil is being recovered by the water-flood and by air and gas repressuring. Copies of the final notice may be had by writing to Dr. Arthur B. Cleaves, secretary, Topographic and Geologic Survey, Harrisburg, Pa.

THE Minnesota Academy of Science held its fifth annual meeting at the University of Minnesota on April 17, with some 300 in attendance. At this meeting a new section, Science Education, planned for those whose interests lie primarily in the teaching of the sciences, was added to the two sections, physics and biology, into which the academy had hitherto been divided. In relation to this movement a Junior Academy of Science plan was initiated. The meeting opened with a general session, at which the speakers were Dean Walter C. Coffey and Drs. D. H. Davis, Irving McQuarrie and A. N. Wilcox, all of the University of Minnesota. The three sections held sessions for the reading of technical papers. In a public lecture before the entire group, Dr. H. K. Hayes, chief of the Division of Agronomy and Plant Genetics, University of Minnesota, closed the session with an illustrated talk, entitled "Some Observations on Life in China." The officers for 1938 include: *President,* Dr. W. S. Cooper, University of Minnesota; *Vice-president,* Professor E. T. Tufte, St. Olaf College; *Secretary-Treasurer,* Dr. H. K. Wilson, University of Minnesota; *Councilors,* Professor G. W. Friedrich, St. Cloud Teachers College; Dr. Louis H. Powell, St. Paul Institute; Dean E. M. Freeman, University of Minnesota; Dr. H. E. Essex, the Mayo Foundation. The academy will hold its next annual meeting at St. John's University, Collegeville, on April 23, 1938.

AN International Congress on the Testing of Materials was opened in London at the Institution of Civil Engineers on April 19. According to a report in the London *Times,* the congress, which was organized by the International Association for Testing Materials, was opened by Sir William Bragg, president of the Royal Society. Visitors from twenty-five countries were in attendance. In connection with this meeting a Joint Committee on Materials and Their Testing was set up to act as the British national organization for these matters. Representatives of twenty-two technical institutions and societies will serve on the joint committee, which has been formed on the understanding that it shall not absorb or replace in any way the activities of existing technical organizations. At the conclusion of the congress this committee took over the work of the British committee that had hitherto been entrusted with the activities in Great Britain of the International Association for Testing Materials. The chairman of the joint committee is Dr. H. J. Gough, and the secretary is C. W. J. Taffs, of the Institution of Mechanical Engineers. An exhibition of testing plant and apparatus was held at the Institution of Civil Engineers throughout the week in conjunction with the congress. The Department of Scientific and Industrial Research provided the largest group of exhibits, illustrating testing methods recently devised or developed in the department.

THE Australian Government has announced, according to *Nature,* that the work of the Council for Scientific and Industrial Research is to be extended in the interests of secondary industry generally. Since its establishment in 1926, the council has restricted its attention to problems of the primary producing industries, though no such restriction is imposed upon it by the Act under which it is constituted. It has always been assumed that an extension was only a matter of time in view of the contraction of world markets for primary products and the consequent pressure to increase the home market by expanding secondary industries. A recent decision to establish aircraft and motor production in the Commonwealth has intensified a growing demand for an extension of national scientific research, and an influential committee, including leading engineers and industrialists, is now at work preparing a definite scheme of work. Existing institutions will be utilized wherever possible, but it is fully recognized by the government that considerably increased financial obligations must be carried by it. A first step is to establish an agency for the maintenance of accurate fundamental standards of measurement and for the testing of master gauges for controlling precision manufacture. It is intended that in all developments intimate contact shall be maintained with, and guidance sought from, established British institutions engaged on work of the same type.

DISCUSSION

ON THE SIGNIFICANT FIGURES OF LEAST SQUARES AND CORRELATIONS

DR. F. R. MOULTON in the issue of SCIENCE for December 25, 1936, pointed out that the number of significant figures in the solution of a set of linear equations can not exceed the number of significant figures in the determinant A of the coefficients. By evaluating this determinant it is possible to make certain statements in advance concerning the reliability of the solution for the unknowns. I propose to add some thoughts to those expressed by Dr. Moulton, also to say a word further on the question raised by Dr. Joseph Berkson in SCIENCE for November 13, 1936.

A large class of problems in which these matters assume considerable theoretical and practical importance is the normal equations of least squares and correlations, though most of what I have to say will apply to any type of linear equations. It is customary to take advantage of the symmetry of normal equations by using Doolittle's method or some modification of it, any of which is but one of many possible algebraically rigorous procedures for "solving" the equations.

The first thing to note is that, given a set of normal equations, a short way of evaluating the determinant A is to proceed with the Doolittle solution or some favored modification of it, exactly as if one were solving for the unknowns. The value of A is then given by the product of certain numbers that naturally occur in the process of elimination; in the illustration to follow, A is the product of the leftmost numbers in the Roman numbered equations. So in dealing with normal equations it is hardly practicable to evaluate the determinant A in advance of solution, but it is helpful to see how its value is tied up with other features of the equations.

The following three normal equations can be used for illustration; a, b and c are the unknowns.

No.	(*a*)	(*b*)	(*c*)	(*Const.*)
I	$1.994009\,a\ +$	$1.998994\,b$	$+1.997000\,c =$	11.982997
2	$[1.998994\,a] +$	$6.004004\,b$	$+0.002000\,c =$	14.013002
3	$[1.997000\,a] +$	$[0.002000\,b]$	$+3.000000\,c =$	11.001000

The Doolittle solution is laid out as follows, six decimals being retained. In practice the bracketed terms would be omitted for brevity, and a sum column would be introduced for a check. The numbering and procedure follow O. M. Leland's "Practical Least Squares" (McGraw-Hill, 1921) Art. 147. In II; a has been eliminated; in III, a and b both have been eliminated.

No.	(*a*)	(*b*)	(*c*)	(*Const.*)
4	....	$-2.003991\,b$	$-2.001992\,c =$	-12.012954
II	0	$4.000013\,b$	$-1.999992\,c =$	2.000048
5	....	....	$-1.999995\,c =$	-12.000971
6	....	....	$-0.999989\,c =$	1.000017
III	0	0	$0.000016\,c =$	0.000046

The solution is practically frozen at the appearance of 0.000016 in column c of III. If but three or even four decimals had been carried, there would have been a complete cancellation at this point. Since 0.000016 contains only two figures, c will be determined to not better than two figures, no matter how many decimals are carried on the right; and the inaccuracy in c will be passed on to a and b in substitution.

The interrelations between the near vanishing of the determinant A, the near vanishing (0.000016) of the coefficient of c in III, the number of decimals carried and the number of significant figures in the values obtained for a, b, c, are now displayed. In the first place, the value of the determinant A is the product of the leftmost numbers in I, II and III, which in this case give

$$A = 1.994 \times 4.000 \times 0.000016 = 0.00013,$$

indicating that when six decimals are carried throughout the solution, not more than three can be significant in the values found for a, b, c, no matter in what order they are solved for.

The values of c found from III, b from II and a from I in turn are

$$a = 1.187844, \qquad b = 1.937500, \qquad c = 2.875000,$$

wherein, though only two figures are mathematically significant, the presence of all figures written is required for satisfaction of the equations to six decimals. This does not mean that we have somehow evolved accuracy out of inaccuracy; we can in fact round c off to 2.9 even, and find that the set of values

$$a = 1.150275, \qquad b = 1.950000, \qquad c = 2.900000$$

satisfies the normal equations just as well. Seeing this, we may wax bold and try $c = 3.15$ without disappointment, for it is a fact that the set

$$a = 0.774586, \qquad b = 2.075001, \qquad c = 3.150000$$

also satisfies the equations to six decimals. Discussion of this apparent vacillation will follow. Dr. Moulton referred to a similar case of instability in a problem in orbits.

The values of a, b and c that satisfy the normal equations absolutely are

$$a = 1, \qquad b = 2, \qquad c = 3,$$

as can be found by solving the normal equations with common fractions (equivalent to carrying all decimals) or by possessing the prescience that these particular ones were built up for illustration from the three equations

$$\begin{aligned} a + \quad b + c &= 6 \\ 0.997\,a + 1.002\,b + c &= 6.001 \\ 0\,a + \quad 2\,b - c &= 1 \end{aligned}$$

to which they are exactly equivalent. The latter set can be solved by inspection. The small angle between the first two planes, purposefully introduced, is responsible for the instability of a, b, c, and the freezing or near freezing of the normal equations.

So long as A differs from zero by any finite number, however small, there is one and only one value of a, one of b and one of c, that will satisfy the normal equations absolutely. These values can be found by holding to common fractions throughout the solution; and when this is done, any two procedures for solution will give identical and perfect results. There will nevertheless be a band of a values, a band of b values and a band of c values, not necessarily of the same width, from which can be picked any number of sets a, b, c that will satisfy the equations to the number of decimals required—the greater the accuracy required, the narrower the bands. Moreover, for satisfaction of the equations to a specified number of decimals, the bands can be made wider and wider as A diminishes; in the limit when A is zero the equations are completely indeterminate, which means that a or b or c—one of them (two if A is of rank one)—can be assigned any value whatever for absolute satisfaction of the equations.

The width of each band is in fact the interval within which the corresponding unknown is significantly determined. Different methods of solution are merely devices for picking out from these bands different sets for a, b, c that will satisfy the equations as far as required. If A is small, the bands will be wide, and the results of two different methods of solution may appear alarmingly discrepant, yet be consistent with the equations to the last decimal. In such a situation the equations are said to be unstable, and we have already seen an example. Instability is identified by rather wide bands, and always occurs inseparably from near-indeterminacy and freezing.

The facts just cited have a bearing on interpolation. Suppose a curve with three adjustable parameters a, b, c, be put through three points in the x, y plane. The coordinates of the three points are exactly sufficient to fix a, b and c; and whether a, b and c enter the curve linearly or not, the three equations for determining them can be made linear by Taylor's series. Let now two of the three points be close together but still distinct; then the determinant A of the linear equations is not zero but is small, and the equations are unstable. The points, two of them being close together, and all with (say) four decimal accuracy in the y coordinate and with, for simplicity, absolute accuracy in the x coordinate, do not fasten a, b and c down to definite values, but only require them to lie within rather wide bands—bands of significance, they might be called.

For every value of (*e.g.*) a picked at random from the a band, values of b and c within very narrow limits are required from the b and c bands to satisfy the equations to the number of figures that are significant. Every set a, b, c so satisfying the given equations determines a curve that passes within the vertical distance ± 0.0001 of each point.

This brings up an angle to the problem easy to overlook: Dr. I. C. Gardner and Mr. L. W. Tilton, of the National Bureau of Standards, mentioned it to me in a recent conversation. For purposes of interpolation we would demand a curve that comes within ± 0.0001 of each point in order to preserve all the information supplied by the points. In other words, we would demand a set a, b, c that satisfies the equations to the full number of significant decimals, and this will require that the three unknowns a, b, c be mutually consistent to more places than are significant. It is not sufficient to take just any values of a, b and c from within their bands of significance; for interpolation, that is, for satisfaction of the equations as far as significant, we must have a set a, b, c consistent with the points to within 0.0001. For instance, in the solutions written earlier, we may not round off a, b and c to two figures on the pretext that they are significant to only two figures; the fact is that six decimals are required, as written, if the equations are to be satisfied to six decimals. It is permissible, even necessary, to pick one unknown blindly from its band, but the equations themselves must then be permitted to choose the other two. A solution by determinants may well require decimals considerably in excess of the number significant, as Dr. Berkson contended in SCIENCE for November 13: on the other hand, some of this extra labor may be avoided if the earlier unknowns are found by substitution (b from II, a from I, as in the above illustration).

One further item in connection with instability. It is advisable to test for it when it is suspected, but where the freezing was not decisive enough to discourage completion of the problem. By hindsight, it is at times advisable to make the test even when instability is not suspected. A test can be made by comparing the results of two solutions; if they are discrepant, yet both consistent with the equations to the last decimal, there is positive evidence of instability. If two solutions agree tolerably well, the equations may generally be considered stable, though this is only negative evidence.

A very sensitive test for stability is the comparison between the direct solution (substitution, as above) and the reciprocal solution, gotten by using the reciprocal matrix A^{-1} as a multiplier. In least squares and correlation work, the matrix A^{-1} is nearly always calculated anyhow as a matter of course, right along with the solution of the normal equations, since the elements of A^{-1} are the respective variance and product-variance coefficients of a, b, c, and are usually needed. The customary way of calculating A^{-1} is to solve the normal equations with the unit matrix

$$\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$$

in the constant columns, writing down the values obtained for the unknowns in the same order. Once obtained, the matrix A^{-1} is readily used as a multiplier for finding the values of a, b and c that satisfy any constant column whatever (*cf.* R. A. Fisher, "Statistical Methods for Research Workers" (Blackie and Son, 1925, 1936) Art. 29). The reciprocal solution, like any other determinant solution, is apt to give spurious results in the case of near indeterminacy, and it is this very fact that furnishes the test of stability.

For the normal equations written above, the reciprocal matrix with two-figure accuracy turns out to be

$$A^{-1} = \begin{vmatrix} 141141 & -46962 & -93922 \\ -46961 & 15625 & 31250 \\ -93922 & 31250 & 62500 \end{vmatrix}$$

The large numbers themselves warn of instability. Used as a multiplier, the top row and the original constant column give

$$a = 141141 \times 11.982997 - 46961 \times 14.013002 - 93922 \times 11.001000 = 2.765577;$$

and in like manner the second and third rows give

$$b = 0.852883, \qquad c = 1.705766.$$

The glaring discrepancies between these values and the earlier solutions are evidence of instability, but of course no test was needed in this instance. If common fractions had been used, or all decimals retained, the reciprocal solution would have given $a=1$, $b=2$, $c=3$, which satisfy the equations absolutely.

Some of the notions here expressed have grown from a few ideas brought to my notice by Dr. A. C. Aitken, of the University of Edinburgh, about two years ago; in particular, the suggestion of the reciprocal solution for comparison is his. Such notions concerning near indeterminacy and instability occur readily enough to mathematicians, but not so readily to other scientists and economists who have use for them. The reader will understand that only the algebraic features of the problem are here dealt with; physical significance of figures is another matter. It is interesting, I think, to see that there are both mathematical and physical aspects to the problem.

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PERANEMA AND "GRANTIA"

CONCERNING the second flagellum of *Peranema*,[1] I can only repeat that after prolonged study of normal active specimens under oil immersion I was not able to see any such structure. However, Lackey[2] and Hall[3] have already pointed out that this second flagellum can not be seen in the living animal and is not used in locomotion. My observations were confined to live specimens, and the flagellum is evidently observable only in stained specimens. I was obviously in error in doubting its existence on the grounds of its non-visibility in life. Since writing the article in question[4] I have been able to observe the vacuolar apparatus in *Peranema* and other euglenoid flagellates, and I find that the recent accounts of this apparatus in *Peranema* are erroneous, as is likewise the standard text-book description of the vacuolar system of the Euglenida in general. The contractile vacuole of *Peranema* is a temporary vesicle which discharges into the gullet base and thus completely vanishes, having no continuity with the succeeding vacuole. As each vacuole reaches diastole there appear near it two or three small vacuoles. These are not, as usually supposed, secondary vacuoles opening into the main vacuole, but are simply the droplets whose fusion forms the next vacuole. As the current vacuole disappears, these droplets tumble together into the space which it occupied and unite to become the next vacuole. A similar state of affairs was found to hold for several other euglenoids, both green and colorless, studied.

Dr. de Laubenfels' correction, in the same number of SCIENCE, of an obvious error in the naming of the common little syconoid sponge of the Woods Hole vicinity is welcome, but unfortunately Dr. de Laubenfels omits to mention that *Scypha* is a synonym of *Sycon*. Sponges with the structure of the Woods Hole form have always up to the present been placed in the genus *Sycon* by sponge specialists, and the erroneous name *Grantia* was already corrected to *Sycon* (on the advice of Professor H. V. Wilson and myself) in the last editions of Pratt's "Manual of the Common Invertebrate Animals" and Drew's "Invertebrate Zoology." It now appears that the name *Scypha* has priority over *Sycon*, and hence it will unfortunately be necessary to change the name *Sycon* to *Scypha*. The form *Scypha* (*Spongia*) *coronata* given by de Laubenfels does not conform to the international rules of nomenclature, for a parenthesis can be used in this manner only to indicate a subgenus, as is certainly not the intention here. Consequently the name of the Woods Hole sponge (assuming that the specific identification is correct) should read *Scypha coronata* (Ellis and Solander) 1786, syn. *Spongia coronata* Ellis and Solander.

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[1] SCIENCE, February 19, 1937.
[2] *Biol. Bull.*, Vol. 65.
[3] *Trans. Amer. Micro. Soc.*, Vol. 53.
[4] *Quart. Jour. Micro. Sci.*, Vol. 79.

A REMARKABLE SABRETOOTH-LIKE CREODONT FROM THE EOCENE OF UTAH

DIRECTOR AVINOFF, of the Carnegie Museum, has kindly sent me for description the lower jaw of a predaceous animal, the nature of which is not apparent