

has proven that Claypole's paper was read before the American Philosophical Society on September 21, 1883, and printed on November 2, 1883.

Hall submitted his paper in his thirty-sixth annual report to the Senate on January 12, 1883 (see Senate document 53). This Senate document, in which Hall's paper with figures is printed, had according to law (Law 1859, Chapter 437) to be printed and distributed before the end of the year. The printer, however, was wont to deliver the Senate documents before the end of the fiscal year, which then was September 30, in order to receive payment, and the bills of the year 1883 show that he also did so that year. That means that copies of the Senate documents with Hall's paper included were distributed before October. A separate reprint of Hall's thirty-sixth report was published and distributed in 1884. This report bears the date 1883 on the title page and that of 1884 on the paper cover. Furthermore, he also had still a reprint of his paper on *Stylonurus* distributed in 1884. These facts have led to the erroneous conclusion that Hall's paper was not in print until 1884, when as a matter of fact it was in print in Senate document 53 sometime in the summer of 1883, long before Claypole read his paper.

This typical case serves to explain numerous cases of doubt of the proper date of publication of the New York State reports. It has been even suggested that

Hall dated his publications ahead. The fact is that the date of publication of many of our earlier museum reports and bulletins is the date of the Senate document, in which they first appeared. This is the date on the title page. Reprints which were often widely distributed bear later dates, but those are not the dates of first publication.

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A NEW WORD

IN the issue of *SCIENCE* for September 25, 1936, page 291, Dr. Robert T. Morris suggests that a new word might be useful, similar in form to *benthon*, *nekton*, *plankton*, etc., to designate the more or less organic mud of *shallow* bottoms on which various fishes, mollusks, birds and other animals feed. Why not "ilyon," from the Greek ἰλύς, -ύς, meaning mud or slime? With various appropriate suffixes, one might speak of "ilyonic" food, of animals living in the bottom mud as "ilyobic" or of those that feed on it as "ilyophagous." If the simple word "mud" is not clear enough, the new term would have to be more accurately defined to meet the requirements of ecological description.

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SCIENTIFIC BOOKS

THE PHILOSOPHY OF MATHEMATICS

Mathematics and the Question of Cosmic Mind and Other Essays. By CASSIUS J. KEYSER. Scripta Mathematica Library No. 2.

THE welcome effort of the Scripta Mathematica Library to encourage discussion of the philosophical and cultural aspects of mathematics is continued in this booklet, which contains some of Professor Keyser's essays collected from various sources. These essays, written clearly and enthusiastically, aim to explain several phases of mathematics to those laymen "who live to think and who are not satisfied with being merely told."

The first three essays form a sequence leading to the question suggested in the title of the book. First, the nature of mathematics is analyzed in terms of its method, which is taken to be strictly postulational. In other words, a mathematical doctrine starts with certain undefined primitive terms, which are really variables, and with a certain number of consistent axioms, and proceeds by defining new terms by means of the primitive ones and by deducing new statements from the axioms. This abstract concept of mathematics is

explained vigorously and lucidly. However, some mathematicians might consider this identification of mathematics with postulational thinking to be incomplete. Does mathematics start with any old system of postulates whatever or does it content itself with any arbitrarily chosen deductions from a given set of postulates? Does the postulational approach account for the fact that, historically, mathematics consists of the elaborate development of a few particular postulate systems? How can the all-important consistency of these postulate systems be abstractly established, especially when Gödel has indicated the great difficulty of formal consistency proofs?

The second essay engagingly explores the "Bearings of Mathematics"—"a certain rich manifold of light-giving relations connecting mathematics with those great human interests . . . in which there is, properly, no question of establishing mathematical propositions." Such relations concern the art of exposition, the universality of the mathematical concepts of change and invariance and the ideal of logical rectitude.

The title essay then reviews the light which mathematics can throw on the question, "Is the world essen-

tially a world of mind, or spirit, or soul?" Professor Keyser recognizes that mathematics alone can not definitely answer this question, but he does exhibit a number of quotations from famous mathematicians who tend to believe that the world is a realm of mind.

In the course of these essays Professor Keyser also presents a challenging definition of science. He says that science has for its characteristic aim "the establishment of categorical propositions in answer to questions about the actual world," while, in sharp contrast, mathematics essentially aims "to establish hypothetical propositions in answer to questions regarding the logically possible." He then explains how a mathematical doctrine can be applied to science by replacing the variable primitive terms by constants and by verifying the resulting postulates. The question arises whether such a definition adequately allows for a sophisticated science, such as physics, where we never verify the postulates but only the results of complicated deductions from the postulates.¹

The other essays include an eloquent eulogy of the American mathematician, philosopher and poet, William Benjamin Smith, a discussion of the use of mathematics in legal science and a forceful plea for the popularization of science, "to mitigate the tragedy of our modern culture." This tragedy, the author takes to be the inability of ordinary intellectually curious people to understand the important implications of mathematics and science. From this point of view, the great art of popularizing science and mathematics is explained. Indeed this booklet is itself an illustration of that art.

Humanized Geometry, An Introduction to Thinking.

By J. HERBERT BLACKHURST, Iowa University Press. 1935.

THIS text-book offers an extremely suggestive and illuminating presentation of high-school geometry. The author is not content with the routine presentation of geometrical proofs; rather he aims "to utilize geometry as a resource in exposing and studying the processes of thinking." Consequently throughout the text he discusses with the student the nature and function of the logical methods used and the possibility of applying these methods to non-mathematical thinking.

For example, the theorem on the equality of vertical angles is first established inductively from measurements in particular cases and only subsequently by a deductive proof. Then the author discusses the errors in measurement present in such inductive proofs and in other parts of science and contrasts this method with the deductive method. There follow some his-

torical remarks on the discovery of the deductive method by the Greeks. The whole discussion consciously aims to help the student understand what deductive "proofs" really accomplish.

Throughout the book the formal treatment of geometry is supplemented by similar discussions: On the nature of abstract thinking, on the disadvantages of memorizing geometry, on the analytic method of finding proofs for theorems, on the syllogism and on fallacies. The author points out the importance of formulating the premises on which an argument rests, and remarks on the danger of the tacit assumptions made in every-day life. Again he considers the ways in which we can solve geometrical problems, compared with the difficulty in solving social problems, and he emphasizes the undesirability of accepting ready-made solutions for such social problems. These discussions should succeed in making the methods of effective thinking interesting to the student.

The geometry itself includes the usual theorems now required in plane geometry (some of the harder ones are placed in an appendix), together with a few sample theorems of solid geometry. The applications of geometry are kept in mind throughout. For instance, similar polygons are introduced in connection with photographic images. Many amusing practical problems are included. Finally, there is a long and excellent section on "Geometry in Art and Practical Affairs."

The formal logic applied by the author is strictly Aristotelian. It seems to the reviewer that some of the more modern logical ideas might have been illustrated in the discussions, especially in connection with definitions, where the genus-species analysis used by the author should be supplemented by the remark that definitions must always explain the unknown in terms of the known. The discussion of necessary and sufficient conditions in locus problems is weak. The treatment of limits is shaky; at one point it is argued that the perimeter of regular polygons inscribed in a circle increases with the number of sides and is always less than the circumference and that *therefore* (!) the perimeter approaches the circumference as a limit. In another place the elementary trigonometric functions are introduced before the logically prerequisite properties of similar triangles have been developed.

These weaknesses (most of them are not original with the author) do not essentially mar the real contribution of this book—a vigorous attempt to rescue geometry from possible desuetude by showing that it can be taught so as to be a real training in effective thinking.

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¹ See H. Weyl, "Philosophie der Mathematik und der Naturwissenschaft," pp. 111-123.