

study of other methods. Special subjects treated in the book include curved beams, eccentric loads on columns and a study of energy of strain with special application to determination of stress under impact.

The book is a worthy addition to the text-books on the mechanics of materials and is especially recommended to the attention of teachers and scholars who

wish to emphasize the mathematical development of stress analysis in their beginning courses in strength of materials. It would make an excellent book for the use of "honor sections" of students of outstanding ability.

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SPECIAL ARTICLES

THE AGGREGATION OF ORTHIC TETRAKAIDECAHEDRA

A MODIFICATION of the regular octahedron in combination with a cube, a figure with fourteen sides, eight hexagonal and six square, all the edges of all the hexagons and of all the squares being equal, was well known to the crystallographers of the eighteenth century. This figure was called the tetrakaidecahedron by Lord Kelvin¹ in a theoretical essay "on the division of space with minimum partitional area." In recent years, with increased emphasis on problems of morphogenesis, this figure has come to be of renewed interest.

The biological significance of Kelvin's figure has been demonstrated by Lewis,² who has shown the tetrakaidecahedron to be a fundamental shape for both plant and animal cells (pith cells of elder and rush, epidermal cells of cucumber and tradescantia, human oral epithelial and adipose tissue and precartilag cells from the toad), when the cells are aggregated into tissues.

The shape of cells may be the result of various factors, such as surface tension phenomena, the law of bipartition, contact and pressure, cell and tissue differentiation and possibly organ configuration. Among the fewer-celled organisms especially, the principle of bipartition is obviously of great significance. And in the higher plants the shape and the arrangement of the cells may well be influenced, in a measure at least, by the adjustments that result from the operation of the law of bipartition in its relation to the ultimate stacking of the tetrakaidecahedra.

It is obvious that fourteen orthic tetrakaidecahedra can be stacked around a central one to produce an aggregate of fifteen. Thus the first layer contains fourteen members surrounding a central one. By stacking paper models of the type described by Matzke,³ it was found that 50 tetrakaidecahedra can be stacked around these fourteen. The second layer has 50 members, making an aggregate of 65 for the whole mass. Similarly, it was determined that 110 tetrakaidecahedra can be stacked around these 50.

The third layer accordingly has 110 members and the aggregate is 175; the fourth layer has 194 members and totals 369. From these data two formulae may be derived, one for the number of members in any given layer, and the other for the total number of members in the aggregate at any given layer. If T_n denotes the number of members in any layer where n represents the number of the layer, then $T_n = 12 \cdot n^2 + 2$. If S_n equals the total number of members in the aggregate at any layer where n is the number of the layer, then $S_n = 4n^3 + 6n^2 + 4n + 1$. Thus for example if $n = 2$, then $T_n = 50$, and $S_n = 65$. Table I gives the aggregate per layer for some of the layers in the tetrakaidecahedron series and also gives the bipartition series.

TABLE I

Tetrakaidecahedron series		Bipartition series
Layer no.	Aggregate per layer	Aggregate
1	15	2
2	65	4
3	175	8
4	369	16
5	671	32
6	1105	64
7	1695	128
8	2465	256
9	3439	512
10	4641	1024
11	6095	2048
12	7825	4096
13	9855	8192
14	12209	16384
15	14911	32768
16	17985	65536
17	21455	131072
18	25345	262144

It is apparent that bipartition and economy of surface relations (the latter illustrated by stacking tetrakaidecahedra) result in series of cells that do not aggregate at the same rate, but converge at first and diverge afterward. Attention is called to numbers in one series which are approximated by a number in the other, such as 15-16 and 65-64. The failure of these series to coincide may well have morphogenetic significance—for instance, in the establishment of polarity—in the development of the organism from the unicellular egg to the multicellular adult.

Additional data and the proof of the formulae given above will be presented in a subsequent publication.

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¹ Lord Kelvin, *Phil. Mag.*, 5s. 24: 503-514, 1887.

² F. T. Lewis, *Proc. Amer. Acad. Arts and Sci.*, 58: 537-552, 1923.

³ E. B. Matzke, *Torreyia*, 31: 129-136, 1931.