numbers than those commonly used in elementary arithmetic. It is a difference in generalization rather than a difference in abstraction. As we advance in mathematical study we deal continually with more general ideas, but it is questionable whether we deal with relatively more abstract ideas. It would be very difficult to prove that arithmetic deals with relatively more concrete quantities than algebra.

It is well known that the terms arithmetic, algebra, geometry, etc., are somewhat vague and that there is no generally accepted line of division between the subjects represented by them. Mathematics is commonly divided into pure and applied mathematics, but here there is also no commonly accepted line of division. Concrete numbers are frequently considered in elementary algebra as well as in elementary arithmetic. It should be noted that numbers are probably among the earliest abstract notions acquired by the human race and that one of the profoundest facts of mathematical history is the very early development of abstract mathematics. It used to be said that the early Babylonian mathematics was mainly concerned with business arithmetic, but it has recently been emphasized by O. Neugebauer and others that this early mathematics is mainly pure mathematics. The first table in the well-known Egyptian "Rhind Mathematical Papyrus" relates also entirely to abstract mathematics.

A more definitely incorrect statement in this dictionary, which also relates to a subject of wide interest, appears under the term "determinant." It is here stated that the consistency or the inconsistency of a system of n linear equations, in n unknown quantities, depends on the non-vanishing or the vanishing of the determinant of the system. It is well known that the consistency or the inconsistency of a system of linear equations can not be determined by the study of the matrix of the system alone but requires also the consideration of the augmented matrix. As the notation employed by G. W. Leibniz (1646–1716) differs so widely from the one which is now commonly employed to represent a determinant it is questionable whether it should be said that he discovered this subject, as is done here, notwithstanding the fact that this is also commonly done elsewhere. Improvement in knowledge is more important than stability.

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PHYSICAL INDETERMINACY AND PHILOSOPHICAL DETERMINISM

HEISENBERG has shown that if we use quantum-mechanical definitions of material particles and their interactions we admit a certain indeterminacy in experimental findings. It follows that, on this basis, it is impossible to prove or disprove the hypothesis that the physical universe is causally connected. It is the purpose of this note to point out that, nevertheless, the "principle of indeterminacy" does not change the status of philosophical determinism for the worse, as some suppose, but rather for the better.

The impossibility of proving strict causality by experiment was, in fact, just as apparent without resort to quantum-mechanical arguments. No careful physicist ever supposed that experiment could be so perfectly controlled as to furnish infinite precision. This meant that experiment could never specify one state of a system so completely that another state (earlier or later) could be calculated precisely, even if the laws of physics were themselves immutable. Otherwise put, no two states could be recorded so completely as to rule out the possibility that non-causal processes had occurred between them. Heisenberg's result merely makes it clear that the spread between calculation and observation may be wide when the systems treated contain individually observable particles. We may conclude that the postulation of a determinate (causal) universe is even farther from the possibility of physical upset than it was a few years ago.

The philosophical implications of Heisenberg's principle would probably not have been misinterpreted if no attempt had been made to build a deterministic philosophy on experimental data alone, without conscious abstractions. Such an attempt is interesting, and any degree of success in it is admirable. We do not, however, expect a philosophy so handicapped to be of the very first quality, any more than we expect one-armed golfers to win national championships, or caves, however neat, to replace more commodious dwellings. The chief defect of such hand-to-mouth empiricism seems to be that it must build upon inconsistent experimental data and has no criterion within itself for resolving such contradictions as thereby arise. Whether or not it uses quantum-mechanical concepts it is foredoomed to chaos. Another difficulty arises in respect to the observer and his observing equipment. Since it seems impossible to write down abbreviations for these in the simple way in which the numerical results of measurement appear in the attempted synthesis, there is a strong tendency to leave them out of the philosophic scheme altogether. Incidentally, this probably precludes such a philosophy from arriving at any ethics whatever. Perhaps its evasion from everything but meter sticks, springs and clocks explains its popularity among experiment addicts.

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