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MATHEMATICS AND SCIENCE[1]

By Professor CHARLES N. MOORE

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THE retiring chairmen of Section A have frequently devoted their addresses to some large phase of mathematical theory connected with their own scientific work. Since their audience and doubtless the bulk of their readers constitute a group whose primary scientific interest is in the field of mathematics, this has been an appropriate procedure. I have chosen to give a somewhat different type of address, for which I think there is also adequate justification. The American Association for the Advancement of Science is an organization that stands for cooperative enterprise among the various scientific groups. I have always felt that an emphasis on the connection between mathematical progress and general scientific progress should constitute one of the most important activities of Section A as a separate entity. For that reason I have elected to speak on the relationship between mathematics and science.

The origins of mathematics and science are lost in the mists of antiquity. As far back, however, as it has been possible for historians in these fields to trace the records, it has been found that science in general and mathematics in particular have undergone a simultaneous and parallel development. A little reflection will convince any thoughtful person why this must have been the case. It is quite apparent that astronomy, one of the earliest fields to attain what may properly be called a scientific form, could not have been seriously advanced without the assistance of a well-developed mathematical apparatus. Even the most descriptive form of scientific development in other fields could hardly have had its inception without making use of counting and calculation. It is well to recall that these elementary phases of mathematics, now become a matter of pure routine, were,

[1] Address of the vice-president and chairman of the Section of Mathematics, American Association for the Advancement of Science, Pittsburgh, 1934.

in the infancy of the subject, mathematical discoveries of a very high order.

The usefulness of mathematics in other fields has sometimes led superficial observers to urge that mathematicians as a class should devote their time exclusively to problems which are of immediate concern to other scientists. Without touching on the question as to whether or not it is beneath the dignity of mathematics to play the rôle of a glorified valet to the other sciences, the whole history of mathematics and science shows that from a purely utilitarian standpoint this course would be eminently shortsighted.

The notion of number undoubtedly had its beginning in man's sense perceptions; the conception of a number as a purely abstract idea, however, was a great triumph of creative imagination and paved the way for much of the vast development of mathematical theory. The operation of counting was closely associated with practical needs, if in fact it did not originate from them. All the fundamental operations of elementary arithmetic can be reduced to a question of counting, if we restrict ourselves to the positive integers in connection with which they arose. When, however, these operations were examined as independent processes, the investigations eventually resulted in a very extensive elaboration of the number class. We find illustrated here one of the most important procedures that lead to mathematical growth; namely, a synthetic process followed by an analytic process. We synthesize a large number of related ideas into a more general idea for the purpose of economy of thought and economy of procedure. Then we analyze the more general idea for all its ultimate implications and, lo and behold, a host of new ideas is found to be contained in our generalization. It has very much the appearance of rabbits tumbling out of a magician's hat, but here there is no hocus-pocus. Creative imagination and strenuous intellectual labor have succeeded in enriching our stock of mathematical ideas.

The process of growth to which we have referred is again illustrated in striking fashion in the passage from arithmetic to algebra. The first synthesis of various arithmetical operations into an algebraic formula goes back into the prehistoric period of mathematics, for we find such a synthesis appearing in the earliest known mathematical writings. It could not have been said to have been properly completed, however, until our modern algebraic notation took form. From that time on the analysis of algebraic operations, which had already been carried forward to a considerable degree, grew apace. We have the so-called imaginary and complex numbers appearing first as purely formal solutions of certain types of algebraic equations, then being recognized as independent entities and given a concrete geometric representation, finally being assimilated into a complete number system and forming the basis of a beautiful mathematical theory with wide ramifications, namely, the theory of functions of a complex variable. It is well known that for some time past this theory has found extensive applications in many branches of mathematical physics. In the newly created spinor analysis complex numbers seem to enter in some essential manner, rather than merely as a tool, as Veblen has recently pointed out.[2] Does any one believe that mathematicians would have arrived at the notion of complex numbers, to say nothing of extensive theories concerning them, if they had confined themselves to problems having immediate practical applications?

The failure of the methods of solution which had been effective for quadratic, cubic and biquadratic equations to yield similar success in the case of quintic equations led finally to Abel's brilliant discovery of the impossibility of such success. A systematic analysis of algebraic equations of higher degree at the hands of Galois led to the notion of groups of transformations. Thus we see emerging the basic ideas which are at the foundation of the theory of groups. In the present century we find far-reaching applications of this theory in the field of mathematical physics by Weyl, Wigner and others.[3] The consideration of sets of linear algebraic equations led first to the notion of a matrix and then by the natural laws of mathematical growth to a comprehensive theory of matrices, including infinite matrices. Quite recently this general matrix theory has been of important service to Heisenberg and other mathematical physicists. It is difficult to see how either group theory or matrix theory could have been evolved by mathematicians who were concentrating their attention exclusively on problems with an immediate physical significance. The mathematicians have created these theories in what seems to have been the only possible way, namely, by developing the science of mathematics along its own inherent lines of growth.

The postulates of Euclid represent the ultimate step in the Greek synthesis of geometric ideas and relationships. On these postulates as a basis a comprehensive geometric theory was elaborated by purely logical processes. An important part of this theory dealt with the curves known as conic sections. Thus an extensive knowledge of the geometric properties of these curves was attained by mathematicians who

[2] Oswald Veblen, SCIENCE, 80: 415–419, 1934.

[3] Herman Weyl, ''Gruppentheorie und Quantenmechanik,'' Leipzig, 1928, 1931 (English translation of 2nd edition by H. P. Robertson). Eugen Wigner, ''Gruppentheorie und ihre Anwendung auf die Quantenmechanik der Atomspektren,'' Braunschweig, 1931.

were advancing their subject in the manner its own nature demanded and not merely for the applications. Centuries later the astronomer Kepler found that the theory of conic sections was precisely what he needed to develop the laws of planetary motion.

The substantial completion of the synthetic operation involved in developing an effective algebraic notation paved the way for the next great step in the development of mathematics. This was the synthesis of algebra and geometry as then known into the analytic geometry of Descartes. This noteworthy advance is usually regarded as the beginning of modern mathematics, and certainly the growth of mathematical theory was enormously stimulated by it. Although this discovery is something less than three hundred years old, it has now so penetrated the thinking of the whole literate group that the newspapers of the day do not hesitate to use graphical representation.

The basic ideas of the differential calculus and the integral calculus developed in somewhat closer touch with the applications than in the case of many other mathematical theories. However, the recognition by Newton and Leibniz of the close relationship between the two methods of investigation and the binding link between their fundamental concepts ranks among the most brilliant mathematical syntheses due to individual effort. This synthesis having been made, the subsequent analysis resulted in forging the most powerful method of mathematical investigation which had yet been known. The calculus thus established enabled Newton to substantiate his theory of gravitation by deducing Kepler's laws from it, and his successors during the next two centuries to develop a comprehensive and majestic theory of the motions of the heavenly bodies.

In connection with the postulates of Euclid we find arising a new type of analytic procedure, namely, the analysis of the logical foundations of a subject to see whether or not they can be improved. The long struggle to prove the parallel postulate from the other postulates finally led to the construction by Bolyai and Lobaschevsky of logically consistent geometries in which the parallel postulate of Euclid was replaced by an essentially different one. Thus we have the origin of the non-Euclidean geometries. Until quite recently the majority of scientists in other fields, if they knew anything about the non-Euclidean geometries at all, must have felt that they should be ranked among mathematical recreations rather than as a serious scientific study capable of application to the physical sciences. The general relativity of Einstein shows that this is far from being the case, and that without the theories which grew out of the logical qualms of the mathematicians it would not have been possible to replace Newton's theory of gravitation by a more general one. For this task there was needed not only the geometric ideas involved in the non-Euclidean geometries, but also the related analytical development known as the absolute calculus.

The importance of trigonometric series seems to have been first indicated in connection with the applications of mathematics to physics, and it was in connection with such applications that Fourier laid the basis for a comprehensive study of them. After Fourier, however, these series were studied by various eminent mathematicians from the point of view of their own mathematical content, aside from their utility in the applications. It has been pointed out in a previous address[4] by a retiring chairman of Section A how this study led to the development of many of the important new notions of nineteenth century mathematics, such as the Riemann integral, the point-set theory of Cantor, etc. In fact, the whole present form of the theory of functions of a real variable has been largely conditioned by ideas which arose in connection with the detailed study of trigonometric series. This is one striking instance of many cases in which mathematics owes a debt to other sciences for suggesting problems which lead to noteworthy advances. I have insisted before that it would be a grave mistake for mathematicians to devote themselves exclusively to problems which arise in other sciences. It would also be a grave mistake to ignore such problems. For in addition to aiding in the development of the other sciences, the complete analysis of the problem from a mathematician's view-point often leads to important new ideas. However remote these ideas seem to be from the original problem, they frequently find application in the same or related sciences. It was by means of trigonometric series that Weierstrass and others succeeded in giving examples of continuous functions which nowhere possess a derivative. Fourier and his contemporaries, to say nothing of mathematical physicists of later generations, would have undoubtedly regarded such functions as excellent examples of the manner in which mathematicians sometimes waste their time. Yet an eminent physical chemist of the day, Jean Perrin, has pointed out that careful studies of the Brownian movement show that the trajectories of the particles suggest nothing so much as continuous functions without a derivative. In Chapter IV of his book "Les Atomes" we find the statement:

> The entanglements of the trajectory are so numerous and so rapid that it is impossible to follow them and the trajectory noted is infinitely simpler and shorter than the real trajectory. Likewise, the mean apparent velocity of a particle during a given time varies wildly in magnitude and direction without tending to a limit when the

[4] E. B. Van Vleck, SCIENCE, 39: 113–124, 1914.

time of the observer decreases, as we see in a simple fashion by noting the positions of a particle in the camera lucida, first from minute to minute, and then every five seconds, or better still by photographing them each twentieth of a second, as has been done by Victor Henri, Comandon and de Broglie, in order to cinematograph the movement. One can no longer fix a tangent, even in approximate fashion, at any point of the trajectory, and we have a case where it is truly natural to think of those continuous functions without derivatives, which the mathematicians have devised and which one would regard erroneously as mere mathematical curiosities, since Nature suggests them as well as functions with a derivative.

In the preface to the same book Perrin takes occasion to justify at some length, and in admirable fashion, the more recondite labors of the mathematicians. Since the statements of a physical chemist can hardly be qualified as mathematical propaganda, I think it will be of interest to repeat some of his remarks.

We all know how, before a rigorous definition is given, we point out to beginners that they already possess the notion of continuity. We trace before them a beautifully smooth curve, and we remark on placing a ruler against the contour: "You see that at each point there exists a tangent." Or again, to communicate the still more abstract idea of the true velocity of a moving body at a point of its trajectory, we say: "You surely perceive, do you not, that the mean velocity between two neighboring points of this trajectory becomes approximately constant when the points approach each other indefinitely?" And many minds, indeed, remembering that for certain familiar movements it appears to be so, do not see that the situation involves great difficulties.

The mathematicians, however, have well understood the defect in rigor of these so-called geometric considerations, and how childish it is, for example, to attempt to demonstrate, by tracing a curve, that every continuous function possesses a derivative. Functions with a derivative are the simplest and the easiest to deal with, but they are nevertheless an exceptional case; or, if we prefer geometric language, curves which have no tangents are the rule, and the very regular curves, such as the circle, are very interesting but very special cases.

At first glance such restrictions seem to be only an intellectual exercise, ingenious without doubt, but definitely artificial and sterile, involving the pushing to a mania of the desire for complete rigor. And, most frequently, those to whom one speaks of curves without tangents or functions without derivatives, begin by thinking that Nature does not present such complications nor even suggest the idea of them. The contrary is nevertheless true, and the logic of the mathematicians has kept them nearer to reality than the supposedly more practical representations of the physicists.

It is well to note here that many of the leading workers of the day in other scientific fields where mathematics is applied realize quite clearly that mathematics has served them best by following its own lines of development. In a recent article by Langevin[5] we find the following statement:

It is nevertheless just and necessary to emphasize here the remarkable fact that among the abstract constructions realized by the mathematicians, while taking for an exclusive guide their need for logical perfection and increasing generality, none seems to remain useless to the physicist. By a singular harmony, the needs of the mind, desirous of constructing an adequate representation of reality, seem to have been foreseen and provided for by the logical analysis and the abstract esthetics of the mathematician.

We also find in a recent paper by Dirac[6] the following remarks:

The steady progress of physics requires for its theoretical formulation a mathematics that gets continually more advanced. This is only natural and to be expected. What, however, was not expected by the scientific workers of the last century was the particular form that the line of advancement would take, namely, it was expected that the mathematics would get more and more and more complicated, but would rest on a permanent basis of axioms and definitions, while actually the modern physical developments have required a mathematics that continually shifts its foundations and gets more abstract. Non-Euclidean geometry and non-commutative algebra, which were at one time considered to be purely fictions of the mind and pastimes for logical thinkers, have now been found to be very necessary for the description of general facts of the physical world.

Finally, in a recent presidential address before the British Association for the Advancement of Science, we find the following statement by Jeans:[7]

Our knowledge of the external world must always consist of numbers, and our picture of the universe—the synthesis of our knowledge—must necessarily be mathematical in form. All the concrete details of the picture, the ether and atoms and electrons, are mere clothing that we ourselves drape over our mathematical symbols—they do not belong to Nature, but to the parables by which we try to make Nature comprehensible. It was, I think, Kronecker who said that in arithmetic God made the integers and man made the rest; in the same spirit, we may add that in physics God made the mathematics and man made the rest.

After such eloquent testimony in behalf of my thesis from distinguished workers in other scientific fields, I can only make suitable acknowledgement by repeating

[5] Paul Langevin, "L'orientation actuelle de la Physique," in "L'orientation actuelle des Sciences," Paris, 1930.

[6] P. A. M. Dirac, *Proc. Royal Society, London*, 133A: 60–72, 1931.

[7] J. H. Jeans, *Nature*, 134: 355–365, 1934.

my previous injunction to workers in pure mathematics that they should keep in touch with the developments in other sciences. It will always be eminently desirable that at least some of the mathematicians should be on the alert for new problems which arise from sources outside of mathematics. Such problems have provided a powerful stimulus of growth in the past and will undoubtedly continue to do so.

At the present stage of development of both mathematics and science, the relationships between them and their mutual services are best illustrated in the fields previously mentioned, namely astronomy, physics and physical chemistry. There exists no branch of science, however, in which some mathematical procedure is not found essential. Moreover, the natural evolution of all scientific theory is in the direction of increasing use of quantitative methods. It seems inevitable that the applications of mathematics in the more descriptive sciences should be enormously extended in the future. Many indications of the processes leading to such extension can be found in recent scientific advances in various fields. For example, the application of statistical methods in the biological and sociological fields is steadily increasing. In view of the fact that in the recent past no special mathematical preparation was regarded as important for workers in these fields, such an increase is noteworthy. We also find cropping up in these same sciences quite unexpected and rather startling instances of the possibilities of mathematical application. An example of this is found in the curve of healing of a wound developed by Carrel and du Noüy[8] during the late war.

In a recent article by Lapicque[9] we have clear indication as to the manner in which the relatively new science of physiology is evolving into a form where mathematical applications will be not only possible but essential. His remarks are as follows:

> Formerly, not very far back in the history of humanity, let us say a century ago, almost everything was unknown concerning the physiology in the labyrinth of a living body. Magendie said: "I wander around there like a rag picker, and at each step I find something interesting to put in my basket." This maxim horrified my teacher, Dastre, who was wont to say: "When one doesn't know what he is looking for, he doesn't know what he finds." For him the ideal of physiological research would have been to conceive in the quiet of one's study a theory explaining such and such a phenomenon, known but not understood (physiology is full of phenomena of this character), then to find, still by meditation, the experiment capable by a yes or a no, of proving or disproving the theory. One would come then some morning to the laboratory, and that very evening the matter would be decided.
>
> These two tendencies, each in its amusingly exaggerated form, seem to me to serve the purpose of characterizing the temperament of naturalists and that of physicists. In proportion as physiology develops, the discoveries for rag-pickers become more rare, and the possibility of working as Dastre dreamed is approaching. The progress of the physical sciences is one of the essential conditions of this development. Physical chemistry, notably the new fashion of interpreting the statics and dynamics of solutions, the rôle of membranes, the infinite variability of colloids, has opened for us new horizons and permits us to understand many phenomena which the older chemistry did not explain. To-day, we tend more and more to explain the vital processes in terms of physical chemistry; we have before us an enormous domain to exploit in this manner.

Certainly if the physiologists are going to explain vital phenomena in terms of physical chemistry, they will need to make extensive use of mathematical methods. Colloids, for example, which are mentioned above, are referred to by Perrin as one of the aspects of nature which suggest continuous functions without a derivative. It seems quite reasonable to suppose that many of the future applications of mathematics in such fields as the biological and sociological sciences must wait on further development of mathematical theory as well as further development of the sciences in question. We should remember that the absolute calculus only reached definitive form some fifteen years before the publication of the first paper on general relativity. The present extensive development of the mathematical theory of atomic structure depends in part on advances in pure mathematics which are of quite recent origin. The inner details of biological phenomena are undoubtedly more complex than atomic structure, and the extensive application of mathematics to biology will in all probability involve mathematical theories as yet unborn.

On the other hand, we have in existence beautiful and extensive theories of pure mathematics which have as yet found no application in other scientific fields. One outstanding example of this sort is the theory concerning the distribution of prime numbers among the other integers. Since all integers can be expressed as the products of primes, and since all the other numbers of mathematics rest on integers as a basis, there is ample justification for the statement of Landau that the prime numbers should be regarded as the "building stones" of mathematics. Recent developments in physical theory suggest that the assumption of a discrete structure for the material world, rather than a continuous structure, is more in

[8] P. Lecomte du Noüy, "Recherches expérimentales et applications des méthodes de mesure et de calcul a un phénomène biologique: la cicatrisation," Paris thesis, 1917.

[9] L. Lapicque, "L'orientation actuelle de la Physiologie," in "L'orientation actuelle des sciences," Paris, 1930.

accordance with physical reality. Perhaps prime number theory, like the conic sections of the Greeks, is waiting for some future Kepler to derive from it important theories concerning the physical universe.

The point that I would like to insist on in closing is the essential unity of all that can be designated as science. One outstanding purpose lies at the basis of all scientific endeavor, as the etymological origin of the word science indicates. We wish to increase our knowledge, both of ourselves and of the world about us. In carrying out this purpose each individual works best along lines dictated by his own tastes and inherent capacities. Some of us are for this reason mathematicians, and more particularly mathematicians working in certain special fields. If the theories we develop had no bearing at all on other scientific work, they would still have a value as exhibiting the capacities of the human mind. But the interrelations of the various scientific fields adds much to the solidarity of scientific interests. We should therefore rejoice that the relationships between mathematics and the other sciences are of such great service in the general development of scientific thought. It may not make our work any more interesting to ourselves, but it adds much to its broad human interest.

AGRICULTURAL PLANNING AS AN ASPECT OF STATE AND NATIONAL PLANNING[1]

By Professor A. R. MANN
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AGRICULTURAL planning is proceeding in this country in two distinct but not unrelated forms. These two forms are perhaps best represented by the production control program of the Agricultural Adjustment Administration, on the one hand, and by the work of the Land Policy Division of the Agricultural Adjustment Administration and of the National Resources Board in conjunction with state planning boards, on the other hand.

The first of these types of planning, economic planning, tends towards a "planned agricultural economy" as that term is currently applied. It seeks to determine the quantity of certain agricultural products which the markets and consumption on the farm will absorb under existing and anticipated conditions, and the optimum carry-over in addition thereto, and to apply inducements, chiefly but not wholly financial, to accomplish such production. Before the inducements can be applied there must be the most thoroughgoing determination of the size of the agricultural plant most likely to yield the desired production under existing conditions without yielding disturbing surpluses. From production control as an immediate instrument this type of planning reaches out toward permanent adjustments in production, commerce and consumption of farm products such as will reestablish a normally balanced and compensating agricultural economy. This form of agricultural planning will receive somewhat extended treatment on the program of this meeting.

The second form of agricultural planning, and the one which is an integral part of state and national planning as contemplated in the title of this paper, is an aspect of the effort to plan the highest economic and social utilization of the natural resources of the nation in the interest of present and future generations. Its essential character is revealed in the declared purpose of the National Resources Board to prepare a program on all "aspects of the problem of development and use of land, water and other national resources in their physical, social, governmental and economic aspects." As the National Resources Board and its predecessor, the National Planning Board, is responsible for the present nation-wide movement for state planning, the work of the state planning boards is designed to accomplish similar purposes with respect to the natural resources within the several states. In the pursuit of these purposes the interests of agriculture are served in highly significant ways. Agricultural planning, in respect of some of its more fundamental features, emerges from the work of the national and the state planning boards.

While the two types of planning proceed from basically different intentions, they inevitably meet at a number of points.

BASIC SOCIAL TRENDS

The requirements of the nation for land and its products, for transportation routes and services, electric power lines, water supplies, stream control and many other utilities and services are affected by certain social trends which require evaluation. The first essential in the making of any plans for the future development of a state or of any other area is an understanding of basic conditions and trends. The

[1] Address of the retiring vice-president and chairman of Section O, American Association for the Advancement of Science, Pittsburgh, 1934.