to hold a similar convention next year, and accepted the invitation of the Maryland Academy of Sciences to convene in its building in Baltimore.

THE University of Wisconsin will receive \$400,000 for the establishment of a cancer research center from the estate of the late Jennie Bowman, of Wisconsin Dells. The entire amount is expected to become available in the autumn.

THE residue of the estate of the late Charles H. Morse, of New York City, is left to Wesleyan University to establish a fund in memory of his mother to be known as the Persis C. Thorndike Fund. The fund is expected to be in excess of a million dollars.

MRS. M. THERESA B. HOPKINS, daughter-in-law of Mark Hopkins, formerly president of Williams College, has given an estate of 1,800 acres to the college as a memorial to her husband. Situated in the northeast hill section of the town, the estate includes the Hopkins residence and several other buildings. It is assessed at about \$100,000.

A GIFT of \$80,000 to establish and maintain a psychiatric unit at the Massachusetts General Hospital, to be operated in conjunction with the Harvard Medical School, has been made by the Rockefeller Foundation. The unit will be under the direction of Dr. Stanley Cobb, professor of neuropathology at the university. Dr. Tracy Jackson Putnam, assistant professor of neurological surgery, has been appointed the first incumbent of a newly established chair of neurology. He is also in charge of the neurological unit at the hospital. There will be close cooperation between the Massachusetts General Hospital and the recently organized McLean Hospital in Waverley, Mass., where the most difficult brain and nerve cases are now treated. At the Boston Psychopathic Hospital Harvard investigators and teachers have long been studying mental diseases under the guidance of Professor C. Macfie Campbell. To his department there will now be added clinics to supply important incipient cases. Of the \$80,000 given by the Rockefeller Foundation, which is to finance the first year's work, \$42,-000 goes to the Harvard Medical School for the expenses of carrying out the psychiatric work and \$38,-000 is given to the Massachusetts General Hospital. The project will be started next September.

THE department of physiology and pharmacology of the University of Nebraska has received a grant of \$1,800 from the Frederick Stearns Company for investigation under the direction of Drs. A. Ross Mc-Intyre, professor of pharmacology, and Ernest L. MacQuiddy, of the department of internal medicine.

## DISCUSSION

## DEFINITIONS OF THE MATHEMATICAL TERM GROUP

THE term "group" with an apparently technical mathematical meaning appears in the literature of various scientific subjects and hence it may be noteworthy that those who now seek a satisfactory definition thereof are apt to encounter unexpected difficulties even in some of the most frequently consulted mathematical works of reference. A brief consideration of some of these difficulties may be of interest not only to those who have been perturbed by them but also to others who wish to clarify their conceptions as regards terms which are said to play an increasingly important rôle in the development of mathematics. The mathematical literature has been greatly enriched during recent decades by the publication of several special mathematical encyclopedias. While no such work has yet appeared in the English language each of the three other principal mathematical languages of the world, viz., German, French and Italian, embodies now at least the beginnings of such a work under the following titles respectively: "Encyklopädie der Mathematischen Wissenschaften." 1898-, "Encyclopédie des Sciences Mathématiques," 1904-, "Enciclopedia delle Matematiche Elementari," 1930-.

While each of these works contains at least one explicit definition of the technical term group none of these definitions is entirely satisfactory as regards the use of this term when the number of the elements is not restricted, as may be seen from what follows: The definition which appears on page 218, volume 1, of the first of these encyclopedias implies not only that the infinite system of the natural numbers 1, 2, 3, . . . constitutes a group as regards multiplication, and hence that the inverse of an operator of a given group does not necessarily appear therein, but also that the infinite system of the natural numbers which exceed an arbitrary given one of them constitutes a group with respect to the same operation, and hence that a group does not necessarily involve the identity. On page 652 of the same volume the term closed group is used with a still wider meaning. Even those who are only slightly acquainted with the literature of this subject realize that it is often assumed that a given group contains both the inverse of each of its operators and also the identity operator. In fact, these assumptions are sometimes explicitly embodied in definitions of the technical term group.

In the third and most recent of the three encyclopedias noted above it is explicitly stated that the natural numbers constitute a group when they are

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combined by multiplication, volume 1, part 2 (1932), page 43. Similar statements appear in many other places. Some writers have employed the term semigroup for such a set of elements, but it is questionable whether it is desirable to establish a neutral zone between the elements which constitute a group with respect to a given definite operation and those which do not have this property. A much wider neutral zone than that suggested by the term semi-group has been advocated by various recent writers who employed the expression a set of elements having "the group property" whenever these elements satisfy the condition that every two of them can be combined and that the result of this combination is an element of the set. Nothing seems to have been gained thus far by the efforts to establish a kind of neutral zone between groups and non-groups. In fact, the difficulties have been only increased thereby since the number of the boundaries to be observed has thus been enlarged with too much vagueness as regards the territory outside the definite group area.

A set of elements which have "the group property" as explained in the preceding paragraph is explicitly defined as a group in volume 2, page 243, of the second encyclopedia noted above. In view of the eminence of this authority it seems almost futile to register here an objection to this use of the technical term group. It is, however, obvious that in the present state of mathematical knowledge it is impossible to develop an extensive theory as regards such a general concept. Few, if any, of the body of more than five hundred theorems which have been developed under the heading of group theory apply to such a general conception of the technical term group. Notwithstanding the diversity of definitions to which we have referred it should be emphasized that the theory of groups involving a finite number of elements has encountered few difficulties as a result of differences in definitions since this theory has always been based on known properties of permutation groups involving a finite number of letters. One of the simplest and most useful definitions of the technical term group, which applies to all groups, is as follows: A set of distinct elements obeying the associative law when they are combined constitutes a group provided it satisfies the condition that if two of the symbols of the equation xy = z are replaced by the same or by different elements of the set the resulting equation has one and only one solution therein.

If such an elementary definition of the technical term group would be universally adopted it would greatly simplify the literature relating to this subject. On the other hand, it is clear that progress is more important than simplicity whenever these can not be combined. As long as experimenting with other

definitions seems to lead to important results such experimentation should be encouraged. The source of the definitions of the technical term abstract group is the use of the term group in regard to concrete elements which obey laws that were not explicitly formulated at the time. The earliest definitions were therefore much simpler than those which aimed to embody these laws and to serve as a foundation of an abstract theory of the subject. Naturally the older definitions were often adopted later, especially by those who made only infrequent use of the subject and failed to familiarize themselves with the more recent advances. Mathematicians are a heterogeneous class of people whose achievements are due to different types of abilities, and who sometimes capitalize their success in certain directions to speak on subjects which they have not mastered. These facts may help to explain the confusion which now exists in the literature as regards the definition of the technical term group and which the present note aims to alleviate.

UNIVERSITY OF ILLINOIS

## A NAME FOR BIO-HYDRAULIC ACTIVITIES

THE volume of literature concerning water and solutions in living tissues is constantly growing, and many investigators are becoming increasingly interested in the amount, state and relatively constant ratio of water and dissolved salts concerned in biological structures and life processes.

Without doubt, most of us would be glad to find a suitable and all-inclusive term to designate the recognized group of phenomena related to the regulation of water and mineral salts (solutes in general, perhaps) in living protoplasm. Those of us who are interested particularly in marine and other aquatic organisms and their maintenance of equilibrium against media of different osmotic value are perhaps more directly concerned, though certainly not necessarily so.

Let us consider such physiological functions as the absorption of solutions by the gills or alimentary tracts of animals, or by the body integument and other tissues of plants and animals, the excretion of fluid wastes by kidney or body integument, the ascent and transpiration of water in plants and the maintenance of a biologically necessary colloidal state in normal protoplasm. Those who investigate the chemical aspects of any of these processes are concerned with such studies as endosmosis, exosmosis, capillarity, imbibition, syneresis, hydrophilia (or the "binding" of water), membrane hydrolysis, the Donnan effect, sol-gel equilibria, the stabilization or coagulation of biocolloids by electrolytes or other solutes and kindred basic phenomena.