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THE AMERICAN ASSOCIATION FOR THE ADVANCEMENT OF SCIENCE

THE DEVELOPMENT OF OUR KNOWLEDGE OF THE LAWS OF FLUID MECHANICS[1]

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IN the absence of special thought on the subject, we are little likely to realize the dependence of our every-day life on the laws of fluid mechanics. Thus the air as a fluid is drawn into and expelled from our lungs in accordance with these laws. Again the blood circulates through our arteries and veins under control of the same laws. The gentle zephyr which cools our face in summer or the hurricane which leaves death and destruction in its path are only the expressions of air moving under the laws of fluid mechanics. The trajectory of a golf ball or of the shell from a 16 inch coast defense rifle are likewise the expression of the laws governing the relative motion under gravity of a solid body in a fluid. So again the sustentation of the airship or of the airplane or again the need for the expenditure of energy to secure continued movement through the air: these are all expressions in different ways of these same laws. The same is true of the flotation of a ship partially immersed in water and of the need for propulsive machinery and the expenditure of energy in order to insure continued movement.

Again the entire collectivity of the phenomena of lubrication is only a special expression of the laws of fluid mechanics. So likewise are such divergent phenomena as the rapid spread of sugar through a cup of coffee when we use the spoon as a stirrer, and the carriage by the Colorado River of a burden of silt amounting to something like 5 billion cubic feet per year. In fact, the present configuration of the earth's surface, in so far as wind and water erosion

[1] Presidential address at the meeting of the Pacific Division of the American Association for the Advancement of Science, Salt Lake City, June 12, 1933.

combined with the fluid transport of solid materials have played a part, is the result of the operation of these same laws.

The basic problem of fluid mechanics is this; what is the behavior of the fluid in the case of relative motion between a fluid and a solid, and what system of mutual forces acts between the two under these conditions?

The phenomena involved in the relative motion of a solid and a fluid must have attracted the attention and challenged the curiosity of our prehistoric forebears, but so far as we have any present record, the discussion of these phenomena of nature first took definite form among the Greeks and with special reference to the movement of bodies through the air. The fact of the resistance of the air to such motion was recognized and accepted, apparently without attempt at explanation. The problem, as it took form in the minds of the Greeks, was not, therefore, whence the resistance to motion, but rather why does motion persist against this resistance after the stone has left the hand or the arrow has left the string of the crossbow.

Regarding the speculations of the ancients on these matters, we have little beyond the brief discussion by Aristotle given in section eight of the fourth book of his "Physics." He is here concerned with an attempt to prove the impossibility of the existence of a vacuum. Recognizing the observed resistance to motion both in air and in water and in the absence of any concept of inertia or energy in the modern sense of these terms, the ancients could only conceive of continued motion as the result of the continued application of a propulsive force. Furthermore, it was held that the motion of a body A was only possible as it was pushed or urged by another body B, and the motion of B was in like manner conditioned upon the action of a third body C, and so on indefinitely.

To this general law, projectiles must, of course, be subject, and to the air was assigned the function of supplying the continuing force required to carry the projectile from the body with which it was first in contact until it had reached the target. From this course of reasoning, Aristotle deduced that, without air, the motion of a projectile would not be possible, and hence, as a conclusion, a vacuum could not exist.

As to the manner in which the air could thus operate to supply a propulsive force, Aristotle did not deeply concern himself. He left rather the working out of these details to his successors, suggesting two hypotheses as possible. One of these adduced the action of the air in rushing violently in behind a body in motion, in order to fill the partial vacuum formed, as furnishing the needful push on the rear face of the projectile. The second hypothesis assumed that the air, by reason of its special fluidity, was able, if once put in motion by the body with which the projectile was first in contact (the crossbow, for instance), to continue its motion and its action on the projectile.

In the sixth century A. D. these ideas were opposed by the Greek grammarian Philoponus, who put forward the hypothesis that, when casting a projectile, a certain impetus was transfused into it by the caster, this impetus being then able to maintain the projectile in motion for a certain length of time.

Little progress was made beyond these rival theories until the later formulation of the concepts of inertia, momentum and energy. With these concepts available, however, all controversy regarding such methods of explanation came to an end and the action of the air was recognized as operating continuously in the sense of a resistance rather than as a propulsive agent.

Referring to this transition period, we may note that Leonardo da Vinci, in his earlier discussion of these matters, assumed, in accordance with the Aristotelian physics, the air as assisting the motion of bodies through it. However, at a later period and definitely in 1506, he abandoned these older ideas and recognized in the air a resisting medium, the resistance of which he ascribed to its condensibility, as he termed it. While he apparently recognized that a part of the total resistance was due to dividing the air and putting it in motion, he considered this the minor part and ascribed the major part to the condensation of the air in front of the moving body.

As a result of this same action, Leonardo deduced the explanation of the lift of birds as due to the condensation of the air under the stroke of the wing. He then considered the possibility of human flight, and, as is known, made a number of designs and left a number of notes bearing on the solution of this problem.

It is one of the tragedies of scientific work that these later researches of Leonardo were lost to the world for so many years. He wrote no books and put down the results of his studies in the form of rough notes, comments and sketches in notebook form. Having in view the background of science in his day, much of this work shows him to have been far ahead of his age and in many ways marvelously near the line of later developments. But hidden and unknown as his work was until relatively modern times, it had, as a matter of fact, little or no influence on the general trend of development in the centuries immediately following his period.

We must ascribe to Galileo the first approach to the foundations of our present interpretation of the phenomena and laws of fluid mechanics. In 1632 he devoted a special section of his "Dialogues on Maximum Systems" in opposition to the Aristotelian theory of the action of the medium, and undertook to demonstrate its action as essentially one of resistance rather than of propulsion. Through the development of his ideas on these matters, and his well-known work on the laws of falling bodies and on pendulums, Galileo, though not always correct in all his conclusions, may most properly be called the father of our modern mechanics.

We must now pass in rapid review some of the great names of the centuries which followed the time of Galileo, with only the briefest of reference to the contributions made by them.

First Huygens, who in the closing years of the seventeenth century announced, as based on experimental evidence, the proportionality of the resistance, in the case of motion through a fluid, to the square of the velocity.

At the same time Newton in his great work on the "Mathematical Principles of Natural Philosophy" (1687) dedicated the whole of the second "book" to the study of what we should now call "fluid mechanics," and in which actual fluids such as water, air, oil and mercury were considered, as well as certain others ideal in character, as defined by special mechanical and physical properties.

Newton recognized that the resistance of a body in motion in a fluid depends on the density of the fluid, on the velocity of motion and on the form of the body. He also recognized the influences of friction and of viscosity as elements in the problem. His broad conclusion was that in the more general case resistance comprises three parts, a first part uniform, a second part proportional to the velocity and a third part proportional to the square of the velocity.

One of Newton's special studies related to the behavior of a hypothetical fluid composed of discrete elastic particles, and it was as a result of the discussion of this ideal medium that he deduced his so-called sine square law—that is, that the action of the fluid on a plane moving obliquely through the fluid is equal to that on the plane at right angles to the direction of relative motion, multiplied by the square of the sine of the angle of incidence. This law gave rise to much controversy. If true, it was shown that aerial flight was not possible. It was soon disproved by experiments carried on by Borda, Dubuat, Hutton and others. Newton has been much criticized in connection with this particular result deduced from his assumption of an artificial fluid medium. It is, however, open to question whether Newton considered this discussion as more than a mathematical exercise. He certainly recognized the artificial and ideal character of the medium assumed, and there seems to be no evidence that he seriously considered these results as applying to actual fluids. In fact, in his scholium commenting on the thirty-fifth proposition, he clearly distinguishes these results from those which pertain to actual or, as he calls them, "continued" fluids, such as water, oil and mercury.

Summing Newton's contributions to this problem, it would appear that he clearly recognized the various factors involved in resistance and that he made some approach to an evaluation of their measure. He also recognized the principle of the relativity of motion—that is, that whether the fluid be considered at rest with the body moving through it or the body at rest with the fluid moving past, the results should be the same, assuming the relative velocities the same.

It is perhaps not too much to say that the contributions of Newton to mechanics in the broader sense and to the beginnings of the branch of mathematics later known as calculus have been, on the whole, more important in the later development of fluid mechanics than his researches on this particular subject. In fact, it is not easy to see now how Newton, with the tools at his disposal, could have made any closely detailed study of fluid motion or of the forces involved between fluids and solids in relative motion. It must be remembered that this was before the concepts of calculus, either as presented by Newton or Leibnitz, had been developed into form suitable for dealing with such problems. In particular, it was before the development of the treatment of problems of continuous change by means of the differential equation, and without the aid of this mathematical discipline it is not easy to see how any effective study could be made of the behavior of continuous fluid media.

However, even with the differential equation, powerful as it is, we are not yet able to cope fully with actual fluids as they exist in nature, and to meet this limitation, the ideal fluid of the mathematician has been substituted for the actual. This fluid is characterized by two special qualities which differentiate it from actual fluids; the absence of viscosity, *i.e.*, perfect fluidity, and incompressibility. As a consequence of the first of these, the fluid possesses infinite mobility; there is no resistance to the sliding of one particle near or past another one.

As a consequence of the second characteristic, incompressibility, no element of the fluid, as a result of changes of pressure incident to relative motion among its parts, experiences change of volume. This is true to a high degree of approximation for liquids, and to this extent such media, for all practical purposes, fulfil this requirement of the perfect fluid. On the other hand, gases and vapors are subject to large changes of volume with change of pressure and hence depart in much greater degree from this requirement of the perfect fluid. However, fortunately for many classes of problems and especially for most of those which present themselves in the domain of aerodynamics in its application to aeronautic engineering, the changes of pressure incident to the motions with which we are concerned are small in comparison with the total pressures involved, and the resultant changes in volume are relatively small and often negligible, thus permitting a very satisfactory treatment of practical problems on the assumption of a complete fulfilment of this specification of a perfect fluid.

At this point, then, we have the perfect fluid of the mathematician and four principal means for dealing with the problems presented by internal motion among its parts, or by relative motion between the fluid and solid bodies or solid boundaries. These four agencies are (1) the differential equation with collateral mathematical disciplines, (2) a sound and rational development of mechanics, that is, the relations of length, mass, time, force, energy, momentum, etc., (3) conformal transformation, and finally, (4) the concept of sources and sinks.

The first of these came out of the original work of Newton and Leibnitz and is directly based on assumption of continuous change and as such is peculiarly adapted to problems involving continuous changes in time or in space, such as those presented by relative motions of solids and fluids, or among and between adjacent parts of the same fluid.

The second of these, due primarily to Newton and as later elaborated with special reference to fluid mechanics by John and Daniel Bernouilli, D'Alembert, Euler, Lagrange and others, makes possible the correct framing of our differential equations and the proper interpretation of their results. Of this group, the two Bernouilli's and D'Alembert were more directly concerned with fluid mechanics as such, while the interest of Euler and Lagrange lay rather in the mathematical aspects of the problem. Euler may indeed be called the father of the mathematical expression of the theory of the perfect fluid, while Lagrange carried on the development in further detail, building largely on the foundation which Euler had laid.

The third (conformal transformation), through the wizardry of geometrical relations, makes possible the transformation of results derived for relatively simple forms of boundary between solid and fluid, to others much more complex in their geometrical character.

The fourth makes possible the building up, constructively, of shapes and forms of fluid masses, in either two or three dimensions of space, and around which a field of fluid flow coming from a distant point will divide as though this constructive mass of fluid were a solid body. In fact, the boundary of this constructive mass of fluid bears the same relation to the remainder of the flow as would a solid body of the same form, and hence we may assume such a body substituted for it, thus obtaining the distribution of force reaction over such body when placed in a field of flow, as well as the lines of flow in the fluid in passing around the body.

These various agencies have thus made possible a very considerable development of fluid mechanics as applied to the perfect fluid. The mathematical control of this domain, is not, however, complete, due chiefly to the difficulty of introducing into our differential equations adequate representation of complex geometrical forms, or otherwise, finding, through tedious and complicated methods of trial and approach, the representation of such form through the use of sources and sinks. However, given any case involving a solid body and a perfect fluid in relative motion, and required the distribution of force reaction between the two, together with the stream lines of flow about the body, and given likewise time and patience, it is fair to say that, through the use of these various agencies, a solution to any reasonable degree of approximation may be found.

Over this same general period of growth in theory, covering the latter part of the eighteenth century and the early years of the nineteenth, there developed a gradual accumulation of the facts of experimental research on these various problems, due to the work of D'Alembert, Borda, Dubuat, Bossut, Duchemin, Robins, Vince, Navier, Robinson and others, thus serving as a check and needful guide on the development of theory alone.

Out of this elaboration of the theory of motion in perfect fluids, there came, however, a most surprising and puzzling result. It will be recalled that the resistance to the relative motion of a solid and a fluid had formed one of the chief objects of interest to Newton and to those who followed him. Likewise, the oblique or lateral force manifested in the case of a body of approximately flat or elongated section, when moving obliquely through a fluid, had formed a major subject of interest. Now with means adequate for the investigation of such problems for the case of the assumed perfect fluid, it appeared that there could be no such resistance; or more generally, no over-all force reaction in any direction, and hence no oblique or lateral force.

It should be noted, however, that the relative field motion assumed between the solid and the fluid was rectilinear and unaccelerated. This would correspond to the case of a solid body moving with a uniform velocity in a straight line through an infinite fluid medium; or, on the other hand, to the flow of such an infinite fluid field past the body, with a field velocity uniform and in a straight line.

However, in actual fluids, the fact of resistance, or of over-all force reaction was obvious. It had challenged the attention and interest of all who had concerned themselves with these matters from the time of Aristotle down. It was then obvious that the explanation of the actual and observed over-all force reactions must be found, either in those circumstances which differentiate actual fluids from the medium known as the perfect fluid, or in departures, in the case of actual fluids, from the simplicity of field motion which had been hitherto assumed. As we shall directly see, both of these differences play their part in furnishing the final explanation.

Before passing to some consideration of this final explanation (at least as now received) brief note should be taken of the experimental researches as well as of the developments in theory which laid the foundation for the solution of the problem.

Of the two characteristics of actual fluids, omitted by mathematical necessity in forming the specifications for the perfect fluid, that of viscosity was recognized from the first as the more important of the two, and indeed as accountable presumably in primary degree for the observed differences between the results for actual fluids and those indicated by theory for the perfect fluid.

If we come now to a period near the middle of the last century, we find the beginning of serious and effective studies relating to viscous fluid media. Among these early writers note may be made of Coulomb, Duchemin, Poisson, Barré de Saint-Venant, and between 1845 and 1856, Stokes, who, in a series of brilliant papers, laid a broad foundation for later studies on this subject. To this same period belong the experiments of Poiseuille (1840–42) which, a few years later, served as a starting point for the brilliant researches of Osborne Reynolds. To Reynolds we are indebted for the definition of the two modes of flow, laminar and turbulent, to the definition of the non-dimensional function of length, velocity, density and viscosity which has properly received the name of "Reynolds Number," and of defining the value of this number which marks the zone of change between these two modes of flow. It would not be easy to point to another single contribution to the theory of fluid mechanics which has exercised a more profound and far-reaching influence on the later studies in this domain, lying as it does at the foundation of the application of the laws of kinematic similitude to the problems of fluid movement, and hence standing as the justification of the enormous extension of model research in recent years to the problems of hydraulics, to those of shipbuilding, of aeronautics and to practically all phases of the problems of fluid mechanics as they bear on the problems of actual life.

To this same general period belong also the epoch-making researches of Helmholtz (1821–1894). To his contributions must be credited the most notable advance in the theory of fluid mechanics since the days of D'Alembert, Euler and Lagrange.

While he illuminated many phases of fluid mechanics, his most notable contributions related to the study of vortex motions in fluid media and to the existence of what he termed surfaces of discontinuity between zones of fluid moving under different physical conditions. With his paper on vortex motions, Helmholtz opened a new field of research, which, carried on in more recent years, has yielded epoch-making changes in our concept of the dynamic relations between solids and fluids in relative motion, and has shed a flood of light on the source of the various force reactions between the fluid medium and the solid bodies.

Again the existence of surfaces of discontinuity in fluid flow was used by Kirchhoff (1869) and by Rayleigh (1876) as a foundation for an explanation of the resistance to the motion of a plane moving through a fluid, a result which, as we have seen, the classical theory had completely failed to explain. Not only was this problem treated in a qualitative sense, but formulae were deduced by Kirchhoff for the case of a plane moving in a direction normal to itself, while Rayleigh independently and later deduced formulae for both the cases of direct and of oblique motion.

It may be remarked at this point that later researches and developments of what may be called the Prandtl school have furnished a more satisfying explanation of the force reaction between solids and fluids in relative motion and a more accurate basis for its evaluation. However, these latest developments go back, for their foundation, to the laws of vortex motion, and hence, in any case, we may ascribe to Helmholtz the credit of laying the foundation for these most recent advances in the explanation and quantitative determination of the force reactions in the case of the relative motion of fluids and solids.

We may now pass to those advances which have especially characterized the present century, and inasmuch as these have centered largely about the problem of the airplane, they may be considered primarily from this point of view. The major problems are here three or perhaps four in number: (1) The source of the lift on the wing of an airplane and its quantitative measure; (2) the source of the resistance to motion and its quantitative measure; (3) the rôle played by viscosity as a factor in problems (1) and (2); (4) the same query with regard to the compressibility of air viewed as an elastic and compressible medium.

As we have seen, the classical theory based on an assumed ideal or perfect fluid gave no explanation of either lift or resistance, and, for rectilinear unaccelerated motion, it gave definitely zero force reaction in all directions between a solid and a fluid in relative motion. The first step forward was taken independently by Kutta in Germany and Joukowski in Russia, who showed that, even with the perfect fluid of the mathematician, assuming a special form of vortex or circular motion in the fluid about the body, combined with the rectilinear motion, a lift would result in a direction at right angles to the line of rectilinear motion. And further, for the measure of this lift, an astonishingly simple and elegant formula was developed.

The term "circulation" has been applied to the line integral of the velocity taken in any path completely around the body. This means simply that if any line, a circle for example, be circumscribed about the body, then the circulation is measured by the summation of all the small elements formed by multiplying each element of length of the path by the velocity along the path at that point. Then the particular type of vortex motion assumed is such that the circulation in all paths about the body remains the same. The formula for the lift is then given simply by the product of the circulation, by the velocity of rectilinear motion and by the density of the fluid.

This law is commonly known as the Kutta-Joukowski law, from the names of its two discoverers, each working independently of the other. The justification for the assumption of this particular law of vortex motion develops from the mathematical theory of potential motion in a perfect fluid and is entirely consistent with the conditions for such motion in such a medium. It is thus seen that the explanation of lateral force, or of lift in the case of an airplane, was developed as a result of superimposing, in a perfect fluid, a special form of vortex or circular motion on the rectilinear field motion which had been hitherto assumed in dealing with such problems.

But while the combination of circulation with rectilinear motion explained lift, it was recognized that there still remained the problem of explaining the explanation. That is, there is no way, in theory at least, of initiating such a form of vortex motion in a perfect fluid devoid of such motion at the start.

However, before proceeding with the discussion of these terminal problems of lift and resistance, we must consider a little more closely the surface and near-by phenomena attendant on the relative motion of a solid body and a viscous fluid. The non-viscous fluid, it will be remembered, is defined as one in which the ultimate particles in gliding or sliding past each other exercise no mutual force reaction. As we must deal with them, all fluids are viscous in varying degrees. Again, the manifestation of viscosity depends in profound degree on the relative velocity of motion. Thus, for very slow motion, certain substances like resin exhibit the phenomena of viscous flow and from such extremes the condition appears continuously in decreasing degree down through such fluids as tar, oil, water and on to and through vapors and gases. Thus, air in popular estimation would not be considered a viscous substance; but it is distinctly so, and where the relative velocity of gliding (or shearing, to use the more technical term) is great, these viscous drags exercise a controlling influence over the attendant phenomena.

It has only become possible, through the aid of modern atomic theory, to form some picture of how and where these drag forces originate. Apparently, we must look for their source in the stray electric fields surrounding the molecules of the fluid and due to the special electronic architecture of the molecule. In any event, the result of these mutual viscous drags acting on two molecules in gliding motion past each other is to impress upon them some degree of rotary or spinning motion. In the case of the non-viscous fluid under the so-called "potential motion", the ultimate particles are assumed to move without any such rotary motion, and, as it develops, this results in a great simplification in the mathematical aspects of the problems of fluid motion.

A point of the highest interest in connection with the phenomena of the relative motion of a solid and a viscous fluid is that a very thin layer of the fluid, perhaps only one molecule thick, is bound to the body and moves with it (supposing, for simplicity, the case of a solid moving in a fluid considered otherwise at rest). Then, passing outward from the body, there develops a continuous lagging of the successive layers of fluid particles, until finally, at some little distance from the body, the effect of these drag forces decreases to the point of vanishing.

We have thus the picture of a continuous series of layers of particles outlying from the body with relative motion between, the result of which is a condition of more or less irregular spinning or vortex motion, throwing the fluid into a state of mixed turbulence quite beyond the reach of mathematical expression in other than some statistical or approximate fashion. Furthermore, this blanket of irregular turbulent fluid of necessity embodies and thereby impounds a certain amount of kinetic energy which streams away to the rear. It is then the continuous generation of this energy which appears manifest as a decrease in the pressure energy of the moving fluid, or broadly as a resistance to the motion.

With this picture, we have, therefore, the body surrounded by a blanket of eddying turbulent fluid, as the physical expression of the action of these minute molecular forces previously referred to. This blanket of fluid is commonly known as the boundary layer, separating, as it does, the solid body from the outlying mass of fluid wherein these effects are negligibly small. It is also a point of great interest and importance that in the region outside this boundary layer, the simpler equations of potential motion very closely apply, so that the effect of viscosity may be viewed as resulting in a virtual change in the geometrical form of the body, consequent upon the addition of this boundary layer, and within which the special phenomena of viscosity are manifest, while outside and beyond, the simpler conditions of potential motion prevail, at least in paramount degree.

In addition to the drag or resistance due to the energy drained away in this surface blanket of eddying turbulent fluid, and notably when the form of the body is rough and with abrupt curvatures or changes in the direction of the surface, there will develop large vortices breaking off irregularly and streaming to the rear, forming a wake of mixed turbulence, which again entails a draft of energy appearing likewise in the sense of a resistance to the motion.

The chief features upon which the phenomena due to viscosity seem to depend are as follows: (1) The character of the fluid; (2) the geometrical form of the body; (3) the character of its surface; (4) the relative velocity of motion.

Too little is known of the exact character and mode of action of the molecular forces producing rotation to permit of the formation of any wholly rational theory of the phenomena of viscous flow. However, here as elsewhere, guided by observation, hypotheses are possible which, developed through the aid of suitable mathematical procedures, are able to give a reasonably satisfactory account of the principal features of such motion.

Further refinement is to be expected and will doubtless be realized, but it hardly seems possible that we can ever rise above the need of following this general plan of attack. That is, it hardly seems possible that we can expect any refinement of theory which will enable us to follow in detail the adventures of any individual particle of the fluid in cases of viscous flow or, even were this possible, to treat other than in some general statistical manner the results in the aggregate for any given case.

These developments mark in a special way the advances made during the past half century in this part of the general field of fluid mechanics. No attempt will be made to discuss in detail the contributions made individually by the various pioneers in this field, but the names of Stokes, Navier, Osborne Reynolds, Rayleigh, Lamb, Prandtl, Blasius, Oseen, Pohlhausen, Karman and Levi-Civita may be mentioned among those who have made notable contributions to the development of the present status of the mechanics of viscous fluids.

Returning now to the problems of lift and resistance, and in particular for the case of the airplane, it results from the basic theory, as applied to the case of abruptly accelerated motion (as at the start of the plane), combined with the influence of viscosity and surface friction, that the initial circulatory motion about the airplane is generated. The existence of such a circulation about the plane has furthermore been demonstrated by clever photographic technique, and measurements on such photographs show that such motion, constituting the circulation referred to at an earlier point, fulfil in close degree the characteristics assumed by Kutta and Joukowski for the circulation in a perfect fluid.

With reference to the terms "circulation" and "circulatory motion," a caution should be noted that we are not to suppose a form of motion in which any given particle makes a circuit about the plane. The facts are rather that at any given instant particles will be moving in paths which form parts of such circulatory paths and, in theory at least, a continuous series of such particles could be found which collectively would form a continuous path about the plane. The circulation is therefore statistical or collective in character rather than actual for any one particle. This form of motion, however, meets perfectly the conditions as assumed by Kutta and Joukowski in their development of an explanation of lift, and the resultant formula measuring its value has been abundantly verified experimentally, especially with airplane wing forms.

It may naturally be asked how it comes about that a formula derived to fit the case of a non-viscous or perfect fluid should give so satisfactory a measure of conditions in actual fluids which are far from meeting the assumption of viscosity zero.

Without tarrying too long over this interesting point, it must suffice to say that in primary degree this result arises from the fact that in the outlying fluid, as noted at an earlier point, the motion of an actual fluid such as air or water very closely follows the laws which govern in the case of the non-viscous fluid. In other words, the immediate effects of viscosity and friction are confined to a relatively thin layer of fluid surrounding the body, and, outside of this, the laws for a non-viscous fluid largely govern the resulting fluid movement. It thus follows that the circulation taken in the fluid well outside the boundary layer will have closely the same value in an actual fluid as for the ideal fluid and hence the close agreement of the Kutta-Joukowski formula with actual measurement.

There is, however, one link in the chain of complete control over the phenomena of lift which yet remains to be filled in. The formula gives the lift in terms of the density, the velocity and the circulation. But there is as yet no general way of determining the circulation having given simply the geometrical form of the body. By means of special assumptions, results can be derived for a certain range of geometrical forms, and these together with experimental results have given a very satisfactory working basis for forms of usual type. There is still lacking, however, the formulation of a general relation between geometrical form and circulation.

Passing now to resistance and its measure, we find a much more complex situation, which has been put on a reasonably satisfactory basis only in relatively recent years.

In the present status of this problem, three types or forms of resistance are recognized—frictional resistance, form resistance and induced resistance. In aeronautic terminology the word "resistance" is replaced by "drag," implying the resistance in the direct line of motion. In the fluid mechanics of the last century, the term resistance was sometimes used in the sense of the total force reaction between the body and the fluid. In more recent times and especially in aeronautics, this total force reaction is decomposed into its two components, at right angles to and along the line of motion, the former called lift and the latter drag.

Of the three types or forms of drag mentioned, that due to friction or skin resistance, so called, finds its explanation, as already noted, in the need for constantly supplying the energy required to maintain the eddying turbulent boundary layer.

The second type, from drag or resistance, finds a similar explanation in the energy drained away in the large vortex and mixed turbulence forming a wake in the case of bodies of irregular or non-stream line form. With well-formed airplane wings, this form of drag is small. Irregular projections or blunt forms, such as found in the wheels of the landing gear, contribute to resistances of this type.

The existence of the third form of resistance, the "induced" drag, was never suspected until relatively recent years, and its measure is a direct result of our better understanding of the details of vortex motion and of the types of vortex or cyclic motion generated by the passage of a body through a fluid.

Without attempting detailed discussion of this interesting point, it must suffice to say that as a result of the mutual reactions between a body such as an airplane wing and the air through which it passes, there is developed a system of vortex filaments trailing from the wing, the reaction of which upon the air flowing to and past the wing is such that the effective direction of flow is no longer in the direction of flight, but becomes inclined from the front downward to the rear. But the "lift," so called, must always be reckoned at right angles to the direction of relative air movement, and this direction will thus be inclined backward or to the rear. This means that the so-called lift force will have a component directed to the rear against the direction of motion, thus constituting a component of the resistance or drag. Our knowledge of the character and magnitude of these vortex filaments further permits of the derivation of very satisfactory formulae for the measurement of this component of the drag.

We may, therefore, sum the situation regarding resistance or at least drag, as involved in the problems of aeronautics, as follows:

The first component, that due to friction, has been placed, through extended experimental research, on a reasonably satisfactory basis.

The second component, that due to form and expressed in a wake of mixed turbulence, is of small importance in well-formed bodies of so-called stream line shape, such as an airplane wing of an airship of typical form—at least so long as the direction of motion is nearly fore and aft along the form. With increasing obliquity of approach, however, this component becomes of rapidly increasing importance. Here, likewise, experimental research has furnished a reasonably satisfactory foundation for estimating the measure of this resistance or drag, at least in such cases as are most likely to present themselves in practical problems.

The third form, the induced drag, as already noted, is satisfactorily subject to measure by means of formulae derived from the theory of vortex motion.

We must not leave the subject of these spectacular advances in the aeronautic applications of the theory of fluid mechanics without at least a mention of some of the names which have made such progress possible. The names of Kutta and of Joukowski have already been noted in connection with the formula for lift.

The dawn of the new vision of the behavior of an airplane wing or indeed of any body of similar form moving through the air goes back to the closing years of the last century when Lanchester came forward with his remarkable physical insight and gave the first picture of the phenomena attendant on such motion substantially in the form in which they are accepted to-day. This work dated from 1891, and in the following years and notably in 1894 he made his first public statement of these new views. These were extended somewhat in later years, but the basic ideas remained unchanged. Lanchester's work was essentially descriptive in character. He was not a formal mathematician and he did not attempt to develop in mathematical form the consequences of his physical picture. This was reserved for Prandtl and those who worked with him. These developments date from 1904 and, during the following years, though unacquainted with Lanchester's work, large advances were made in expressing much of Lanchester's picture in mathematical form. At a later time Prandtl became acquainted with Lanchester's work and recognized its priority in time to his own, calling attention, however, to the need of quantitative expression such as that which he and his coworkers had sought to supply.

The work of Prandtl and of what may be called his school thus stands as the great achievement of the present century in the extension of our understanding of the phenomena of fluid motion and in the development of this understanding in mathematical form, thus serving to give a quantitative measure to many of the chief features of this complex picture.

We turn now and finally to a brief consideration of the influence of compressibility, the second point of difference between the perfect fluid and actual fluids in nature.

The effect of compressibility is of gradually increasing importance with increasing speed. It affects chiefly the magnitude and distribution of the force reactions between a fluid and a solid body, between which there is relative motion. The general criterion of velocity in these respects is that of sound in the fluid medium. For velocities not exceeding one half that of sound in the medium, the fluid is only slightly compressed by the force reactions developed, and for most practical purposes this effect can be neglected. As the relative velocity rises, however, approaching the velocity of sound in the medium, this effect rapidly increases in magnitude and at velocities near and beyond the velocity of sound, all force reactions undergo large increase, and these effects can be no longer discarded.

In air, if the velocity of sound be taken at 1,100 feet per second, this is equivalent to 750 miles per hour, and the more common modes of translation, whether by railroad train, automobile or airplane, will be so far removed from this figure that for such cases the fact of the compressibility of air has no significant influence on the attendant phenomena. On the other hand, in the case of an airplane propeller 9 feet in diameter and turning 2,400 revolutions per minute, for example, the speed at the tip of the blade will be about 1,130 feet and under these conditions the fact of compressibility becomes of importance and can no longer be neglected. The same is true in a still more emphatic manner when dealing with the problems of exterior ballistics. The velocities of projectiles fired from heavy guns are now rising to initial values of 3,000 feet per second and above, and for all such cases the compressibility of the air and its influence on the attendant phenomena will be of the highest importance. The same conditions will also be of controlling importance in all problems connected with the flow of compressible fluids in long-closed conduits, as, for example, the flow of natural gas over long distances, accompanied with a continually reducing pressure and corresponding change of volume.

The general equations for cases of flow or of relative motion with compressible fluids have been pretty well developed as a phase of what may be called the classical period and represented in particular by the work of the last century. These developments have been refined and extended somewhat in the years of the present century, and in their present form represent a reasonably satisfactory stage of advancement so far as abstract theory is concerned.

Practical applications, however, are still hampered by the difficulties met with in attempting to include the geometry of the form of the solid body with which the fluid is in contact, and also the effects arising from viscosity and skin friction. It results that, for practical problems, recourse must be had to experimental results and to various partial and empirical hypotheses, so adjusted as to represent the best information available at the moment.

It will thus be realized that while theory has gone far in explaining the phenomena of fluid mechanics and has provided useful and reasonably accurate formulae for certain portions of the force reaction, it is far from having furnished such formulae in an entirely general form, and for many of the features of interest in a quantitative sense, formulae are quite lacking. It may be noted that one of the principal difficulties in generalizing the formulae of fluid mechanics lies in the fact that we have no adequate method for connecting the geometry of form of a solid body with the phenomena to be anticipated in the case of relative motion between such a body and actual fluids, exhibiting, as they do, both viscosity and compressibility. A second difficulty, and perhaps the more serious of the two, is found in our lack of knowledge of what goes on in the interior of an actual fluid moving near or under the influence of a solid body. The mechanism of the development of minute vortices; their laws of growth or change, the establishment of mixed turbulence—of all these we have only the most sketchy knowledge in matters of detail. In consequence, our quantitative control over such phenomena must be primarily empirical in character or based on theory admittedly incomplete and inadequate.

Further extension of our knowledge of these details and a wider generalization of our mathematical control over these two major types of present-day limitation present an inviting field for further study in the years to come.