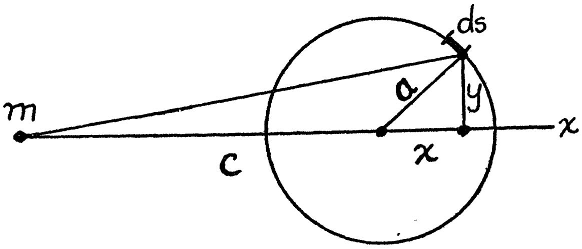
DISCUSSION

THE ATTRACTION OF A SPHERE

IT is surprising to find in SCIENCE for April 14, 1933 (page 371), a denial of the well-established theorem that under the Newtonian law of gravitation the attraction between two homogeneous spheres is the same as if the entire mass of each were concentrated at its center. Since the proof of this theorem, while entirely elementary, is often omitted in the text-books, it may be permitted to reproduce it here.

Consider a thin spherical shell of radius a and surface density ρ, attracting a particle of mass m at distance c from the center. If the surface is regarded as generated by rotating a circle about the axis of x, as shown in the diagram, an element of arc ds, rotating through a small angle $d\vartheta$, will generate an element



of area $yd\vartheta ds$, which may be regarded as a particle whose mass is $\rho\, yd\vartheta ds$.

According to the Newtonian law, the force exerted by this element on the particle m is $G\,[m\rho\, yd\vartheta ds]\,/\,[(c+x)^2+y^2]$. This force is directed toward the element, and the component along the x-axis (which is all that concerns us) is obtained by multiplying by the cosine of the angle, namely, $(c+x)\,/\,[(c+x)^2+y^2]^{\frac{1}{2}}$. Hence, noting that $yds=adx$, and $y^2=a^2-x^2$, the element of force along the x-axis is

$$(c+x)\ [Gm\rho\, ad\vartheta dx]\ /\ [c^2+2cx+a^2]^{3/2}.$$

Integrating from $\vartheta=0$ to $\vartheta=2\pi$, and then from $x=-a$ to $x=a$, we have as the total force acting on the particle m along the x-axis,

$$F=\int_{-a}^{a}\frac{(c+x)\ Gm\rho\, 2\pi\, adx}{[c^2+2cx+a^2]^{3/2}} \qquad (1)$$

Now suppose that the mass of the attracting spherical surface, namely $4\pi a^2\rho$, were concentrated at a distance D from the particle m, where D is as yet undetermined. Then, by the Newtonian law, the force acting on the particle m would be

$$F'=Gm\ (4\pi a^2\rho)/D^2$$

Equating these values of F and F', we have the following equation for determining D:

$$\frac{2a^2}{D^2}=\int_{-a}^{a}\frac{(c+x)\ adx}{[c^2+2cx+a^2]^{3/2}} \qquad (2)$$

To evaluate this integral, make the substitution $t=(c^2+2cx+a^2)^{\frac{1}{2}}$, and note that when $x=a$, $t=c+a$, and when $x=-a$, $t=|c-a|$, where the vertical bars indicate the absolute value of $c-a$. Then we find

$$\frac{2a^2}{D^2}=\frac{a}{2c^2}\int_{|c-a|}^{c+a}\left(1+\frac{c^2-a^2}{t^2}\right)dt. \qquad (3)$$

Two cases must now be distinguished, according as the particle m is outside or inside the spherical shell.

Case 1. If $c>a$, $|c-a|=c-a$, and we readily find $2a^2/D^2=(a/2c^2)\ (4a)$, so that $D=c$.

That is, *the attraction of the spherical shell at an external point is exactly the same as if all its mass were concentrated at its center.*

Case 2. If $c<a$, $|c-a|=a-c$, and we find $2a^2/D=0$, so that D is infinite. That is, the attraction of the shell at an interior point is zero.

Finally, a solid sphere in which the density at any point is a function only of the distance of that point from the center, may be regarded as made up of a series of concentric homogeneous shells. Since each shell attracts an external particle as if all its mass were concentrated at its center, the same will be true of the sum of the shells. Hence we have the theorem:

On the basis of the Newtonian law of the inverse square, a homogeneous sphere, or any sphere in which the density is a function only of the distance from the center, attracts an external particle exactly as if all the mass of the sphere were concentrated at its center.

In view of the recent misunderstanding of this theorem, I trust that this reproduction of the standard proof may not be wholly superfluous.

EDWARD V. HUNTINGTON

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THE MOVEMENT OF DIFFUSIBLE SUBSTANCES IN FOOD PRODUCTS

WHEN fluids diffuse through substances of capillary structure, they tend to carry solutes with them. Where water is the diffusing fluid, phenomena of hydrolysis, chemical reaction, selective adsorption and differential diffusion may become manifest.[1] Colloids may diffuse, albeit more slowly than crystalloids—a

[1] Jerome Alexander, *Jour. Am. Chem. Soc.*, 39: 84, 1917; SCIENCE, 54: 74, 1921.