SCIENCE

VOL. 77 FRIDAY, MARCH 24, 1933 No. 1995



SCIENCE: A Weekly Journal devoted to the Advancement of Science, edited by J. MCKEEN CATTELL and published every Friday by

THE SCIENCE PRESS

New York City: Grand Central Terminal
Lancaster, Pa. Garrison, N. Y.

Annual Subscription, $6.00 Single Copies, 15 Cts.

SCIENCE is the official organ of the American Association for the Advancement of Science. Information regarding membership in the Association may be secured from the office of the permanent secretary, in the Smithsonian Institution Building, Washington, D. C.

# THERMODYNAMICS AND RELATIVITY[1]

By Professor RICHARD C. TOLMAN

CALIFORNIA INSTITUTE OF TECHNOLOGY

## (1) INTRODUCTION

WE have met to do honor to the memory of Josiah Willard Gibbs. By the labors of this master, the classical principles of thermodynamics were given their most complete and comprehensive expression. As the subject for the tenth memorial lecture, it seems appropriate to discuss the extensions to these classical principles which have since been made necessary by Einstein's discovery of the special and general theories of relativity.

The need for an extension of thermodynamics to relativity arises in two ways.

In the first place the classical thermodynamics was—perhaps unintentionally but nevertheless actually—only developed for systems which were tacitly assumed to be at rest with respect to the observer, and further investigation is necessary for the treatment of thermodynamic systems which are moving relative to the spatial coordinates in use. This further investigation must be carried out with the help of those principles for the intercomparison of measurements—made by observers in uniform relative motion to each other—which form the subject-matter of the special theory of relativity.

In the second place, the older thermodynamics tacitly assumed that the behavior of thermodynamic systems could be described with the help of ideas as to the nature of space and time which we now know to be approximately valid only for a limited range of space-time and in the absence of strong gravitational fields. The considerations of the classical thermody-

[1] The tenth Josiah Willard Gibbs Lecture, delivered at Atlantic City, December 29, 1932, under the auspices of the American Mathematical Society, at a joint meeting of the society with the American Physical Society, and Section A of the American Association for the Advancement of Science.

namics were thus actually limited to the treatment of small enough systems and weak enough gravitational fields so that the deviations from this kind of space time could be neglected, and the Newtonian theory of gravitation could be applied as a close enough approximation. In order, however, to investigate the thermodynamic behavior of large portions of the universe as we may wish to do in connection with cosmological problems, and in order to obtain, even in the case of small systems, more precise expressions for the thermodynamic effects of gravity, it becomes necessary to extend thermodynamics to general relativity, and make use of the more valid ideas as to the nature of space and time and the more precise theory of gravitation which Einstein has now provided.

### (2) The Character and Validity of Thermodynamics and Relativity

In carrying out these proposed extensions of thermodynamics to relativity, it proves possible to combine the known principles of thermodynamics with those of special and general relativity in a very natural manner with only small and apparently rational additions in the way of new hypothesis. Hence the character and validity of the system of relativistic thermodynamics that we obtain is largely dependent on the character and validity of the two component sciences.

In character, the classical thermodynamics may be regarded as a macroscopic, phenomenological science, which has no actual need for that interesting kind of support that can be furnished by the microscopic atomic considerations of statistical mechanics, but which attempts to treat the gross behavior of matter with the help of those generalized descriptions of the results of numerous gross experiments on the mechanical equivalent of heat and on the efficiency of heat engines, which we call the first and second laws of thermodynamics.

As to the validity of thermodynamics, we have feelings of great confidence, on account of the extensive experimental verification which exists, not only directly for the two laws themselves, but for an extraordinary number of consequences which have been drawn from them—often by elaborate but logical trains of deductive reasoning. Further additions to the principles of thermodynamics may be found, such as the newer so-called third law of Nernst and Planck, but we can not escape the conviction that, so long as the human mind retains its present ideas of rationality, these additions are likely to prove—as in the case mentioned—supplementary rather than destructive.

Relativity, a science which changes as it does our very ideas as to the nature of space and time, has much more fundamental and far-reaching implications than thermodynamics and can not be so easily characterized. There are, however, certain similarities between the two sciences which may be emphasized.

In the first place, at least in its present stage of development, relativity must also be regarded as a macroscopic theory dealing with ideas as to the nature of space and time which have been directly derived from macroscopic experiences. Indeed, in view of Heisenberg's uncertainty principle and the great difficulties which have been encountered in all attempts to construct a satisfactory relativistic quantum mechanics, we may even doubt whether these ideas as to space and time are even suitable for microscopic considerations. This, however, offers no difficulties if we are to combine with another macroscopic science such as thermodynamics.

In the second place, although we are often inclined to be specially impressed by the wonderful conceptual content of the theory of relativity, we may here emphasize its not negligible character as a phenomenological or descriptive science.

Thus the first of the two postulates of the special theory of relativity may be regarded as a generalized description of many failures to detect the absolute velocity of the earth's motion. And the second postulate may be regarded as a mere empirical statement of that constancy in the velocity of light, which is specially clearly demonstrated in the case of distant double stars by the lack of any effect from the changing motions of the members of the doublet on the time needed for their light to reach the earth.

Turning, moreover, to the two postulates necessary for the general theory of relativity, the principle of equivalence may be regarded not unfairly as a reasonably generalized description of Galileo's discovery that all bodies fall at the same rate. The principle of covariance, however, is on a somewhat different footing, since, as first pointed out by Kretschmann—given sufficient mathematical ingenuity—any physical law whatever could undoubtedly be expressed in covariant language the same for all coordinate systems, so that the principle of covariance can imply no necessary physical consequences. Nevertheless, as emphasized by Einstein, the actual phenomena of physics must themselves be independent of the choice of coordinate system which is a conceptual introduction on the part of the scientist which may be made in any way that may suit his convenience or please his fancy. Hence the actual employment of invariant forms of expression in searching for the appropriate axioms of physics is desirable in order to avoid the introduction of unsuspected assumptions which might otherwise be insinuated by the use of special coordinates. We can then also, somewhat facetiously, emphasize the phenomenological character of the principle of covariance, by regarding it as a generalized description of the familiar phenomenon, that the purely conceptual activities of man—in inventing imaginary coordinate systems—are likely in first approximation to have no immediate effect on the laws of physics.

As to the validity of the theory of relativity, we have to rely on three different kinds of evidence.

In the first place, we may put its agreement with a great range of diverse facts from different branches of science, which as isolated phenomena can often be attractively explained in terms of pre-relativistic notions, but which as a whole have only been successfully correlated with the help of the theory of relativity.

In the second place, we must put those special observations which distinguish as uniquely as may be between the predictions of relativity and those which would result from other points of view. Here we have in the case of the first postulate of special relativity the demonstration of the Lorentz contraction by the Michelson-Morley experiment and all its now numerous repetitions, if we may include the extensive work of Professor Miller as demonstrating this contraction at least as the primary effect. And we also have the remarkable demonstration of Einstein's time dilation by the beautiful experiments of Kennedy and Thorndike. In the case of the second postulate of the special theory, we have as most important the precise analysis of double star orbits by de Sitter. And turning to the general theory of relativity, we have the entirely satisfactory results of the three crucial tests provided by the rotation of the perihelion of Mercury, the bending of light in passing the sun, and the shift in the wave-length of light originating on the surface of the sun and on that of the companion to Sirius.

Finally, as a third kind of evidence, for judging the validity of relativity we must not neglect the bearings of that wonderful internal coherence of the theory, with its simple foundation and elaborate but logical superstructure, which so well attests the genius of Einstein. Although such qualities can of themselves provide no guarantee as to correspondence with external phenomena, we can, nevertheless, regard them as indicating that such correspondence—when found for our present limited range of observation—is likely to persist over a much wider range of possible experience.

Like all parts of science, the theory of relativity will presumably be subject to future modifications and additions, such for example as might be provided by a successful unified field theory. Nevertheless, just as the Einstein theory has retained the Newtonian theory of gravitation as an exceedingly satisfactory first approximation, we may expect at least for a long time that such changes will here—as well as in the case of thermodynamics—be supplementary rather than destructive.

The character of the two sciences of thermodynamics and relativity, which we are going to combine, is thus sufficiently similar that we may have no hesitations, on that score, and may expect the resulting relativistic thermodynamics to be itself a macroscopic theory suitable for use in the description of the gross phenomena of the external world. And the validity of the two component sciences is sufficiently established so that for the present we may concentrate attention, as we shall in what follows, on the rationality of that small amount of additional hypothesis which we must introduce to effect the combination.

### (3) THE EXTENSION OF THERMODYNAMICS TO SPECIAL RELATIVITY

We are now ready to consider the actual procedure adopted in the extension of thermodynamics, first to special relativity and then to general relativity. The extension to special relativity, so as to obtain a suitable thermodynamic theory for moving systems, was made by Planck[2] and by Einstein,[3] as early as the year 1907, in that brilliant period of development which was initiated by Einstein's publication of the elements of special relativity only two years previous.

(*a*) *Special Relativity and the First Law of Thermodynamics.* In order to appreciate the nature of this extension, let us begin by seeing what happens to the first law of thermodynamics when the extension is made.

In the classical thermodynamics for systems at rest with respect to the observer, we have found it important to distinguish two ways in which there can be an interchange of energy between a system and its surroundings, namely, through the flow of heat into the system from its surroundings and through the performance of work by the system on its surroundings. Making use of this distinction, and making use of the principle of the conservation of energy, which requires that any alteration in energy content can only result from interchange with the surroundings, we then write the first law of thermodynamics in the form given by the equation (1)

$$\Delta E = Q - W \tag{1}$$

where $\Delta E$ is the increase in the energy of the system which accompanies the influx in heat Q and the performance of work W against external forces.

In *form* this equation can be taken over without modification into the thermodynamics of moving sys-

[2] Planck, Berl. Ber. (1907) p. 542; *Ann. der Phys.*, 26: 1, 1908.

[3] Einstein, *Jahrb. f. Rad. u. El.*, 4: 411, 1907.

tems, in the first place since the special theory of relativity has done nothing to upset the principle of the conservation of energy, and in the second place since we shall still wish to distinguish between the energy transfer W corresponding to work done against macroscopic external forces and the other modes of transfer which we call the flow of heat Q.

In the application of this equation to moving instead of stationary systems, however, an important difference—which would not have been suspected in prerelativistic days—now arises on account of the relations between mass, energy and momentum made clear by Einstein's work. To illustrate this difference, let us consider—as we usually do in thermodynamics—only very simple systems consisting of a given amount of thermodynamic fluid or working substance which exerts a pressure on its surroundings.

If such a system is *at rest,* the only way it can do work on its surroundings is by a change in volume under this pressure, and the application of the first law equation (1) then gives us simply

$$dE_0 = dQ_0 - p_0 dv_0 \tag{2}$$

where the subscript ($_0$) has been added to indicate that the quantities involved are all referred to coordinates in which the system is at rest.

If such a system is *in motion,* however, its momentum will in general change with its energy content even though we hold the velocity constant, owing to the special relativity relation which associates mass with energy. Hence in applying the first law equation to moving systems, even in the simple case of constant velocity, we shall have to include, in addition to the work done against external pressure, the work done against the external force involved in the change in momentum. We must then write in general

$$dE = dQ - p\,dv + \bar{u} \cdot d\bar{G} \tag{3}$$

where the last term is the scalar product of the velocity of the system $\bar{u}$ and its change in momentum $d\bar{G}$. Moreover, in making use of this equation we must employ the special relativity relation connecting the momentum of the system with its energy flow

$$\bar{G} = \frac{E + p\,v}{c^2}\bar{u} \tag{4}$$

where c is the velocity of light, and the term $\frac{E\bar{u}}{c^2}$ gives the momentum due to the transport of the energy of the system as a whole, and the term $pv\bar{u}/c^2$ corresponds to the additional flow of energy resulting from the work done on the moving volume by the action of the external pressure.

With the help of these two expressions for the first law (3), and for the momentum of a moving system (4), we can then obtain transformation equations which will give us expressions for all the quantities, involved in the application of the first law to moving systems, in terms of the analogous quantities as they would be measured by a local observer moving with the system. In accordance with the known equations for force, and the Lorentz contraction for moving volumes, we are already provided by the special theory of relativity with the simple transformations for pressure and volume

$$\begin{aligned} p &= p_0 \\ v &= v_0 \sqrt{1 - u^2/c^2} \end{aligned} \tag{5}$$

Furthermore, considering first an adiabatic acceleration in which the velocity of our system is changed without flow of heat or change in internal condition as measured by a local observer, and then considering more general processes in which flow of heat is permitted, we readily obtain as the transformation equations for energy and heat the two expressions

$$\begin{aligned} E &= \frac{E_0 + p_0 v_0}{\sqrt{1 - u^2/c^2}} \\ dQ &= dQ_0 \sqrt{1 - u^2/c^2} \end{aligned} \tag{6}$$

This gives all the apparatus necessary for the application of the first law of thermodynamics to moving systems. It is to be specially noted that so far no new assumptions, beyond those already present in the mechanics of special relativity, have been introduced into our systems of thermodynamics, except, if you wish, our procedure in still giving the name heat to that part of the energy transfer which does not take place through the work done against macroscopic external forces.

(*b*) *Special Relativity and the Second Law of Thermodynamics.* Let us now turn to the more characteristically thermodynamic considerations involved in the application of the second law of thermodynamics, and examine the fate of this principle when the extension to special relativity is made.

In the classical thermodynamics the full content of the second law could be conveniently condensed into the very simple expression

$$\Delta S \geqq \int \frac{dQ}{T} \tag{7}$$

where the left-hand side gives the increase in the entropy content S when a system changes from one state to another, and the right-hand side is to be obtained by dividing each element of heat dQ absorbed by its temperature T, and summing up for the whole process by which the system changes from its initial to its final state.

The sign of equality (=) in this expression applies to reversible processes which take place with that highest possible efficiency, which would just be sufficient to permit a return *both* of the system and its surroundings to their original state. And the sign of inequality (>) applies to those less efficient, actual processes which we ordinarily encounter in nature. With the help of the relation of equality we can then calculate the entropy of any system by considering an ideal reversible process by which it could be brought from its standard state to the state under consideration. And with the help of the two relations of equality and inequality, we codify all that extraordinary range of information as to the equilibrium and efficiency of physical-chemical processes which is subservient to the second law.

In making the extension to special relativity, it was found possible to take over this expression for the second law of thermodynamics as a postulate without any change at all in form. And this was evidently a rational thing to try to do, since it preserves the constancy of entropy for purely mechanical processes, makes the increase of entropy for reversible thermal processes dependent on the transfer of energy in forms other than work, and retains with the help of the sign of inequality those opportunities for irreversibility and spontaneous increase in entropy which lie at the heart of thermodynamic considerations.

In applying this expression to moving systems we must of course substitute values for entropy, heat and temperature which are appropriate for a moving system, and hence shall desire transformation equations which will permit us to calculate these quantities in terms of the analogous quantities which would be directly measured by a local observer traveling with the system in question.

In the case of heat, we are already provided by the application of the first law with the transformation equation

$$dQ = dQ_0 \sqrt{1 - u^2/c^2} \tag{8}$$

In the case of entropy, we are then directly led by the postulate itself to the conclusion that the entropy of a system must be an invariant for the Lorentz transformation

$$S = S_0 \tag{9}$$

owing to the possibility of changing the velocity of a system by a quasi-static reversible adiabatic acceleration, which leaves the internal state and proper entropy $S_0$ unaltered on account of the quasi-static character of the acceleration, and leaves the entropy S unaltered on account of the reversible and adiabatic character of the acceleration. This invariance of entropy is, moreover, in evident agreement with the statistical mechanical interpretation which relates the entropy of a system to the probability of its state, a quantity which could hardly be a function of the velocity with which the observer happens to be moving past the system.

Finally in the case of temperature, by combining the requirements of the postulate itself with the two transformation equations already obtained, it is evident that we are now necessarily led to the relation

$$T = T_0 \sqrt{1 - u^2/c^2} \tag{10}$$

in order that the postulated law (7) may apply to the description of a given change in state both from the point of view of a local observer moving with the system and from the point of view of other observers with respect to which the system is in motion.

(*c*) *Discussion of the Extension to Special Relativity.* This completes all that is necessary for the extension of thermodynamics to special relativity.

It will be seen that the additions in the way of new hypotheses, beyond what is already contained in the special theory of relativity and in the classical thermodynamics, have really been very small and apparently rational. Indeed, it seems fair to say that these additions consist solely in the assumption that the second law of thermodynamics, as expressed in the usual well-known form given by (7) will not break down when we turn to the consideration of moving systems, and that the quantity dQ occurring in this expression must still be interpreted as that part of the energy transfer which can not be considered as work done against macroscopic external forces.

It should also be noted that the results which are given by this extended theory are entirely coherent with the accepted body of theoretical physics. For example, the application of this theory to determine the dynamical properties of a moving enclosure filled with black-body radiation leads to the same results as were originally obtained by Mosengeil[4] from strictly electromagnetic considerations. And the transformation equation given for heat (6), which we have regarded as derived from an application of the mechanics of special relativity to the behavior of a portion of fluid, agrees with that which can be derived from electromagnetic considerations for the Joule heating effect in a moving electrical conductor. Most important of all, however, it should be noted that the extension has been so devised that any predictions, which we make with its help as to the behavior of a given system moving with a *constant* velocity u, will completely agree with those which would be made with the help of the classical thermodynamics by a local observer who moves along with the system in question.

It is important to emphasize these qualities of rationality and coherence, since our judgment as to the validity of this extension of thermodynamics must be largely based thereon. Any direct test of the extension would for the present be out of the question,

[4] Mosengeil, *Ann. d. Phys.*, 22: 867, 1907. The results of Mosengeil were employed by Planck in his method of obtaining the extension of thermodynamics to special relativity.

since all the various thermodynamic quantities for moving systems were found to differ from the analogous ones for stationary systems only by terms of the order of $u^2/c^2$ or higher, and we could only expect differences of this practically undetectable order for any thermodynamic theory of moving systems that might be proposed.

The usefulness of the extension consists partly in the ease with which we can now treat problems by simple thermodynamic methods which would otherwise involve complicated kinetic theory or electromagnetic considerations, as in the case of the moving enclosure filled with radiation. The usefulness of the extension depends mainly, however, on the increased insight which we now have into the nature of thermodynamics and thermodynamic quantities. Thus the invariance to the Lorentz transformation for entropy and for the ratio of heat to temperature provided by the special theory of relativity prove essential for the further extension of thermodynamics to general relativity to which we must now turn.

### (4) The Extension of Thermodynamics to General Relativity

In the general theory of relativity, the space-time continuum in which physical events take place is regarded as characterized by the formula for interval

$$ds^2 = g_{11}\,dx_1^2 + 2\,g_{12}\,dx_1\,dx_2 + \ldots + g_{44}\,dx_4^2 = g_{\mu\nu}\,dx^\mu\,dx^\nu \qquad (g_{\mu\nu} = g_{\nu\mu}) \tag{11}$$

where $x_1$, $x_2$ and $x_3$ are the three spatial coordinates that are being used, $x_4$ is the temporal coordinate, and the $g_{\mu\nu}$ are the ten gravitational potentials. The dependence of these gravitational potentials on the distribution of matter and energy is given by Einstein's ten field equations

$$-8\pi T^{\mu\nu} = R^{\mu\nu} - \tfrac{1}{2} R g^{\mu\nu} + \Lambda g^{\mu\nu} \tag{12}$$

where $T^{\mu\nu}$ is the energy-momentum tensor, $R^{\mu\nu}$ and $R$ are obtained from the Riemann-Christoffel tensor by contraction, and $\Lambda$, the so-called cosmological constant, is a quantity which is observationally known in any case to be exceedingly small when expressed in reciprocal square centimeters, and may well be zero. Finally, the motion of free particles and light rays in this space-time continuum is determined by the equation

$$\delta \int ds = 0 \tag{13}$$

with ds greater than zero for material particles and equal to zero for light rays.

The results predicted by these fundamental equations of general relativity are in satisfactory agreement with all the facts that are now at our disposal, and in particular agree with the astronomical observations which have furnished the three crucial tests of relativity.

In order to include thermodynamics within this framework, we must now inquire into the analogues in general relativity of the ordinary first and second laws of thermodynamics.

(*a*) *The Analogue of the First Law in General Relativity.* In the case of the first law the procedure to be adopted is clear. In the classical thermodynamics the first law was an expression of the principle of the conservation of energy as applied to small stationary systems in the absence of a gravitational field, and in relativistic thermodynamics we must evidently use as the analogue of the first law the more general energy-momentum principle provided by relativistic mechanics.[5]

This principle can be expressed by the very simple *tensor* equation

$$(T^{\mu\nu})_\nu = 0 \tag{14}$$

and may be regarded as an immediate result of Einstein's field equations (12), since the tensor divergence of the expression there given for the energy-momentum tensor $T^{\mu\nu}$ can be shown to be necessarily identically equal to zero. For purposes of computation it is often more convenient to rewrite this equation in the *tensor density* form

$$\frac{\partial \mathfrak{T}_\mu^\nu}{\partial x^\nu} - \tfrac{1}{2}\,\mathfrak{T}^{\alpha\beta}\frac{\partial g_{\alpha\beta}}{\partial x^\mu} = 0 \tag{15}$$

and to obtain an insight into the nature of the principle, it is sometimes useful to rewrite it in the form of an ordinary divergence as expressed by the nontensor yet nevertheless *covariant* equation

$$\frac{\partial}{\partial x^\nu}(\mathfrak{T}_\mu^\nu + \mathfrak{t}_\mu^\nu) = 0 \tag{16}$$

where the pseudo-tensor density of potential energy and momentum $\mathfrak{t}_\mu^\nu$ is defined for *all systems* of coordinates in such a way that we can substitute $\partial \mathfrak{t}_\mu^\nu / \partial x^\nu$ for the second term of (15).

To remind us of the physical significance of these familiar equations of relativistic mechanics, it will be recalled that the equations reduce in the absence of a gravitational field to the ordinary principles of special relativity for the conservation of the energy and momentum directly associated with matter and radiation. In general, however, in the presence of gravitational fields, it will be evident from the third form (16) in which the equations have been written that they will lead to conservation laws only when we include—along with the energy and momentum directly associated with matter and radiation—the potential energy and momentum of the gravitational field, which corresponds to the presence of the pseudotensor density $\mathfrak{t}_\mu^\nu$ in the equation in this form (16).

This general result proves to be of great importance

[5] Tolman, *Proc. Nat. Acad.*, 14: 268, 1926; *Phys. Rev.*, 35: 875, 1930.

for relativistic thermodynamics by permitting—even in the case of isolated systems—an increase in the energy directly associated with matter and radiation at the expense of the potential energy that we assign to the gravitational field. For example, if we consider a system composed of a perfect fluid having the proper macroscopic density of energy $\rho_{00}$ and proper pressure $p_0$ as measured by a local observer at the point of interest, and having no flow of heat, it is known that we can write

$$T^{\mu\nu} = (\rho_{oo} + p_o)\frac{dx^\mu}{ds}\frac{dx^\nu}{ds} - g^{\mu\nu} p_o \qquad (17)$$

as an expression for the energy-momentum tensor. And if we substitute this expression into the above equations of relativistic mechanics, we can obtain for any infinitesimal element of the fluid of proper volume $\delta V_0$, the relation

$$\frac{d}{dt_o}(\rho_{oo}\,\delta V_o) + p_o\frac{d}{dt_o}(\delta V_o) = 0 \qquad (18)$$

From one point of view there is nothing surprising about this result since it merely states that a local observer who examines the behavior of an element of the fluid small enough so that the gravitational "curvature" of space-time can be neglected will find the rate of change in energy content related in the expected way to the work done against the external pressure. From another point of view, however, as this same equation can be applied to each one of all the elements into which the total fluid of the system can be divided, the result may seem somewhat surprising, since it leads to the possibility of systems in which the proper energy of every element of the fluid may be simultaneously decreasing or increasing, according as the system is expanding or contracting. Moreover, since it is this proper energy immediately associated with matter and radiation which determines the possibilities for entropy increase, we shall later find in relativistic thermodynamics an escape from certain restrictions imposed in the classical thermodynamics by the usual form of the principle of the conservation of energy.

Just as in the previous case of special relativity, we note that the extension of thermodynamics to general relativity involves, so far as the first law is concerned, no new hypothetical material beyond that already contained in relativistic mechanics. And we may now turn to the relativistic analogue of the second law of thermodynamics.

(*b*) *The Analogue of the Second Law in General Relativity.* To guide us in obtaining a suitable postulate to serve as the relativistic second law of thermodynamics, we must make use of the two fundamental ideas of general relativity which are expressed by the principles of covariance and equivalence. In accordance with the principle of covariance, our postulate must be expressed in covariant form the same for all coördinate systems, to avoid the danger of being influenced in its selection by a spurious simplicity when referred to some particular system of coordinates. And in accordance with the principle of equivalence, our postulate must reduce to the thermodynamic requirements of special relativity, when applied to an infinitesimal element of fluid using natural coordinates for the point of interest.

These two principles have been sufficient to lead with considerable confidence to the expression[6]

$$\frac{\partial}{\partial x^\mu}\left(\phi_o\frac{dx^\mu}{ds}\sqrt{-g}\right)\delta x_1\,\delta x_2\,\delta x_3\,\delta x_4 \geqq \frac{\delta Q_o}{T_o} \qquad (19)$$

as the appropriate postulate to take as the relativistic analogue of the ordinary second law of thermodynamics. The quantity $\phi_o$ in this expression is the proper entropy density of the thermodynamic fluid under consideration as measured at the point of interest by a local observer; the quantities $dx^\mu/ds$ are the components of the macroscopic "velocity" of the fluid at that point; and the other quantities on the left-hand side of the expression have their usual significance. The significance of the right-hand side of the expression is more difficult to grasp, and will be specially treated in a forthcoming article by Professor Robertson and myself.[7] The quantity $\delta Q_0$ may be taken as the heat—measured by a local observer at rest in the fluid at the point of interest—which flows into an element of the fluid having the instantaneous proper volume $\delta V_0$ during the proper time $\delta t_0$ where these quantities are so chosen as to make

$$\delta V_o\,\delta t_o = \sqrt{-g}\,\delta x_1\,\delta x_2\,\delta x_3\,\delta x_4 \qquad (20)$$

and the quantity $T_o$ is taken as the temperature ascribed to this heat by the local observer.

The two signs of equality (=) and inequality (>) in the expression refer respectively to the two cases of reversible and irreversible processes, and in applying the principle to irreversible processes we are to regard an increment in coordinate time $\delta x_4$ as positive when taken in the direction to correspond to a positive increment in proper time $\delta t_0$ as measured in the ordinary manner by a local observer.

To show the agreement of this postulated expression for the relativistic second law with the principle of covariance, we have merely to note that it is a tensor equation of rank zero—both sides being scalar invariants—and hence is true in all coordinate systems if true in one. To show its agreement with the principle of equivalence we must see what it reduces to in natural coordinates for the point of interest. Intro-

[6] Tolman, *Proc. Nat. Acad.*, 14: 268, 701, 1928; *Phys. Rev.*, 35: 896, 1930.

[7] Tolman and Robertson, submitted to the *Physical Review* for publication.

ducing such coordinates x, y, z, t, *and making use of the transformation equations for entropy, heat and temperature provided by the special theory of relativity,* we find, however, that our principle then reduces to

$$\left[\operatorname{div}(\phi\bar{u}) + \frac{\partial\phi}{\partial t}\right]\delta x\,\delta y\,\delta z\,\delta t \geqq \frac{\delta Q}{T} \qquad (21)$$

where $\phi$, $\bar{u}$, $\delta Q$ and T are now the quantities referred to our present coordinate system which we ordinarily designate as entropy density, velocity, heat absorbed and temperature. And we see that this result does relate the change in the entropy of the element of fluid, instantaneously contained in the coordinate range $\delta x\ \delta y\ \delta z$, to the absorbed heat and temperature in the way

$$\frac{d}{dt}(\phi\,\delta x\,\delta y\,\delta z)\,\delta t \geqq \frac{\delta Q}{T} \qquad (22)$$

which is required by the second law of thermodynamics in special relativity.

At the present stage of observational knowledge, our belief in the validity of the proposed postulate is primarily based on this agreement with the two principles of covariance and equivalence. In addition, however, it may be emphasized that the principle has been chosen so as to be simply the immediate covariant re-expression of the special relativity form of the second law (21); and past experience has shown, notably, for example, in the cases of the fundamental formulae for space-time interval and geodesic trajectory, that these simplest possible covariant generalizations when feasible are likely to be correct. Furthermore, it may be remarked that the conclusions which have so far been drawn from this extension of thermodynamics to general relativity appear—at least after due reflection—to be reasonable and illuminating.

It must be emphasized, nevertheless, that these qualities are not sufficient to prove the validity of the postulate, since other covariant expressions might be found which would also reduce to the special relativity law in natural coordinates. Hence the postulate must be regarded as a real generalization with a range of validity to be finally determined only by the correspondence between observation and prediction.

*(To be concluded)*

# STUDIES IN NUCLEAR PHYSICS

By Dr. JNO. A. FLEMING

ACTING DIRECTOR, DEPARTMENT OF TERRESTRIAL MAGNETISM, CARNEGIE INSTITUTION OF WASHINGTON

## STUDIES IN NUCLEAR PHYSICS

THE Carnegie Institution of Washington announced on February 2 in a lecture by Dr. M. A. Tuve before the Franklin Institute on "The Atomic Nucleus and High Voltages," the results of the past year's work at the high-voltage laboratory of its Department of Terrestrial Magnetism in Washington. This program of investigations constitutes the present expression of a long-continued policy of the department of undertaking laboratory studies of the basic physical phenomena which underlie all large-scale manifestations of magnetism and electricity, as complementary to its field and observatory operations and theoretical investigations. These laboratory studies are now directed toward studies of the simplest cases of the interaction at close distances of the known primary material particles—the electron and proton, which have both electric charge and magnetic moment—and their simplest combinations.

During the past year Dr. Tuve and his colleagues, L. R. Hafstad and O. Dahl, have obtained results covering a variety of experiments in nuclear physics, including a verification of the existence of the recently discovered neutron, observations on the resonance-disintegration of aluminum by polonium alpha-particles, and studies of the disintegration of lithium and boron, using high-speed protons from a high-voltage tube. The Van de Graaff type of electrostatic generator, a simple metal sphere charged to a high voltage by a silk belt, was tested, developed and used for the atomic disintegration-studies. A special building is now under construction at the department to house a large (2-meter) generator of this type which was built at the department and tested during May, 1932. This equipment promises to give an intense source of artificial neutrons, this being one of the most interesting of its numerous potentialities. It will provide 10 microamperes or more of protons or helium-ions having energies above 1,500,000 electron-volts.

The existence of the non-classical phenomena at present explained on the neutron-hypothesis was verified in the department's laboratory last September by a repetition of the main features of Chadwick's experiments. Using a 3-mc polonium source bombarding beryllium, recoil-nuclei of nitrogen were found to produce a maximum of nearly 50,000 ions in the 15-mm ionization-chamber of a valve-counter of the type used by Chadwick. Recoil-nuclei were observed with this instrument and with the FP-54 pliotron connected to a small chamber and used in the same way as Pose has used the duant electrometer. The