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THE CONSTITUTION OF THE STARS[1]

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To number and name the stars is easy enough—or would be so if there were not so many of them; but to determine their real nature, and discover how and why they shine, is a task which, though well begun, is not more than half done.

(1) THE PROPERTIES OF THE STARS

We have many ways of gaining information about the outside of a star. First and foremost, by collecting its light with a telescope and feeding it into a spectroscope, we learn that the stars, like the sun, are self-luminous incandescent bodies surrounded by atmospheres which contain the familiar chemical elements in a gaseous state. All the elements which are most abundant on earth have been found in the stars, and many of the rare ones—more and more as more powerful instruments can be applied—and few unidentified spectral lines remain, so that we can be sure that the stars are essentially similar in composition to our own world. No conclusion of science is more significant than this. The poet Stedman has expressed its meaning better in verse than any technical prose could render it.

> White orbs like angels pass
> Before the triple glass
> That men may scan the record of each flame,—
> Of spectral line and line
> The legendry divine
> Finding their mould the same, and aye the same,
> The atoms that we knew before
> Of which ourselves are made,—dust, and no more.

The materials of nature, and her laws, are the same everywhere. Upon this foundation we build.

[1] First Maiben Lecture before the American Association for the Advancement of Science, given at Atlantic City on December 30, 1932.

Applying these laws to our observations of the stars' light, we may calculate the temperatures of their surfaces—for the hotter an incandescent body is the more blue light it emits in proportion to red. Comparison with laboratory sources shows that the stars begin about where these leave off—very few having surface temperatures less than 3,000° and the great majority ranging from three up to fifteen or twenty thousand degrees (on the Centigrade scale). Some exceptional bodies may reach 50,000°. These conclusions are checked by other methods and entitled to confidence. At even the lowest of these temperatures, all but the most refractory chemical compounds are decomposed into their elements—which immensely simplifies the physical situation—and, higher up in the scale, the atoms themselves lose one or more of their outer electrons, becoming ionized and giving new and different spectra. The conspicuous differences between the spectra of the hotter and cooler stars originate from changes of this sort, combined with other and equally well-understood effects of temperature. When allowance is made for these, it is found that, despite the apparent diversity, the stars, or, at least their atmospheres, are remarkably similar in composition. The relative proportions of atoms of different kinds are very similar to those which are found in the earth's crust, with one important exception. Hydrogen, which forms but a small proportion of the earth's mass, is more abundant in the sun and stars than anything else. Helium is also abundant. It is reasonable, by the way, to suppose that these light gases escaped from our planet (which has not enough gravitative power to hold them) shortly after its birth as an independent body.

Finally, from the temperature and degree of ionization in stellar atmospheres, it follows that the pressure must be very low—something like a thousandth part of that of the air which surrounds us at sea-level. The pressures are different, of course, in different stars, but are always small.

Why we do not see down into regions of greater density and pressure was once puzzling; but it is now known that an ionized atmosphere, full of free electrons and charged atoms, will scatter light like a thin fog. This prevents us from seeing down deeper. The calculated temperature represents an average for the partly foggy layers toward the bottom, from which most of the observable light comes.

All this information could be obtained by spectroscopic means, if we knew nothing about the distances of the stars. Measurements of distance, which have now been made in various ways, for thousands of them, give us new data. The real brightness, or luminosity, shows a remarkably wide range. Our sun is not far from the middle of this range, and forms a convenient standard. Stars 100 times as bright are common. Many are known to be more than 1,000 times as bright, and a few give more than 10,000 times the sun's light. On the fainter side stars down to 1/100 and even 1/1,000 of the sun's brightness are common and a few are known which give but 1/10,000 of its light. Only the very nearest of these are visible to us, even with good-sized telescopes, so that there is not much chance of picking up the still fainter ones which probably exist. Very bright stars, on the contrary, can be seen at enormous distances, and our failure to find them indicates that very few such objects exist. There are, of course, various ways of measuring luminosity—with the eye, which is sensitive mainly to yellow and green light, with photographs, which work in the violet, and with heat-measuring apparatus, which is sensitive to all radiations alike. The corrections necessary to reduce measures on the other scales to the last (which is obviously the best for theoretical studies) can be calculated when the star's temperature is known.

If we know the luminosity and temperature of a star, we can at once calculate its size—its actual diameter in miles. Once more our results can be checked, by Michelson's beautiful interferometer method, and by various others, and it is found that the unavoidable assumption that the hazy indefinite lower atmosphere behaves like a hot solid surface, of standard properties, at a suitable average temperature, leads to reliable results. The stars differ less in diameter than in brightness. The general run of stars have diameters ranging from three or four million miles to about half a million (comparable with the sun's 866,000). Some are much larger, from ten million miles to 500 million in extreme cases, while a few are abnormally small, from 50,000 miles to 10,000 or even less.

Direct information about the interior of a star can be gained in only one way—by means of gravitation. Here, and here alone, the inner portions exert their full action, quite independent of their envelopement by great masses of matter. At ordinary interstellar distances, the mutual gravitation is quite negligible; but, fortunately, there are numerous binary pairs in orbital motion controlled by gravitation. For many such systems we can derive reliable values of the masses. These differ far less widely than the sizes or luminosities. The greatest mass so far measured is 113 times that of the sun, the least 0.18—a ratio of 700—as against 50,000 for the diameters, and more than 100,000,000 for the luminosities. Why this should be true is a problem for the theorist, which, as we shall see, has been successfully answered.

In density, the stars differ amazingly. Some stars have more than 100,000,000 times the sun's volume, and probably less than 100 times the sun's mass. The density of such a star is of the order of a thousandth part that of ordinary air. Those of smallest diameter are massive, and some of them have densities as high as 50,000 times that of water—almost a ton per cubic inch! How such tenuous bodies as the first can hold together and shine and how any material substances can be as dense as the second are other problems which have been satisfactorily solved.

These calculations tell us only the average density of the material inside a star. How much denser it is in some parts than others can be determined only in a few favorable instances—double stars whose components revolve in an eccentric orbit and, being separated by but a few diameters, are pulled out into ellipsoidal shapes by their mutual attraction. In this case the periastron (or point of closest approach) moves slowly forward at a rate which depends not only on the sizes and masses of the stars, but on the extent to which the density at the center exceeds the average. Only one system (Y Cygni) has so far been carefully studied. In this it appears that the concentration of density toward the center is large.

Barring this rare good fortune, all that we can find out observationally about a single star, in addition to its chemical (or atomic) composition, may be expressed by three numbers defining its mass, its radius and its luminosity. From these the density and the surface temperature may immediately be computed.

(2) RELATIONS BETWEEN THESE PROPERTIES

These properties of the stars are not distributed at random, but are related among themselves in very striking ways. The simplest and perhaps the most important of these correlations is that between mass and luminosity, discovered by Eddington. Stars of large mass are always bright, and stars of small mass faint. A star of the sun's mass has nearly the sun's brightness; one of 4 times the sun's mass gives about 80 times as much radiation, and one of 16 times the mass 1,600 times as much, while one of ¼ the sun's mass radiates only about 1/200 as much heat.

Strangely enough, the size of a star seems to have very little influence on this relation. Red stars, fifty times the sun's diameter, and white stars only two or three times as big as the sun, if of the same mass, have nearly the same luminosity. Indeed, if a curve is plotted, representing the relation just indicated, with points corresponding to the data for individual stars, the deviation of the latter from the smooth curve averages hardly greater than the effects of the outstanding error of observation. There is but one exception. The stars of very small diameter, and very great density, are much fainter than others of equal mass—from 1/10 to 1/100 as luminous.

The luminosity and surface temperature (or the closely related spectral type) of a star are also correlated, but in a more complicated way, best illustrated by a diagram. Fig. 1, prepared from observations

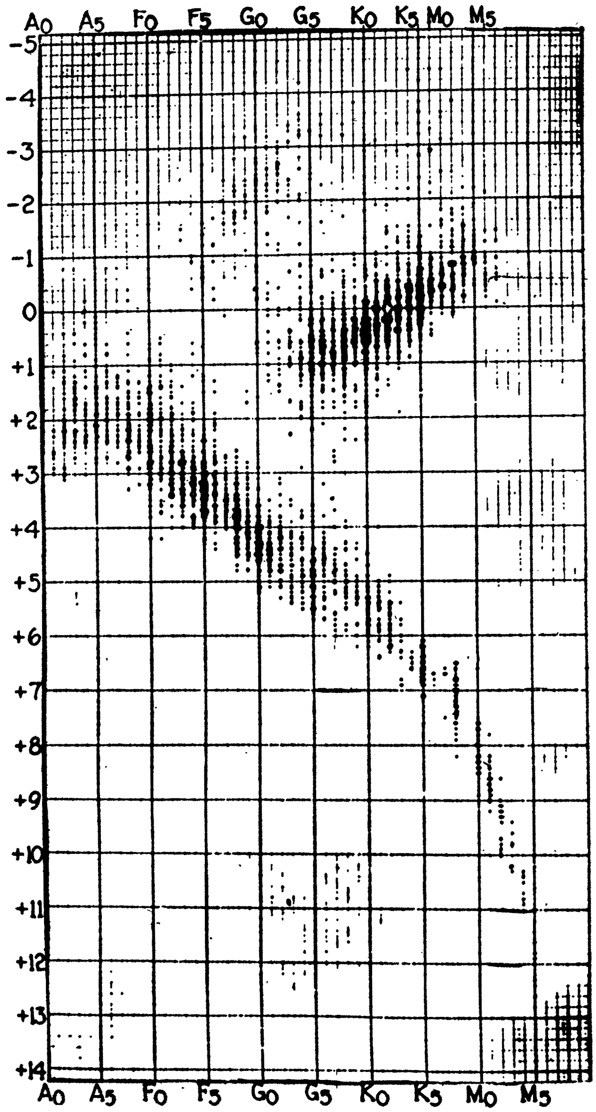

FIG. 1

of about 4,000 stars by Adams and his colleagues at Mount Wilson, shows the brightness (on the familiar scale of stellar magnitude) plotted against the spectral type—each dot representing one star. The points are very far from scattered at random. The majority of them congregate in a narrow band, crossing the diagram downward and to the right. Along this line, called the *main sequence,* we pass from bright, hot and massive stars, at the top, to faint, cool bodies of small mass at the bottom.

The hottest stars of all are not shown in the figure. They extend the sequence upward and to the left.

The sun belongs to this sequence, and is a fairly typical member. Though so different in brightness and in mass, these stars are fairly similar in size. A member near the top might be of 10 times the sun's mass, 4½ times its diameter, 1/10 its density, 800 times its luminosity and 2½ times its surface temperature. One near the bottom might have about ¼ the sun's mass, 3/10 its diameter, 9 times its density, 1/180 of its luminosity and half its surface temperature. There is an unbroken series of stars all the way between these types, and even beyond them, and the whole obviously forms a natural family. The great range in brightness within it arises mainly because the radiation from equal areas varies as the fourth power of the surface temperature.

A second sequence of stars is represented on the upper right-hand part of the figure. Hertzsprung's name of "giant stars" is very appropriate, for they are of gigantic size. Capella, which belongs at the lower left of this sequence, has in round numbers 4 times the sun's mass, 125 times its brightness, 14 times its diameter, 90 per cent. of its surface temperature and 1/700 of its density. A red giant at the upper end would have some 10 times the sun's mass, 1,500 times its luminosity, 150 times its diameter, ½ its surface temperature and less than 1/200,000 of its density. Here is evidently another natural family of stars. According to the data represented on our diagram, they are rather sharply separated from the main sequence, but Strömberg's calculations, based on other data, indicate that the gap is partially filled up.

A number of very bright stars are represented by scattered dots near the top of the diagram. These are the "super-giants" of which examples are found in almost every class of spectrum. A fourth important class of stars, the white dwarfs, would be represented by points in the lower left-hand corner. They are far fainter than those main-sequence stars of the same temperature, and so must be much smaller—indeed, their computed diameters range from 30,000 to 8,000 miles, no larger than the planets. Their masses, which can fortunately be determined in a few critical cases, are considerably larger than those of main sequence stars of the same luminosity, and their densities come out extraordinarily great, up to a ton per cubic inch, as was already mentioned. Few of these stars are known; but they are so faint that only those which lie very near the Sun (as stars go) can be adequately observed. There must be thousands of others, visible in great telescopes, but lost among the countless swarms of faint stars. This is typical of the effects of observational selection which meet the student of the stars wherever he turns, and demand eternal vigilance, lest they lead him astray.

(3) The Problem to be Solved

The great task of theoretical astrophysics is to interpret these properties of the stars—individual and collective—by means of the general physical properties of the matter of which they are composed. Here we are greatly aided by the high temperatures of the stars. Chemical compounds are decomposed; hence we have to deal only with the properties of atoms, and of course of electrons and protons, and of light-quanta. Hence, the more we know about atoms, the more we can find out about the stars. The minutest things in nature give us the key to unlock the secrets of the greatest.

We must ask the physicist to answer three main questions:

(1) What is the *equation of state* inside the star—the relation connecting the pressure with the density and temperature?

(2) What is the *opacity of the material* which regulates the flow of heat from the intensely hot interior to the surface, and supplies the loss by radiation into space?

(3) By what processes, and according to what laws, is the *internal supply of heat* kept up, so that the star does not gradually cool down and go out?

We have now a satisfactory answer to the first question, and a fairly good one to the second; but only a beginning has been made with the third, though there is hope that we may know more soon. If we had this third answer we could predict, from general principles, what the stars ought to be like, and, if our knowledge were adequate and our reasoning sound, our predictions would agree with the facts. As things are, our partial knowledge enables us to interpret many of the most important properties of the stars, though not all.

(4) First Success. The Mass-Luminosity Relation

Let us begin with the equation of state. Since the material is gaseous, we have by the familiar gas laws

$$p = R\,\varrho\,T/m \qquad (1)$$

(p is the pressure, ϱ the density, T the absolute temperature, m the average molecular weight, and R a constant—8.31×10^7 in c.g.s. units.) The density inside a star will be greater at some places, and less at others, than the mean. To get an idea what to expect, suppose we had an imaginary star of the sun's size and mass, and with uniform density throughout. The pressure at the center then comes out 9,800,000 tons per square inch, and the corresponding temperature, by the gas laws, is 11,500,000 m degrees Centigrade. Under ordinary laboratory conditions, the molecular weight m would vary enormously with the composition of the gas. But inside the star the atoms must be almost completely ionized, and the heavier an atom is the more parts it is split into. The *average* weight of one of these parts (which is what we need) is 0.5 for hydrogen, 1.3 for helium and about 2 for all the other elements, except the heaviest (which are too rare to be of importance). The general average thus depends almost entirely on the proportion of hydrogen inside the star. We shall see later that there is evidence that this is such as to make the value of m about 1.

For a homogeneous body of M times the sun's mass and r times its radius, the central temperature must be multiplied by $\frac{M}{r}$. If the body is not homogeneous, let x be the ratio of the density at any point to the mean density, and y the ratio of the pressure at this point to the pressure at the center of a homogeneous sphere of the same size and mass. A very simple calculation then shows that the temperature T is given by

$$T = 11{,}500{,}000 \frac{M}{r} \frac{y}{x} m. \qquad (2)$$

We can now calculate the temperature at any point inside the star, if we know what it is made of, that is, the value of m, and on what model it is built, that is, how the quantity x varies as we pass from the center to the surface. (If this is known y can be calculated.)

The density may increase greatly toward the center, making x very large there, but in this case y also increases. For example, there is a famous "model," discussed by Eddington, in which, at the center $x = 54$, that is, the central density is 54 times the mean density. But at the same point $y = 93$, so that the temperature is only 1.72 times as great as for a homogeneous sphere. Nearer the surface x and y both fall off, and the temperature decreases. It follows from this that, unless the central condensation is much greater than suggested above, the central temperature of the sun is about 15 or 20 million degrees. A generation ago no one would have dared to say much about the properties of matter under conditions so far removed from experience. But we know enough about atoms now to be sure of our ground.

At these high temperatures, the flying quanta of radiation, as well as the flying atoms and electrons, tend to drive the atoms which they hit outward and exert a powerful pressure. At 20 million degrees this radiation pressure amounts to 3,000,000 tons per square inch—enough to support a considerable fraction of the gravitational pressure inside a star. This was first realized by Eddington. If β is the fraction of the whole pressure which is carried by the gas, leaving $1 - \beta$ for the radiation, he derived the remarkable equation

$$\frac{1-\beta}{\beta^4} = 0.0033 \frac{y^3}{x^4} m^4 M^2 \qquad (3)$$

which enables us to compute the effect at any point in any star. The temperatures previously calculated must be multiplied by β. For the sun's center (with $m = 1$), $\beta = 0.97$ for the homogeneous model, and 0.997 for Eddington's model, so that the correction is small. For massive stars, it may be considerably greater.

The outcome of this discussion is that we have gained a good idea of the central temperatures of the stars. The value for the sun is probably 15 to 20 million degrees; for a faint star at the bottom of the main sequence, 12 to 15 million; for a bright star at its top, 25 to 35 million (allowing for radiation pressure). For the giant stars, the values are much lower, about 5 million for Capella and one million for a red star at the other end of the group.

For a star built on any given model, we can now find the outward temperature gradient at any point. This, with the opacity of the material, determines the rate of outward flow of heat—and the latter evidently supplies the radiation from the surface, which can therefore be calculated. The opacity arises mainly from the capture of radiation by atoms, which hold it for a moment before they re-emit it, and its law can be determined from the principles of quantum- and wave-mechanics. The researches of Kramers, Gaunt and others have now put the theory of opacity on an apparently reliable basis, though it is not yet exact. Passing over many details, it is found that the luminosity L of a star built on any model is given by the equation

$$L = C \frac{M^{5\frac{1}{2}}}{r^{\frac{1}{2}}} (m\,\beta)^{7\frac{1}{2}} \qquad (4)$$

where the coefficient C depends upon the model on which the star is built, but does not change rapidly with changes in this.

The equation shows that the luminosity of a star should depend but little on its size, and very greatly on its mass. For large masses, when β begins to diminish, the change is proportionately slower. This is an excellent qualitative description of the mass-luminosity relation, and numerical calculations show that the quantitative agreement is remarkably close. Eddington, who first worked the theory out, determined the constant C (which could not at that time be accurately calculated theoretically), from a single star, Capella, and found that the resulting formula represented the data for all reliably observed stars (except white dwarfs) in an almost uncanny fashion.

Theoretically, a star of given mass may have *any* luminosity, adjusting its size to suit. But to halve L multiplies r by 4, and reduces the surface temperature to 42 per cent. of its original value, so that any large change in luminosity, up or down, would place our hypothetical body outside the observed range of stellar surface temperatures. Within this range a rather small correction (carefully made by Eddington) serves to reduce the observed brightness of a star to the value it would have if shrunk or expanded so that its surface temperature was equal to the sun's. This correction hardly ever amounts to one stellar magnitude, while the observed range in brightness is fully 20 magnitudes.

Changes in the model on which we assume our star to be built do not greatly affect its luminosity. Eddington, Vogt and Biermann have discussed a large number of cases, and found that the effect very rarely exceeds one magnitude. Changes in the mean molecular weight, on the contrary, produce great differences. Recent discussions by Strömgren and Eddington agree in showing that stars of the same mass, size and density-model, differing only in the proportion of hydrogen which they contain, may differ in brightness by a factor of a thousand—more than seven magnitudes. With no hydrogen present, m is about 2, and the central temperature high. This drives a rapid flow of heat to the surface. As the hydrogen percentage increases, m falls, and so does the internal temperature, while the opacity gradually diminishes. At first the falling temperature prevails, and the luminosity diminishes. Finally, when the gas is almost all hydrogen, the opacity diminishes so rapidly that the escape of radiation becomes greater again. The minimum luminosity is reached with about 85 per cent. of hydrogen by weight. Agreement with the observed brightness is reached at a hydrogen content of 30 to 35 per cent., for the sun, Capella and Sirius, independently. There would also be agreement if about 99¾ per cent. of the gas were hydrogen; but this appears to be less probable. This interpretation of the mass-luminosity correlation is the first great triumph of the theory of stellar constitution. Strangely enough, it is almost *too* successful, for it shows that the observed facts result from very general properties of matter, almost independent of the internal constitution of the stars. Concerning the specific model on which they are built, it tells us practically nothing—but it does give us important information regarding the composition of the interior, which was wholly unexpected. Finally, we may note that this part of the theory does not even suggest why a star of given mass should have any particular size or surface temperature.

(5) Second Success. The White Dwarfs

We have so far assumed that the familiar gas-laws can safely be applied to stellar material, even though it is much denser than any known gas. If the atoms were in their normal condition, this would not be true; they are bodies of known size, and would get jammed together at densities little greater than those of ordinary liquids, so that the material was no longer compressible. But, inside the stars, the atoms are highly ionized, and their remaining fragments, as well as the liberated electrons, are exceedingly small, so that there is no danger of their "jamming" even at thousands of times the density of ordinary matter. The realization of this led to the first intelligible interpretation of the white dwarf stars. It had been known for some time that all the data indicated that their densities must be enormous, but this conclusion was supposed to be physically impossible till Eddington pointed out that it was a natural consequence of the high ionization.

The later investigations of Fowler, Sommerfeld and other students of quantum theory show, however, that there is a limit to the degree to which a gas can be compressed. Nature abhors overcrowding, though she does not really abhor a vacuum.

Inside an atom the quantum laws provide only two places for electrons close to the nucleus, eight further out, and so on. Similar quantum restrictions apply to the free particles in a gas. Only a fixed number of *slow-moving* particles can be crowded into a given volume—say a cubic centimeter. To get more in we must add faster moving ones. This increases the average energy of a particle, and hence the pressure. Only the excess of the total energy of motion of the particles, above that tied up in this way, is available to count as heat. The lightest particles—the electrons—show this effect of "degeneracy" first. Being very numerous in the stars, they dominate the situation. When degeneracy has gone to the limit, the equation of state is

$$P = K_1 \rho^{5/3} \qquad (5)$$

Here K_1 is a constant (equal to $9.16 \times 10^{12} \mu^{-5/3}$, where μ is the average mass per free electron, which is 1 for pure hydrogen, and a little greater than 2 for heavy atoms). This equation is safe to use whenever it gives a considerably higher pressure than the perfect-gas law. For a density 1,000 times that of water, this happens when the temperature is less than two million degrees—while at ten million the gas-law is still a fair approximation. At intermediate temperatures the pressure gradually changes from one value to the other. The exact formula for the change has been worked out, but is complicated.

At extremely high densities, the velocities of the electrons become nearly as great as that of light, and the equation of state becomes

$$P = K_2 \rho^{4/3} \qquad (6)$$

where $K_2 = 1.23 \times 10^{15} \mu^{-4/3}$. The transition between this form and the last has also been accurately worked out. It is under way when the density is about two million times that of water.

We can now answer the question, What would become of a star which had lost heat by radiation until all possible sources of internal energy had been exhausted? There would be no more available heat in it, and the matter would therefore be in the degenerate state. The equation (5), together with the law of gravity, then enables us to work out in detail all the properties of the body, since (given the composition, and therefore μ) there are no adjustable constants left. This has been done by Milne. A body of given mass settles down to a definite size and density. With 30 per cent. of hydrogen and a mass equal to the sun's, the radius comes out 1/62 of the sun's, making the diameter 14,000 miles, and the mean density 240,000 times the sun's. The central density is six times as great, or two million times that of water.

Smaller masses are of larger diameter—being less compressed by their own gravitation—so that the volume is inversely proportional to the mass. For large masses, the central density is so high that equation (6) becomes applicable. The compressibility increases, and the size of the body becomes still smaller. For masses greater than twice the sun's, practically the whole interior is in the second stage of degeneracy. The law of this state no longer assigns any limit to the possible contraction and any greater mass would presumably shrink until the protons and electrons themselves were "jammed." The resulting density can only be roughly guessed, but it is probably many billions of times the sun's, so that the shrunken mass would be only a few hundred miles in diameter!

Such bodies, whatever their masses, would be dead stars—cold on the surface, and quite invisible. But a star may be nearly but not quite dead, retaining a small amount of internal heat. In the deep interior this would contribute in but small proportion to the total energy and pressure, so that the inside of the star would be built very much as has been described. Towards the outside, there would be a gradual transition to more familiar conditions, and the degenerate core will be surrounded by an envelope of ordinary gas. The outer diameter should be larger, but not much larger, than has just been computed. The internal temperature will be much lower than that of a non-degenerate star, and hence the luminosity will be much smaller for a given mass than that of a normal star. Since the surface layers are composed of ordinary gas, the spectrum will differ little from that of a normal star.

Now these are exactly the properties which are actually exhibited by the white dwarfs, and there is no longer any doubt that these actually have degenerate centers. Milne points out that we should have not only white dwarfs with degenerate cores, but still fainter orange and red dwarfs, stages of approach to the dead "black dwarf." Such stars should be excessively faint, and only the very nearest of them would have any chance of being observed.

Of the few white dwarfs for which good data are available, four, by great good fortune, are components of visual binaries, and we can find their masses. These range from 0.85 to 0.37 times the sun's mass, and all lie within the interval for which even a completely degenerate mass finds a limit set for its contraction. It is possible that we do not find white dwarfs of large mass, because any such objects, if they existed, would be of such excessively small size that we could not see them at stellar distances, whatever their surface temperature.

A degenerate gas is known to be highly transparent to radiation. Hence, as Milne points out, the differences in temperature within it must be relatively small. The layer of non-degenerate gas near the surface is much more opaque, and acts as a blanket to keep the interior warm. Milne estimates that the temperature in the interior of the companion of Sirius does not exceed 15 million degrees.

The white dwarfs have, within the last few years, changed their rôle from most perplexing to the best understood class of stars. The present theory of their nature (which we owe to Milne) is the second notable triumph of the application of general physics to stellar constitution.

The properties of gas masses with degenerate cores have been the subject of lively discussion during the last couple of years, and a great deal of work has been done on the subject—notably by Milne and his colleagues at Oxford. To save a great deal of algebraic labor, he has worked on the assumptions that the rate of liberation of energy is uniform all through each mass (though differing from one mass to another), and that the opacity is constant in the gas phase, and also in the degenerate state, though much smaller. The transition between the two states is supposed to be sharp, but without discontinuity.

These assumptions do not lead to a close numerical representation of the properties of actual stars; for example, they give a change of luminosity with mass which is similar to, but only about 60 per cent. as great as, the actual change. However, they were not designed for this purpose, but to enable an exhaustive survey of the consequences of at least *one* set of assumptions to be made. The problem remains difficult, but Milne has solved it by methods of great mathematical elegance.

The results are interesting. For bodies of small mass (less than about twice that of the sun), the size and brightness steadily diminish as the degenerate core grows larger. For large masses, the brightness increases slightly until the core is of about a quarter of the star's diameter, and then falls off. There is no great loss of light (exceeding a magnitude) until the core occupies most of the volume. All these configurations, except a few of very large mass, are smaller than those in which the center is just beginning to be degenerate. The latter diminish slowly, with increasing mass, from the size of Saturn to a little smaller than Uranus. The final completely degenerate forms are from one third to one fifth as big.

Milne has not yet extended his calculations to include the second stage of degeneracy, but it is clear that this would make the radii still smaller. His calculations, therefore, appear to represent transitions between ordinary stars and white dwarfs.

Professor Milne has expressed the opinion that the exact solution of certain equations might lead to much larger computed diameters, and hence to an explanation of main-sequence stars, and even giants, as bodies possessing dense, degenerate cores; but later calculations by Cowling are unfavorable to this view. If, however, Kramers' opacity law is substituted for the simpler assumption, keeping the same law of energy generation, Jeans has shown that bodies of large mass must inevitably be greatly concentrated toward the center. Further computations on this basis are to be desired; meanwhile the question whether actual stars (other than white dwarfs) may have degenerate cores must remain open.

(6) Sources of Stellar Energy Synthesis or Annihilation of Atoms

So far we have considered only the escape of heat from a star, without inquiring how the supply is maintained. The rate of loss is almost incomprehensibly great. We can come nearest to realizing it by remembering that, according to the theory of relativity, heat, like other forms of energy, possesses mass. It is as proper to speak of a pound of heat as of a pound of ice; but a pound of heat is a very large amount—enough, in fact, to melt 30 million tons of rock and turn it into white hot lava. The sun radiates heat away into the depths of space at the rate of 4,200,000 tons per second—and the sun is a smallish star! Upon what vast stores of energy can it draw to keep going?

The first approximately adequate suggestion was made by Helmholtz, and developed by Kelvin. All parts of the sun attract one another strongly. If it was once larger than now and has contracted to its present size, the gravitational forces must have done an enormous amount of work during the contraction, which would necessarily reappear as some other form of energy. Part of this is doubtless still stored as heat in the interior. The rest, escaping to the surface, would maintain the radiation. The whole supply made available by contraction from a very large size may be calculated easily, though not exactly, since it depends on the way in which the density increases toward the center. On the most liberal assumptions regarding this it would suffice to keep the sun shining at its present rate for about 20 million years. Before the great extent of geological time was realized, this appeared to be an adequate explanation. But now we have conclusive evidence that the earth's surface has had very nearly its present temperature for 1,000 million years or more, which means that the sun has been shining about as strongly as at present through all these ages. Some greater supply must be sought, and the only present hope is in changes in the atoms themselves, involving a loss of mass.

Two principal suggestions deserve consideration. The first, and boldest is that, under certain rare conditions, a pair of the positive and negative charges out of which matter is built—proton and electron—may annihilate one another and disappear, leaving the whole amount of energy represented by their masses to escape, presumably as radiation of very high frequency, which inside a star would soon be absorbed, its energy reappearing as heat in the gas. Could such a thing happen, the annihilation of one per cent. of the sun's mass would keep it going for 150,000 million years. To consume the whole mass, if possible, would take much more than a hundred times as long, for as the mass decreased, the luminosity, and with it the rate of wasting, would diminish rapidly. The second hypothesis derives the energy from the transformation of hydrogen into other elements. The atomic weights of all the heavier elements (that is, of the separate isotopes of the complex ones) are very nearly integral multiples of the unit ordinarily used by chemists; but that of hydrogen is greater by one part in 130. If in any way hydrogen atoms could be broken up and their parts redistributed to form a heavier atom, the protons and some of the electrons condensing into the nucleus, and the remaining electrons forming the rest, there would be a loss of mass which would presumably escape as energy into the surrounding gas. Transformation of the sun's whole mass from hydrogen into other atoms would liberate heat enough to supply its present radiation for 100 billion years (10^{11}). This far more than meets all the demands of geology; but the maximum life, on this hypothesis, for the brightest known stars comes out less than one billion years, which is inconveniently short.

Radioactivity—which is a special case of the transformation of one heavy atom into others—also liberates energy at the expense of loss of mass. The known cases supply heat too slowly to keep the brighter stars going, and in too small a total amount to meet the requirements. Suggestions that heavier and more powerfully radioactive atoms than any known on earth may exist in the stars are purely speculative.

Very little can yet be said about the possibility of the annihilation of matter inside the stars; but that little indicates that it should not occur except at temperatures of many billions of degrees. More progress has been made in the discussion of the building up of heavier atoms out of hydrogen.

According to Heisenberg's new and very attractive theory, atomic nuclei are built up of protons and the recently discovered neutrons. The incorporation of a proton into a nucleus would in many cases change an atom of a known element into one of the following element (Be^9 to B^{10}, B^{11} to C^{12}, etc.). The introduction of a neutron would change an atom into an isotope of the same element, but with atomic weight greater by 1. (Li^6 to Li^7, B^{10} to B^{11}). By alternation of these processes the heavy elements might be built up, step by step. When a proton is built in, about 1/130 of its mass disappears and must be represented by heat liberated in the process. Building in a neutron probably liberates heat also.

At distances greater than 1/1,000 of the outer diameter of an atom, protons and nuclei repel one another. A fast-moving proton, rushing directly at a nucleus might, however, get so near it that attraction succeeded repulsion, and thus penetrate the nucleus and become a part of it. Neutrons would not be repelled, and would probably have a better chance of going in. We do not know enough about them yet to estimate the chances; but a tolerable idea of the probability of penetration of a proton can be obtained by means of wave-mechanics. The chance is greatest for the lightest nuclei, which have the smallest charges and repulsive forces. Calculations by Atkinson and Houtermans show that such penetrating collisions would begin to become important when the temperature of the gas rose above a few million degrees. Their predictions have been strikingly confirmed by the recent experiments of Cockcroft and Walton, who found, upon bombarding lithium with protons accelerated by a potential drop of 300,000 colts, that alpha rays—helium nuclei—were given off with a total energy corresponding to 16,000,000 volts. Evidently a proton penetrates a lithium nucleus (charge 3, mass 7) producing an atom of charge 4, and mass 8 (an isotope of beryllium) which breaks up into two alpha particles, charge 2, mass 4, while the energy represented by the loss of mass sets these in very rapid motion. The number of cases, per million atoms hit, in which this process occurs is of the order of magnitude predicted theoretically a year earlier.

There is then no room for doubt that the synthesis of heavier elements out of lighter ones and hydrogen may actually occur within the star and liberate great quantities of heat.

(7) PROMISE OF THIRD SUCCESS. THE MAIN SEQUENCE

We have already seen that the interior of a star probably contains about 30 per cent. of hydrogen by weight. In the outer parts of the star a large fraction of the remainder are atoms of carbon, nitrogen and oxygen.

If the same is true in the interior (as appears probable), we may apply formulae recently given by Steenholt and calculate how hot the sun's interior should be to provide heat enough, by atom-building, to balance the outward leakage. Assuming that the central "furnace" contains one tenth of the sun's mass, and is 15 times as dense as the mean, we find that the necessary temperature is 15 million degrees if the active atoms are carbon, 21 million for nitrogen, and 28 million for oxygen. An increase of 30 per cent. in these temperatures would increase the rate of heat production a hundredfold; a decrease of 22 per cent. would diminish it in the same ratio.

The theory is still rough; but further refinements are not likely to alter the conclusion that, if the inside of the stars is at all like the outside in composition, a sufficient supply of heat for their needs will be developed if the central temperature is from 20 to 40 million degrees. The latter would suffice even for a very massive and luminous star.

Now we have already seen that the central temperature of the sun is probably 15 or 20 million degrees. We appear to have here, at last, an explanation why it is of its actual size. If it were twice as big it would be about half as hot inside, and far too little heat would be generated to keep it going; if it were half as big, far too much.

The same is true for the whole main sequence. The great white stars at the top radiate far more heat per gram than the red dwarfs at the bottom. They should therefore be hotter inside, especially as their lower mean densities diminish the chance of atomic collisions. Now there are eclipsing binary stars—whose diameters and masses can be found with considerable accuracy—which belong in all parts of the main sequence. If they are built on anything like the same model, the big white stars are actually hotter inside than the little red ones, to fully the extent demanded by theory.

If this theory is even approximately true, no star could reach an internal temperature as high as 100 million degrees, unless either all the hydrogen, or all the other elements from lithium to sodium, had been completely used up. It is noteworthy that there is no known star for which the data compel us to believe in so high a central temperature (as they do compel the belief that the sun's central temperature is several millions of degrees).

Another striking success of the theory of atomic synthesis is Atkinson's explanation of the relative abundance of the lighter elements in the sun. The strength of the spectral lines shows that lithium and beryllium are present in very small proportions in the sun's atmosphere. Boron is more abundant, and carbon, nitrogen and oxygen increasingly so. Now the lightest elements, if synthesis occurs, should be the shortest-lived, and we might expect that lithium and beryllium would be quite gone, having been all built up into heavier atoms. There is still helium there—plenty of it—but there is good theoretical reason to suppose that, though a proton might easily enter a helium nucleus, it would be very unlikely to stick there, so that the building of lithium would be slow. The lithium atoms thus made would not last long before they were in turn changed into beryllium, so the number present at any instant would be relatively small. The "lives" of heavier atoms would be longer, and they would therefore be increasingly abundant.

(8) Unsolved Problems

Notable as the successes of this theory have been, it is still in a provisional stage. It can not yet treat quantitatively of the entrance of a neutron or an electron into a nucleus—though it has to assume that one process or the other must occur. Nor can it account for the presence in the sun and stars of the heavier metallic atoms, which would not be synthesized in appreciable amounts until after a very long time. Atkinson, who has discussed the matter very thoroughly, believes that some other process, not yet tractable theoretically, must have operated to produce them, and suggests that some of the heavy atoms thus built up may break down with liberation of helium, and thus supply material for further syntheses.

On the astrophysical side, the important sequence of giant stars remains unexplained. They radiate about as much heat, for equal masses, as the white stars of the main sequence; yet their internal temperature and density must both be much lower, unless the central condensation is exceedingly great. They are still a puzzle. It may be, as Atkinson suggests, that they are supplied by the conversion of hydrogen into lithium and other very light elements; or, as Milne believes, that they have small dense cores of high temperatures; or, as the speaker has suggested, that some unknown atomic or sub-atomic process is at work which is actually favored by low density. It is tempting to imagine that a proton or neutron may enter a nucleus and remain loosely bound there, but be liable to be drawn in deeper, with liberation of energy, after a time. Collision with another atom, during this waiting period, might knock the particle out, and prevent its permanent binding. But this is pure speculation, and need not be taken too seriously.

Despite these difficulties, good progress has certainly been made in interpreting the properties of the stars; and it is remarkable that so much success has been achieved while we are still ignorant of the density-model on which any star, except a few white dwarfs, is built. There is an intimate connection between this problem and the other incompletely solved problem of the source of stellar energy. Given either of them, we could deduce the other, provided that we knew the composition of the material at every point of the interior. "There's the rub"; for the composition involves the specification of the relative proportions of more than two hundred kinds of atoms, including the various isotopes. With regard to the internal temperature and opacity, all elements except hydrogen act very much alike, so that only the percentage of hydrogen is really important. But, when it comes to the liberation of energy, different elements, and even different isotopes of the same element, probably behave differently, and this would make prediction difficult, even though we knew the laws of atomic synthesis and disintegration exactly. To get anything that we can work with, we must cut the Gordian knot by assuming that the composition of all parts of a star's mass is the same. If this assumption was made solely to facilitate our analysis our position would be embarrassing; but, fortunately, there are good physical reasons to believe that inside a star there will be slow currents, mixing the gas and tending to make its composition uniform.

(9) Model Stars. Vogt's Theorem

Material of exactly specified composition must have perfectly definite physical properties, so that, if the temperature and pressure are given, the density, the opacity, the rate of liberation of atomic energy, and so on, must be calculable by equations, which (whether we know them or not) are exact, and leave no room for ambiguity. If we knew these equations, we could calculate all the properties of stars of the given composition in the following way.

We may assume the star to be rotating so slowly that it is practically spherical. (What happens otherwise is an additional problem, to be solved later, if we can.) The pressure will certainly be greatest at the center, since the overlying mass is greatest there, and the temperature, too, will be greatest, for otherwise heat would flow from the hotter parts to the center and could not be accounted for except by the grotesque assumption that the central material consumed heat instead of liberating it.

Start the calculations by guessing at any plausible values of the central pressure and temperature. Then the density, opacity and rate of heat-liberation are determined. Within a small sphere described about the center these quantities will change very little, so we may calculate the force of gravity at its surface, and the rate of flow of heat outward through it. From these we can find the outward gradients of pressure and temperature at the surface of our sphere, and so estimate their values at the surface of a sphere twice as big—which enables us to extend all our calculations to this second sphere. Proceeding thus, step by step, we may carry our calculations on as far as we please. This is known as the "method of quadratures." Though very laborious, it is powerful, and will solve a great variety of problems. Proceeding outward in this way, we find the temperature and pressure steadily decreasing—and incidentally calculate the mass inside each successive sphere, and the total outward flow of heat through it.

Finally, the pressure may fall to zero, denoting the outer limit of the mass. Knowing the radius and the heat flow, we can calculate what the surface temperature should be to permit the radiation of this amount into space. The temperature derived from our calculations should be 81 per cent. of this—to make allowance for the fact that the "surface temperature" is an average including hotter layers. This condition will usually not be satisfied—which means that we have made a false start with our guesses at the central temperature and pressure. By changing one of these—say the pressure—and repeating our calculations, we may succeed—usually after a great deal of labor—in getting a satisfactory solution—a model star, made of our given material, for which we know the mass, radius, luminosity, density-model, etc. Then we start with a new central temperature, repeat the whole process and get another model star,—and so on.

In this way, by a great but not impossible amount of calculation, we find out exactly what stars of the specified composition would be like.

One thing about them follows at once from the nature of the process of calculation. If we plot any two of their characteristics—say mass and luminosity—in a diagram, all the points representing our solutions will lie on a definite curve—that is, there will be an exact mass-luminosity relation. Similarly, if we plot luminosity against surface temperature, we will get a definite curve. These curves may be of complicated shapes, or even consist of two or more branches; but our points will be on lines, and never be scattered over an area in our diagram. In mathematical language, our model stars form a one-parameter family.

This is Vogt's theorem—the most important general proposition which has yet been established regarding stellar constitution. It has important practical applications. For example, the stars of the main sequence lie so close to one line on the diagram that they may all be of the same composition. The giants lie close to another line and may be similar in composition *inter se*. Indeed it is possible, though it looks unlikely, that the loci for the giants and main sequence may be two parts of the same curve. But the super-giants are represented by points which are apparently scattered here and there over a wide area in the upper part of the figure. Hence they can not all be of the same composition.

Vogt's theorem would fail for bodies cool enough to permit liquid drops to condense out of the gas—a contingency not to be feared in the stars—and also in the case of a star which derived its heat supply from gravitational contraction—for here the rate of heat production in a given part of the interior depends not merely on the temperature and pressure there, but also on the rate of shrinkage of the star as a whole. But its range of applicability is very wide.

A good many individual models of stars have been computed. In some of these, physically improbable assumptions were made to simplify the mathematical work—for example, that the rate of generation of heat per gram is the same in all parts of a star, though different in different stars. Such models, though of great value in the development of the theory, need not detain us here.

A curious class are the "point-source" models, in which it is assumed that all the heat is generated in one point at the very center—the rest of the star acting simply as a blanket to keep it from escaping. This is not so crazy as it sounds—it represents the limit of the quite plausible assumption that the heat is generated in a small central region of high temperature.

In some of these solutions, the density diminishes near the center—the matter being driven away by the enormous radiation pressure. Even with this extreme assumption, the luminosity for a given mass differs by less than one stellar magnitude from that of a model in which the heat-liberation is almost uniform—as Eddington first showed. Biermann, who has calculated several such models, concludes that they may be "finished off" into physically possible stars by replacing the point-source with a small, hot and very dense core—so dense that it is relativistically degenerate. This idea of a dense core was first suggested, long before, by Jeans, who showed that its existence was probable in stars of large mass.

The first calculations of stellar models with a law of heat-generation derived from theory (atomic synthesis) have recently been published by Steensholt. In the cases so far discussed, the density is less at the center than half way out to the surface. So much heat is produced in the inner regions, where the temperature is highest, that the point-source model is approached. Some of the theoretical models, therefore, have a partly empty center, others a very dense center. A great deal more calculation will be required before the subject is cleared up. The only observational data, on Y Cygni, indicate that in this star the central condensation is high.

(10) Stability

The assumption that a star is in internal equilibrium, which has been the basis of the whole discussion, is justified by the observed constancy of the brightness, spectral peculiarities and other observable characteristics of the great majority of actual stars. But, before any theoretical model can be accepted as satisfactory, we must inquire what would happen to it if its equilibrium should be disturbed.

Possible disturbances may be of two kinds, mechanical, involving changes in the size or shape of the star, and thermal, involving changes in the rate of generation of heat inside, or loss from the surface.

The most important mechanical disturbance is a change in the size of the body. Suppose, for example, that a body, of low enough density to be gaseous throughout, were suddenly reduced to 9/10 of its previous diameter (remaining built on the same internal model). All parts of the mass, being nearer together, attract one another more strongly, and the internal pressure is increased. To maintain equilibrium, the temperature must rise, according to Lane's law, to 10/9 of its previous value in each part of the body—which means that a large amount of energy must be communicated to the gas. During the contraction, however, a great deal of work is done by the gravitational forces, and gravitational potential energy is lost. In the simplest case, when no changes occur in the atoms of the gas, and no compounds are present, an easy calculation shows that just half the change in gravitational energy would suffice to furnish the necessary heat. Hence the star can not actually contract unless some way is found to get rid of the other half. Radiation from the surface would ultimately bring this about, but the process is very slow. It would take the sun, for example, about 1½ million years to lose enough heat at its present rate to permit it to contract by 10 per cent.

If, by some hypothetical force, such a star should be rapidly compressed to (say) 90 per cent. of its present diameter, the freed gravitational energy would all appear as heat in the interior, the internal pressure would be too great to balance the weight of the overlying layers, and, the moment the constraint was removed, the body would begin to expand. When it reached its original size, all parts of it would be moving outward, and this expansion would overshoot the mark, carrying it to about 10 per cent. larger than normal. The internal temperature and pressure would then be too small, and contraction would ensue, to be followed by another expansion, and so on. The star would therefore show periodic changes in diameter, or pulsations, which would continue for a long time—though, after a great many oscillations, the "leakage" of heat from one part of the star to another might damp them out. The period of pulsation depends mainly on the density of the body. It would be a few hours for the sun, a few days for the yellow giants and many months for the enormous red giant stars.

If, however, the gas of which the star was composed became more heavily ionized at the higher temperature after the compression, part of the surplus energy would be used in bringing this about, and less would be left over to produce the pulsation—which would then be slower for the same density, but the body would still be stable, unless the ionization demanded more energy than the gravitational changes made available. In such a case, the pressure, after contraction, would be inadequate to sustain the load, and the mass would contract further and further, and never return to its original state. In a star made up wholly of one kind of atoms, there might be some chance of this—though not in all parts of the mass at once. But with a mixture such as is actually found in the stars there is no danger, especially as the abundant hydrogen is completely ionized except in a very thin skin at the surface.

The radiant energy and radiation pressure, by the way, adjust themselves so as to take up just their share of this gravitational energy—leaving the properties of the gas to determine the stability of the body.

A pulsating star should vary periodically in diameter, and also in surface temperature, brightness, color and spectrum. Now there are hundreds of stars—the Cepheid variables—which change in all these respects, repeating their variations regularly for thousands of periods. The pulsation theory gives a good account of almost all their properties—though it is not yet well understood why the rise in brightness should almost always be more rapid than the fall. They are brightest, and hottest outside, not when they are smallest (and hottest inside), but when they are rapidly expanding, as if the heating effect took time to work out to the surface. They are all super-giant stars, of exceptional luminosity, and their periods, ranging from a few hours to a couple of months, are in good agreement with their probable densities.

Forced changes in the natural spherical shape of a star would also lead to oscillations—of shape, this time, and not of size—but these would involve much smaller changes in internal energy, since some parts of the mass would be hotter while others were cooler, and are much less likely to lead to trouble.

One more type of disturbance must be noted. If a mass of the hot gas deep down should be suddenly raised nearer to the surface, it would expand to the degree permitted by the lowered pressure, and be cooled by a calculable amount. If the temperature gradient in this region is high enough, such a mass might find itself hotter than the surrounding gas, and therefore less dense, so that it would continue to rise of its own accord. In any such region, ascending and descending currents would be set up, and continued until the too-rapid change of temperature had been smoothed out. This effect would operate to change the model on which the star was built, and may cause a good deal of trouble to the theoretical calculator, but it is not likely to upset the stability of a star—though, if the convection currents are irregular, it might cause small irregular variability in brightness, as Rosseland suggests.

Thermal disturbances within a star have to do with the balance between the production of heat within it and the loss by radiation. They act slowly, because the store of heat already inside a star is so great. For example, the sun's interior contains, in the form of actual heat (disregarding potential energy of all kinds), enough to supply the present rate of radiation for some ten million years. If the supply of heat from subatomic sources were completely cut off, a slow contraction would ensue; if it were doubled, the excess would be consumed in forcing an expansion against gravity at the same rate. Even if 100,000 years' heat supply should be suddenly liberated (not at one point, of course, nor too near the surface), the effect would only be to increase the radius by about one per cent.—accompanied by pulsations about the new value. This expansion, by Lane's law, would lower the internal temperature. We have here an apparent paradox. A gaseous star, considered as a whole, has a negative specific heat; liberation of heat inside it cools it. The explanation is that the addition of this heat energy forces a much larger transfer of heat into gravitational potential energy—reminding one of the effect of a resisting medium in increasing the orbital velocity of a comet.

Suppose now that we have a star in radiative equilibrium—and that the hypothetical disturbing agency not only diminishes its diameter, but at the same time abstracts from it the excess heat freed by the contraction—so that there is no longer any tendency to pulsate about the new radius. Under the changed conditions both the income of heat from internal sources and the expenditure at the surface will be altered. If the two still balance, the new configuration will persist unchanged. If the income exceeds the outgo, the star will slowly expand, and return toward its initial state; it is secularly stable. In the opposite case, it is unstable, and will depart further and further from its original state. The condition for stability is therefore that the rate of heat production shall increase more rapidly, with increasing temperature and density, than the rate of outflow. It is easily verified that this also secures stability in event of a forced expansion.

These changes are unlike the mechanical pulsations in two respects. First, they are vastly slower, with a time-scale of millions of years (tens of thousands for the brightest giants), instead of days or months. Second, the return to "normalcy" is steady, without oscillations.

Heat-liberation by atomic synthesis increases very rapidly with the temperature, and satisfies the condition for stability; but heat production at a rate independent of pressure and density (such as arises from radioactivity) does not.

There is, however, another more subtle danger to which our star is exposed. Any disturbance is likely to set it pulsating. When smallest, it will be hottest inside, and the rate of heat production will be greater than the average. This tends to make the rebound a little greater than the contraction which preceded it, and so to build up the pulsation with steadily increasing amplitude. This effect has a long time-scale, like the last—as Jeans has shown—but it is ultimately serious. Eddington, who first pointed it out, suggests that it may be avoided by assuming that the heat-liberating process does not act instantly, but with a time-lag greater than the period of pulsation. This would "iron out" the irregularities and prevent trouble. Such a lag may easily occur in processes of atomic synthesis, provided that, as Atkinson suggests, the main liberation of heat occurs when the products at first synthesized break down.

An increasing pulsation need not necessarily grow till it breaks up the star altogether—it is entirely possible that, when the amplitude becomes considerable, secondary effects may slow up, and finally prevent, further increase of range. Stars for which the stability condition was not satisfied might then be started pulsating by the least disturbance, but the pulsation would increase only to a certain limit. The Cepheid variables behave in very much this way; they are never more than three or four times brighter at maximum than at minimum, and the change in diameter (derived from the observed radial velocities of the expanding or contracting surface) does not exceed 20 per cent. on each side of the mean. It is probable, though not proven, that these stars are actually unstable, in the sense here discussed, but that their pulsations are self-limited.

Jeans has shown that the tendency to pulsatory instability increases with the mass of a star. The Cepheids are among the most luminous, and hence presumably the most massive, of all stars. The period-luminosity relation among them—which has been so important in determining the distances of remote star-clusters and nebulae, may well represent the relation between the mass of a star and the density at which this instability begins to appear. A really satisfactory theory of the energy-supply of giant stars should explain this relation as a by-product.

(11) History of a Star

The probable life-history of a star presents the most difficult problem of all. It is natural that the path of advancing knowledge regarding the physical constitution of the stars is strewn with the corpses of dead theories of their evolution. We are not far enough along yet to possess the basis for really constructive work on the subject.

The conclusions which can be drawn from our present limited knowledge depend greatly upon our assumption regarding the source of stellar energy. If the greatest available supply is the synthesis of other elements from hydrogen, a star's mass will change very little during its whole life. Beginning, we may imagine, with large radius and low density, it will at first have to draw on its gravitational energy and contract after the manner discussed by Kelvin, until it gets hot enough inside to start the process of synthesis. It will then approach a steady state, in which it will remain, with little change in size or other observable properties, until the active material —whatever it may be—is nearly exhausted. Then it will again begin to contract, and grow hotter inside, pausing again, perhaps, if some new energy-liberating process is "turned on" at a higher temperature. Finally, after all the available sources are exhausted, it will contract until degeneracy sets in at the center, and then pass through intermediate stages of the type discussed by Milne, into a state of great density. If its mass is small, it will be successively a white dwarf, a yellow dwarf and a black dwarf, finally extinguished. If its mass is large, it may contract to exceedingly small dimensions and be lost to all observation.

During this process its luminosity will increase slowly, till degeneracy sets in, and will fall rapidly as it becomes advanced. The maximum length of a star's life as a luminary—between the times when it gets hot enough on the surface to be visible, and when it reaches the degenerate stage—may be estimated by assuming that it was composed of almost pure hydrogen at the start, all of which becomes transformed. The result varies greatly with the star's mass, being 10^{11} years for the sun, and not more than 500 million for a super-giant of mass 50 times and light 10,000 times the sun's. This last figure is uncomfortably small; it makes the star's life shorter than geological time, and only four or five times longer than the interval during which light from the more distant nebulae has actually been on the way to our spectroscopes.

The relative lengths of time which a star will spend in different stages will depend upon the amount of transformable matter available to keep it going in each, and this, again, may depend on the breakdown of atoms formed in earlier stages or earlier in the existing stage (as Atkinson suggests). Knowledge of these is essential to any detailed prediction of the stages in which the stars will linger long, and in which therefore the majority of them will be found. Till this becomes available, we may well hesitate before pronouncing one star to be older or younger, in years, than another.

Suppose, for example, that there were many stars, of different masses, all composed of hydrogen and oxygen, or hydrogen and nitrogen, and all initially of very low density, and that the only process of atomic synthesis is that discussed above. They would perform "Kelvin contractions" till their central regions reached, approximately, the temperatures possessed by the various stars of the main sequence, and then would settle down with the corresponding radii, densities and surface temperatures. A massive star might become stabilized, with spectrum of class B, in a few hundred thousand years. A star of small mass might take scores of millions to reach a spectrum of type K or M; but, after they had done so, both would remain substantially unaltered for a long interval.

The white dwarfs show apparently definite signs of senility; but even here we must be on our guard against dogmatism in attributing this to greater age, for we do not know whether they started with the same composition as the others, or what has happened to them in the meantime. The fact that many of the known white dwarfs are components of binary systems, whose other members belong to the main sequence, suggests that, after all, they may not be much older than other stars—only more decrepit. If they have passed through such intermediate stages of contraction with very hot centers as have previously been described, it is hard to see how any hydrogen can be left inside them—though their atmospheres might still contain it, as their spectra indicate. But it is well to be cautious.

If annihilation of matter takes place within the stars on a large scale, their lives may be hundreds of times longer, and a star, during its history, may diminish greatly in mass and in luminosity. The possible life-histories may be much more complicated, depending on how much mass is lost by the consumption of various kinds of expendable material in different stages. The early stages of large mass and great luminosity will be relatively short, and the final faint stages of very long duration. Provided that any indestructible atoms remain, the final state is a black dwarf, as before.

The great problem of the longer or shorter time-scale may find its solution in another way. The theory of the expanding universe indicates that at a relatively recent time—something like 10^{10} years ago—the universe was probably either just beginning to expand perceptibly after a previous eternity of sluggishness, or very much smaller than at present, and expanding rapidly. The latter type of theory—adopted in different form by Lemaitre, Tolman and de Sitter—leaves hardly time enough for much change even in a massive star, and certainly none for a star of smaller mass. If such an epoch of intense cosmic activity occurred in the relatively near past, the stars may be indeed all of the same actual age, and may, with few exceptions, still be almost fresh from the mint—" in their first innings" as Eddington puts it. On Tolman's hypothesis, the universe previously to this "age of confusion" was contracting after a long history. It is perhaps permissible, and certainly attractive, to suggest that dense bodies like the white dwarfs may be survivors of this earlier world, which have come unscathed through the cataclysm.

Once again, we lack many data necessary for a definite solution. We do not know whence the stars arose, nor what their histories have been. But so much of the nature of the greatest material objects in nature has already been revealed from the study of the minutest that there is better reason than ever to hope, with Eddington, "that in a not too distant future we shall be competent to understand so simple a thing as a star."

OBITUARY

ELIAKIM HASTINGS MOORE

FOR forty years Moore has been a leading figure in the mathematical world. His fundamental researches have been recognized by many honors bestowed here and abroad. He played a leading rôle in the conversion of a local mathematical society into the present national one and in the initiation of the research journal of the nationalized society. He inspired and started on their research careers many of the younger generation. It is not possible to exaggerate Moore's profound influence on the development of mathematics in America.

He was born in Marietta, Ohio, on January 26, 1862. He graduated at Yale in 1883 and received his Ph.D. there in 1885. At the University of Berlin in 1885–86, he was most influenced by Kronecker, who like Moore had an abstract type of mind.

Young Moore taught at Yale for two years and at Northwestern University for four years. In 1902, President Harper selected Moore to initiate the department of mathematics at Chicago. With the effective support of two colleagues, Bolza and Maschke, who were seasoned research men, Moore began with great enthusiasm to develop a research center.

His plans matured so favorably at Chicago that Moore could extend his influence elsewhere. He took an important part in the conversion of the New York Mathematical Society into the American Mathematical Society and in making the latter truly national. Prior to 1899, the society published a single journal, its *Bulletin.* Moore was the first to foresee the rapid growth of research in mathematics in America and hence the need of an additional journal devoted exclusively to research. His influence was now so great that he was able to overcome the opposition to this expansion. Having been the father of the *Transactions* of the society, it was natural that he should serve as its editor-in-chief for the next eight years. Later he became president of the society. Ten years later (in 1921), he was elected president of the American Association for the Advancement of Science, the subject of his retiring address being "What is a Number System?"

Moore received numerous honorary degrees: Ph.D., Göttingen, 1899; LL.D., University of Wisconsin, 1904; Math.D., Clark University, 1909 (where he lectured on "The Rôle of Postulational Methods in Mathematics"); Sc.D., Yale, 1909; University of Toronto, 1921; Northwestern and Kansas Universities, 1927.

Since 1908, he has been associate editor of *Rendi-*