Orleans for his work on nerve growth and repair. An account of Dr. Speidel's prize paper is given in SCIENCE, for February 5, 1932.

A second general session will be held on Wednesday afternoon in the Smoking Room, Municipal Auditorium, for an address by Dr. Dayton C. Miller, of the Case School of Applied Science. In speaking on "The Physics of the Flute," Dr. Miller will describe the air currents producing musical sounds and will play a flute to illustrate his lecture.

The tenth Josiah Willard Gibbs Lecture will be delivered by Dr. Richard C. Tolman, of the California Institute of Technology, at a general session of the association and the American Mathematical Society on Thursday afternoon at 4:30 P. M. in the Vernon Room of Haddon Hall. Dr. Tolman, who is well known as a physicist, chemist and mathematician, has chosen the subject "Thermodynamics and Relativity."

A second lecture on Thursday afternoon at 4:45 P. M. in the Belvidere Room, Traymore Hotel, will be given by Dr. Mel T. Cook, director of the Insular Experiment Station at Rio Piedras, Puerto Rico, on the subject, "Personal Experiences in West Indian Hurricanes." Dr. Cook will give a botanical background to this lecture and will exhibit some lantern slides, showing the paths of cyclones in recent years. In addition, Dr. Cook will propose a theory to explain the origin and cause of hurricanes.

On Friday afternoon at 4:45 in the Benjamin West Room, Haddon Hall, Dr. Russell W. Bunting, professor of oral pathology at the University of Michigan and director of dental caries research under the Children's Fund of Michigan, will deliver a general address on "Recent Developments in the Study of Dental Caries." Every one interested in the present status of our knowledge of tooth decay will be interested in Dr. Bunting's lecture.

In addition to the above general addresses, many of the retiring vice-presidents and chairmen of the various sections of the association will deliver addresses of general interest. The titles of these as now known are as follows:

(1) "The Spontaneous Heating and Ignition of Hay and Other Agricultural Products," by Dr. C. A. Browne, Bureau of Chemistry and Soils, U. S. Department of Agriculture.

(2) "The Rôle of Analysis in Scientific Investigation," by Dr. Douglas W. Johnson, Columbia University.

(3) "Genetics and Embryology," by Dr. Charles Zeleny, University of Illinois.

(4) "Crops and Civilization," by Dr. E. D. Merrill, New York Botanical Garden.

(5) "Photographic Records of Changes in Vegetation," by Dr. G. E. Nichols, Yale University.

(6) "The New Anthropology: Twenty-five Stages of Vertebrate Evolution from Silurian Chordate to Man," by Dr. W. K. Gregory, American Museum of Natural History.

(7) "The Historical Development of Response Psychology," by Dr. Herbert S. Langfeld, Princeton University.

(8) "Mediaeval Guilds of Medical Interest," by Dr. Howard T. Karsner, Western Reserve University.

(9) "Science and the Problem of Value," by Dr. Ernest Horn, University of Iowa.

(10) "Tendencies in the Logic of Mathematics," by Dr. Earle R. Hedrick, University of California at Los Angeles.

CHARLES F. ROOS,
Permanent Secretary

SCIENTIFIC APPARATUS AND LABORATORY METHODS

A LEAST-SQUARES CURVE-FITTING MACHINE, USING SPRINGS

THE machine described herewith has been developed to meet a special need. Some features may be of enough general interest, however, to warrant an account of the problem and solution.

Breeders of dairy cattle are accumulating a large body of yearly records of production based on the milk yield and fat test of individual cows by calendar months. The problem is to measure the *rate* of milk secretion for each cow by fitting the data of the first eleven full months of lactation with the equation $y = Ae^{-kx}$, in which y is milk-energy yield per day, and x is time in months with origin at one month after calving.[1] The observed y's are computed by use of a special slide rule,[2] wherein the stops of the machine sketched in Fig. 1 replace the runner of the slide rule. When the eleven stops, representing the observed y's, are set the machine immediately indicates the fitted A and k constants, as explained in Fig. 1.

The present machine has been worked out by a sort of "cut and try" process,[3] without knowledge of any previous device along the same line. We have since learned that the spring principle of fitting a straight line to irregular observations was demonstrated some years ago by M. D. Hersey.[4]

1 The use of this type of equation in the study of lactation was introduced by S. Brody and coworkers, *Jour. Gen. Physiol.*, 5, 441–443, 1923.

2 *Jour. Agric. Res.*, 34, 373–383, 1927.

3 The first model was constructed by the Hayes Machine Company, Urbana, Illinois. This was revised in the shops of the Department of Physics, University of Illinois, in which connection we wish to acknowledge the assistance of C. W. Fieg, shop mechanician.

4 *Jour. Wash. Acad. Sci.*, 296, 1913.

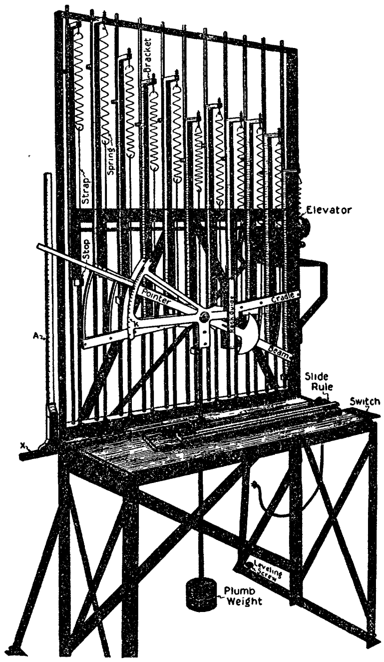


FIG. 1. Curve-fitting machine. (Description and operation of machine for fitting the equation $y = Ae^{-kx}$ to 11 observed y's).

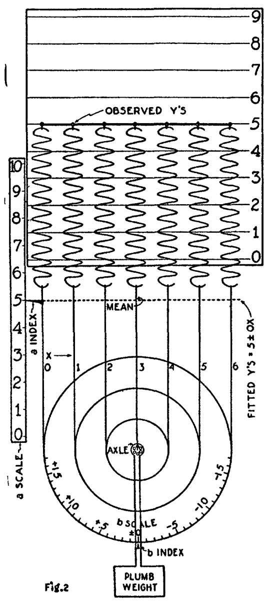
The machine has an erect framework carrying 11 brackets adapted to independent vertical movement. Each bracket carries one of a set of 11 uniform coil springs having no initial tension. The springs support a free-hanging cradle beam by thin metal straps rolling over arcs built into the beam. The beam has a central axle which carries by ball bearings a frame provided, at the left, with a k scale and, at the right, a rack guide. This frame, while free to move vertically with the beam, is held from rotational movement by a heavy plumb weight rigidly attached to it. The axle of the cradle beam also carries through a ball bearing a balanced pointer. A gear segment attached to the cradle beam meshes with a rack moving vertically in the rack guide. The rack is connected to the pointer by means of a pin operating in a center-line slot in the pointer. Rotational movement of the beam is thus communicated to the pointer in the ratio of angle and tangent.

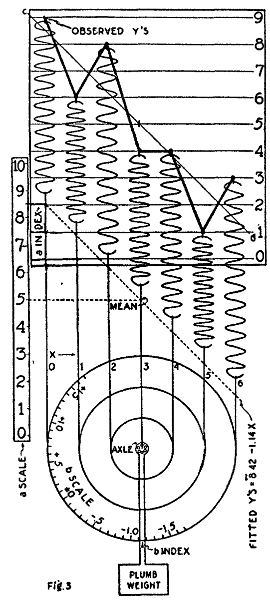
In operation, the whole fitting assembly is raised to the top of the framework by the electric elevator, controlled by a switch. Each bracket is provided with a hanging stop. With the assembly elevated, the first stop is removed, placed on the 20-inch slide rule, the first observed y computed directly on the stop, and the stop replaced; similarly, for each of the other y's in succession. The assembly is now lowered, whereupon it comes to rest supported by the stops or observed y's, and the fitted constants are immediately indicated by the pointer on the k and A scales. The vertical A scale is a duplicate of the slide rule, and is movable laterally on the horizontal x scale to accord with the time origin.

The machine is adjusted as follows. Adjust leveling screws to cause the plumb weight rod to hang in the clear in the circular hole in the table top through which it passes. Unhook all the springs from their straps. Fasten the cradle beam in a horizontal position, that is, the pointer reads $k = 0$. Remove all the stops and set one of them at a suitable value. Proceeding one bracket at a time, hang this stop in place supporting the bracket, attach a weight to the spring sufficient to stretch it about one-half its working range, adjust screw from which spring hangs at top of bracket until lower hook of spring meets its point of attachment in strap. Having thus adjusted each spring, hook up all springs. Set all stops at the same value (say 5), hang in place supporting fitting assembly, and so adjust the A scale vertically in the slot at its base attachment that the pointer reads $A = 5$, (the k reading will be 0).

As a test on the accuracy of the machine fitting, 50 actual records were selected at random, but to include a wide range of k. These records had previously been fitted by least squares, algebraically. The range of k was from .170 to $-.018$. The results by the machine were then compared with the algebraic results by determining the Pearson product-moment coefficients of correlation. The A's showed, $r = .99992$; and the k's, $r = .99979$. The accuracy appears to be comparable with that of a 20-inch slide rule. It is of interest to note a similar comparison[2] of the k's as determined algebraically and graphically (that is, by plotting and eye fitting) which gave, $r = .956$.

The diagrams and explanation of Figs. 2 and 3 are given to further clarify the principle, and also as suggestive of a simpler design.





FIGS. 2 and 3. Diagrams to illustrate the use of springs in curve fitting. (Least-squares fit of the equation $y = a + bx$ to seven observations).

Seven uniform coil springs, without initial tension, suspended from the observed y's, support the uniformly stepped wheel and heavy plumb weight, by light bands which may be considered as weightless. In Fig. 2 the observed y's are all 5 and it is clear that $a = 5$ and $b = 0$, as indicated on the respective scales. Each spring supports 1/7 of the suspended weight and each undergoes the same stretch, l. The combined stretch of the springs is Σl or $7l$.

In Fig. 3 the observations are irregular, 9, 6, 8, 4, 4, 1, 3 from $x = 0$ to 6, respectively. The sum of the y's is 35 and the mean is 5, as before. The center of the wheel remains stationary since its position is dependent on Σy. The fitted line passes through the center x at the mean of the y's, one characteristic of the least-squares principle.

The stretch of the springs in Fig. 3 varies. The line cd is drawn to make this more apparent. If all the observed y's were on cd the stretch of each spring would be the same, l. But, Σl remains constant and hence, the fitted line takes a position such that the sum of the plus deviations equals numerically the sum of the minus deviations, or the algebraic sum of the deviations is zero, a second characteristic of the least-squares principle.

The essential characteristic, the sum of the squares of the deviations is a minimum, may be explained in terms of energy, as developed by Hersey.[4] The coil spring with no initial tension and no stretch has no stored energy. As the spring is stretched the stored energy increases as the square of the stretch. In Fig. 2 the stretch of each spring is l and the stored energy of all the springs is Σl^2. In Fig. 3 the stretch of the springs is variable, $l+\Delta$, where Δ is the deviation of the respective observed y from the fitted y. The stored energy of the springs is $\Sigma(l+\Delta)^2$, or

$$\begin{array}{lll} \text{for the } +\Delta\text{'s}, & \Sigma l^2 + 2\,\Sigma l\Delta + \Sigma\Delta^2 \\ \text{for the } -\Delta\text{'s}, & \Sigma l^2 - 2\,\Sigma l\Delta + \Sigma\Delta^2 \\ \text{for all } \Delta\text{'s}, & \Sigma l^2 \qquad\qquad + \Sigma\Delta^2 \end{array}$$

The $l\Delta$ terms cancel since l is constant and the sums of the $+\Delta$'s and $-\Delta$'s are numerically equal. The system comes to rest with stored energy at a minimum and since Σl^2 is constant it follows that $\Sigma\Delta^2$ is a minimum.

In reaching this equilibrium the wheel is rotated through an angle proportional to b. The b scale on the circumference of the wheel is linear, and dependent on the ratio of the x to the y spacing. In the diagram this ratio is 1.146, and 5° corresponds to .1 on the b scale ($1.146 \times 2 \times 3.1416 \times 5 \div 360 = .1$).

Three features of the least-squares principle of fitting the equation $y = a + bx$ are thus shown mechanically:

1. The fitted line cuts the center x at the mean of the y's.
2. The algebraic sum of the deviations is zero.
3. The sum of the squares of the deviations is a minimum.

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PALEOGEOGRAPHIC WALL MAPS

In practically all the present-day text-books of geology, more or less use is made of paleogeographic maps. These maps constitute a most excellent teaching device, but it is the opinion of the writer that the maps as published in most of the text-books are of but little value to elementary students. In all the text-books in use at the present time such maps are published in black and white and seas are shown by means of lined areas or some such device. All these devices so obscure the boundaries of land masses, political boundaries and so forth, which are also shown by black lines, that it is almost impossible to determine the relation of the seas to the present continents and political divisions. This difficulty is increased by the small scale necessarily used in the text-book publications. The result is that the average student gets only a hazy idea of the paleogeography of the past in relation to the present-day continents, and even the exceptional student who endeavors to study out these maps is soon hopelessly lost in the maze of lines and boundaries and soon gives up in discouragement. Thus a very excellent teaching device is rendered practically useless. The difficulty of the small scale can be partially overcome by the use of lantern slides of these maps, but this is of little help, since the confusion of lines and boundaries still exists.

The writer has for some time been using a method of making paleographic wall maps which is both rapid and inexpensive and which gives maps on which students can see at a glance the position of past geographic features in relation to the present-day continents. An outline map of the continent in question is first made by tracing from a political wall map of suitable size the political boundaries and boundaries of land masses on tracing paper or tracing cloth. All boundaries are traced in black India ink with a lettering pen which makes a stroke of about an eighth of an inch wide. Lines of this width can usually be distinguished anywhere in the ordinary lecture room. Care should be taken not to use too much detail, but enough should be included so that no student will have any difficulty in locating any place on the map with regard to its present-day political boundaries or the boundaries of the land masses. From this outline map a large number of outline maps are prepared by tracing them on tracing paper of the kind used by architects in making preliminary plans of buildings. These outline maps are then used as needed in preparing paleographic maps of any period or epoch. Using the same kind of a lettering pen the boundaries of the seas are drawn in with some sharply contrasting color such as bright red. In this way the boundaries of the seas are never confused with the political and other boundaries drawn in black on the outline map. The areas of the seas are then roughly colored with soft blue chalk (not crayon) and then carefully rubbed with a small piece of cotton cloth. This rubs the color into the paper sufficiently that it is not rubbed off by handling and distributes the color evenly over the map. The seas now stand out in strong contrast to the land areas, and the political and other boundary lines are not in the least obscured by the coloring. As a result students grasp the situation immediately as soon as one of the maps is shown to them and get a clear conception of all the geographic features of the past with their relation to the present-day configuration of the continents. The tracing paper map when finished is mounted on muslin in the usual way and makes a very durable map. After the preliminary outline maps are made a