THEOREM V: If R_0 does not vanish, as in the case of sliding friction, the ratio of the curvatures is $1:3+2\frac{R_0}{F}$ where F is the acting force.

We observe that Theorem V includes Theorem I as a special case.

If a particle starts from a given point in a given general direction (not the force direction) with varying speed v, we study, in the last part of the paper, the variation of the successive radii of curvature r, r_1, r_2, . . ., and the loci of the successive centers of curvature C, C_1, C_2, . . . of the successive evolutes of the trajectories. The results for r_1, (quartic law) and C_1, (parabolic locus), and also Theorem I about the ratio 1:3, were given in earlier papers by the writer (Trans. Amer. Soc. Math. 1905–1910; Princeton Colloquium Lectures, *Differential geometric aspects of dynamics,* 1913; an application of Theorem I is given by W. H. Roever, *Bull. Amer. Math. Soc.,* 1915, p. 456). For r and C the results are obvious. The general results are as follows:

THEOREM VI: *The locus of C_n is a rational curve of order $n+1$. The radius r_n of the nth evolute of the trajectory varies as a polynominal of degree $2n+2$ in the speed v.*

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SOIL WEIGHTS BEFORE AND AFTER DISPERSION

THERE appears to be a rather common belief among soil chemists and physicists that a soil, after dispersion, may weigh more than before dispersion, the dry weight at 105° to 110° C. being used as the basis of comparison. In this laboratory it was first observed, some ten to twelve years since, that, in the quantitative separation of colloidal material from soils, sometimes the total weight of the fractions into which the material was separated apparently exceeded the weight of the original sample. A somewhat similar observation has more recently been made in carrying out mechanical analyses of soils by the pipette method.[1] In this procedure the soil sample, after dispersion, is wet-sieved so that the clay and fine silt are collected in the sedimentation cylinder. By deducting the dry weight of the sands and coarse silt from the dry weight of the sample before dispersion, the weight of the material in the sedimentation cylinder may be obtained. The weight of this material may also be estimated by removing an aliquot from the sedimentation chamber, evaporating to dryness, and weighing. In practically every case where this check was made it was found that the direct determination by the aliquot method gave a slightly higher value than did the determination by difference. It is to be noted that such difference may be occasioned by an error in the aliquot used, which error is magnified by the aliquot multiple. However, it has been usual to attribute such assumed increases of weight to combination with water when the colloidal aggregates are broken apart by dispersion and a greater total surface is exposed. The added water is sometimes spoken of as "water of hydration."

The assumption of weight increases is not wholly absurd. If, as is the usual thought, soil colloids are produced by hydrolysis of minerals, the "water of combination" of the colloids has been added by just such a process. Further, from experiments with silica gel and with charcoal, it appears that microscopic particles are exceedingly effective in condensing and holding vapors. The dispersion procedure breaks down the structure of many of the colloidal particles formed by repeated wetting and drying in the field. New particles are formed when dispersed soil is dried, and it is possible that these new particles may have a different holding capacity for water than the field particles.

It seems, indeed, quite inconceivable that the time required for soil dispersion can effect any measurable increase in the hydrolysis of unhydrolyzed minerals above that effected in the field during the almost infinite period of formation of the soil. It seems quite improbable that any addition products would be formed by dispersion, and remain stable at 105°, which can not be formed and remain equally stable under field conditions. Nevertheless, the belief in the weight increase, as noted above, seemed so general that it appeared worth while to attempt to determine whether it actually does occur. For this purpose the work reported in this report was carried out.

THE SOILS EMPLOYED

Three soils were used. The sample of Cecil clay loam was taken from the soil profile of the Erosion Experiment Station at Statesville, North Carolina. The B horizon, 6–32 inches, was used. It contains 12.1 per cent. silt and 57.1 per cent. clay. It is a lateritic soil, low in organic matter, low in base content and in exchange base capacity. The finer particles are cemented together rather firmly and it disperses with difficulty.

The sample of Beckett silt loam is a part of a profile taken in a forested area in Massachusetts. The B_2 horizon, 13–24 inches, was employed. It has 30.2 per cent. silt and 9.0 per cent. clay. The Beckett is a podsol soil and is exceptionally acid (pH = 4.1) in this horizon. While it has a fairly high exchange base capacity, it is low in bases. The organic matter and iron-oxide content are both high.

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[1] U. S. D. A. Tech. Bul. 170, p. 19.

The sample of Amarillo silt loam is from a profile obtained in Potter County, Texas. The stratum used was that from 10 to 20 inches in depth. It has 41 per cent. silt and 47.4 per cent. clay. The Amarillo is a chernozem soil fully saturated with bases.

The descriptions, though brief, are sufficient to show that the soils selected were developed under a wide range of climatic conditions and offer a fair range of physical and chemical characteristics. It is believed that if increase in weight, as a result of dispersion, is a phenomenon of common occurrence, it should be shown by one or more of these soils.

EXPERIMENTAL DATA

Duplicate air-dry samples, of 2.5 grams, were weighed into Erlenmeyer flask-shaped weighing bottles of 30 cc capacity. The weighing bottles were then put in a vacuum desiccator and the air removed with a vacuum pump having a rating of 0.01 mm pressure. The pressure remained constant at 4 mm for several hours under continuous operation of the pump. After 24 hours evacuation the pressure became too low to read on the manometer and was probably less than 0.1 mm. The samples were weighed on successive days until the weight became constant. Twenty-two cc of water, slightly alkaline with ammonia, were added and the flasks were shaken for 50 hours in a reciprocating shaker. The flasks were then placed in a desiccator and dried under half an atmosphere pressure at 35°. The drying was exceedingly slow. When all visible moisture had been removed, the drying *in vacuo* was completed exactly as in the initial drying. This method of drying before and after dispersion was used in order to avoid the known alteration of weight of organic matter at 105–110°. The initial and final weights are given in Table 1. After the final vacuum drying the samples were allowed to stand for several days in the laboratory and then dried over night at 105° C. The changes in weight from the previous drying are given in the last two columns of Table 1. The small increases noted for the Cecil and Amarillo samples are probably due to air absorbed and the loss in the Beckett to change in the organic content.

TABLE 1

VACUUM-DRIED SOIL WEIGHTS BEFORE AND AFTER DISPERSION

Lab. No.	Soil series	Initial weight by vacuum drying		Final weight by vacuum drying after dispersion		Change in weight after dispersion		Change in weight at 105° C.	
		Sample 1	Sample 2	Sample 1	Sample 2	Sample 1	Sample 2	Sample 1	Sample 2
		gm	gm	gm	gm	gm	gm	gm	gm
6978	Cecil	2.4583	2.4580	2.4599	2.4597	+.0016	+.0017	+.0031	+.0036
4447	Beckett	2.4596	2.4607	2.4590	2.4600	–.0006	–.0007	–.0055	–.0038
4575	Amarillo	2.4169	2.4166	2.4207	2.4211	+.0038	+.0045	+.0039	+.0039

It will be observed that the results are essentially negative. That is, the weight changes are very slight and of the order of magnitude of the unavoidable errors of the experiment. The maximum increase is but 0.16 per cent.

This experiment, having failed to demonstrate the point at issue, under conditions presumably favorable to increase in weight after dispersion, if such increase actually occurs, might seem conclusive. However, soils are usually dried at 105–110° C. and therefore the experiment was repeated in modified form. For this purpose four samples of each of the soils were brought to like condition of humidity by standing in a desiccator over sulfuric acid, with a relative humidity of 65 per cent., until they ceased changing in weight. They were then accurately weighed. Each sample weighed approximately three grams. One sample of each was dispersed with 22 cc of distilled water. One of each was dispersed with 22 cc of 0.2 N. ammonium hydroxide. They were dispersed by shaking for 32 hours on a reciprocating shaker. The dispersed samples were then dried on a water bath until visible water had disappeared and were then placed in an oven at 105° The remaining two samples of each soil were placed in the same oven under identical conditions and dried over night. The undispersed samples were in duplicate in order to show the degree of accuracy obtainable. The experimental results are given in Table 2.

Instead of recording in the table the actual initial weights of the samples, the data are recalculated on the basis of exactly 3 grams each and the oven-dry weight corresponding to this given in column 5. The last column in Table 2 gives the difference between the oven-dry weight and the mean value of the undispersed samples. It may be observed that the deviations of the dispersed samples are of the same order of magnitude as those of the single undispersed samples, the largest being but 3 mg. The total dry weight of the dispersed samples is but 1 mg. less

TABLE 2
CHANGE IN WEIGHT OF SOILS AT 105° C.

Lab. No.	Soil series	Sample treatment	Initial weight	Oven dry weight	Deviations from average oven dry wt. of moisture samples
			gm	gm	gm
6978	Cecil	Air dry (1)	3.0000	2.9231	+.0003
“	“	“ (2)	3.0000	2.9225	−.0003
“	“	Dispersed with ammonia	3.0000	2.9258	+.0030
“	“	Dispersed without ammonia	3.0000	2.9241	+.0013
4447	Becett	Air dry (1)	3.0000	2.9294	+.0005
“	“	“ (2)	3.0000	2.9304	−.0005
“	“	Dispersed with ammonia	3.0000	2.9300	+.0001
“	“	Dispersed without ammonia	3.0000	2.9278	−.0021
4575	Amarillo	Air dry (1)	3.0000	2.8106	−.0015
“	“	“ (2)	3.0000	2.8136	+.0015
“	“	Dispersed with ammonia	3.0000	2.8114	−.0007
“	“	Dispersed without ammonia	3.0000	2.8110	−.0011

than that of the undispersed samples. On the basis of this experiment, therefore, if any weight change occurs as a result of dispersion, it does not exceed 0.1 per cent. It is to be observed that a very small increase in weight should occur in acid soils dispersed with ammonia, since ammonia adds itself directly to acids and this addition has been repeatedly demonstrated in soils. In the case of the Cecil sample such an increase is noted. The total base capacity of the Cecil is about 1 milliequivalent per gram and the absorption of this quantity would mean an increase in the sample used of 2.5 milligrams. No such difference should occur in the Amarillo soil and indeed the difference between the weights of the sample dispersed in ammonia and that in water is less than that of the undispersed samples. The Beckett soil is also unsaturated like the Cecil, and shows a difference of about 2 mg between the sample dispersed in ammonia and that in water. Both show negative values which are probably properly ascribed to loss of organic matter during the evaporation of the dispersed material. The differences in the weight of the undispersed samples, while not large, are probably due to actual lack of uniformity in the samples themselves which were sieved to pass a 2 mm sieve. Closer duplication might have been secured by finer grinding of the material.

The results obtained are essentially negative. There is no appreciable increase in weight of dry samples as a result of dispersion in water. The writers realize, of course, that as soils are ordinarily dispersed, much greater dilutions of soil by water are employed. The concentrations employed were used in order to minimize any possible loss in handling. It seems improbable that greater dilution would give greater hydration. Also, while in the normal process of dispersion the colloid and coarser material are dried and weighed separately, there would seem, especially in soils so high in clay as these employed, to be no reason why separate drying should alter the matter. The writers are also aware that these experiments do not prove that hydration of soils by dispersion never occurs. They do show that if such result were common and of appreciable magnitude, it should have occurred in the soils examined. It seems desirable to put the results on record and to request that, if any soil worker can furnish reproducible data to show that increase in the dry weight of soils results from dispersion, he will likewise communicate the results. It is worth while to determine whether the thing is mythical or if it constitutes a real excuse for inaccurate data.

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BOOKS RECEIVED

BÉHAR, M. F. *Fundamentals of Instrumentation.* Pp. xiii + 109. 48 figures. Instruments Publishing Company, Pittsburgh. $2.00.

BAITSELL, GEORGE A. *Manual of Animal Biology.* Pp. xiii + 382. Illustrated. Macmillan. $2.50 .

CURTIS, HARRY A., Editor. *Fixed Nitrogen.* Pp. 517. 82 figures. Chemical Catalog Company. $12.00.

LASHLEY, K. S., Editor. *Studies in the Dynamics of Behavior.* Pp. xiv +323. Illustrated. University of Chicago Press. $5.00.

MITCHELL, PHILIP H. *A Text-Book of General Physiology for Colleges.* Second edition. Pp. xviii + 799. 196 figures. McGraw-Hill. $6.00.

Report of the National Research Council of Japan, No. 8–9, 1928–1930. Pp. 229–350. The Council, Tokyo.

ZUCKERMAN, S. *The Social Life of Monkeys and Apes.* Pp. xii + 356. 24 illustrations. Harcourt, Brace. $3.75.