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MODELS OF THE PHYSICAL UNIVERSE[1]

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INTRODUCTION

FIRST of all, I wish to express my thanks to Professor Boodin and to Professor Miller for suggesting that the Philosophical Union might be interested in some of the recent work of theoretical physicists in the field of cosmology. More than two thousand years ago, at the instigation of Aristotle, scientific expeditions were financed by Alexander the Great to explore the countries which he had conquered and report their findings to the great philosopher and his school[2]; and in coming as a scientist to report to you—who are philosophers—I feel that I am acting in accordance with an honorable and desirable tradition.

Nevertheless, certain changes have occurred in the relations between science and philosophy since the golden time of Aristotle which must not be overlooked.

In the first place, philosophy and science are no longer a united study, and it is not the philosophers but the scientists themselves who now organize and direct their own explorations. Indeed, I think it would have been an unhappy day for science if Galileo had not broken from the Aristotelian tradition, and in any case, whether for better or worse, science has long since issued a declaration of independence from philosophy which can not now be disregarded.

In the second place, the developments of philosophy and science have in the meantime been so extensive and complicated that both disciplines and even their separate branches have been forced to invent special technical languages for the discussion of their subject-matters. Indeed, I have heard a humorous, and I trust untrue, definition of philosophy which

[1] Address before the Philosophical Union, University of California at Los Angeles, February 17, 1932.

[2] See Pliny, "Hist. Nat.," VIII, 16.

reads: "Philosophie ist der systematische Misbrauch einer eigens zu diesem Zwecke erfundenen Terminologie."[3] However this may be, it is abundantly evident that scientists and philosophers now have almost insurmountable difficulties in talking with each other, and indeed among themselves, because of a new Babel of tongues.

This afternoon I am particularly impressed by this latter difficulty, since I wish to describe to you, in ordinary English, results which can only be obtained by using the language of a special and somewhat complicated branch of mathematics. For this reason I shall confine my mathematics to a few equations which will be projected on the screen, and shall be forced, I am afraid, to say some things which are not very detailed and precise but which will I hope convey a moderately correct impression.

THE PROBLEM OF COSMOLOGY

For the scientist the problem of cosmology lies in an attempt to describe and understand the structure and behavior of the physical universe as a whole, looked at from a large-scale point of view which considers the average distribution and properties of the matter and radiation of which the universe is constructed, but neglects local details such as the structure of our own earth and solar system, or even the particular arrangement of stars in our own galaxy. Indeed, the scale of view is so large that the contents of the universe are in some ways treated like a fluid with the stars taking the place of atoms and the galaxies or nebulae playing the rôle of molecules.

The method of attacking this problem of cosmology is twofold, observational and theoretical.

For the observational material we must depend on the astronomer, in particular on the remarkable work of Hubble at the Mount Wilson Observatory. With the 100-inch telescope it has been possible to penetrate to a depth of about 10^8 (one hundred million) light-years, way beyond the limits of our own galaxy of stars, whose diameter is only of the order of 10^5 (one hundred thousand) light-years. Scattered throughout all this region at great distances from one another we find numerous nebulae, some millions in all, which themselves appear to be great collections of stars similar to our own milky way. These nebulae are grouped to a certain extent in clusters, but from a large-scale point of view they may be regarded as distributed with a reasonable uniformity, and in particular show no tendency to decrease in concentration as we go farther and farther out into space. A specially important characteristic in the light from the nebulae is a shift towards the red in the position of their spectral lines, this red shift increasing proportionally to the distance out to the nebula. As to the contents of extra-galactic space other than the nebulae, for example, dust and radiation, we can say very little. Owing to the great volume of space between the nebulae, Hubble estimates that the amount of matter spread out in the universe in the form of dust could be several thousand times as great as that concentrated in the nebulae themselves, without producing effects which so far would have been detected. And on the other hand, I have calculated that the temperature of intergalactic space would only have to be 12 degrees absolute to give a density of radiation as large as Hubble's averaged out density of matter.

[3] Told to me by my colleague, Professor Fritz Zwicky. "Philosophy is the systematical misuse of a terminology specifically invented for this purpose."

The theoretical method of treating the problem of cosmology is the one which we are to discuss this afternoon. It has three tasks: first, the interpretation of the findings of the astronomer in terms of accepted or acceptable theory; second, the extrapolation of the findings of the astronomer farther out into space and forward and backward in time together with the prediction of additional phenomena; and third, investigations to determine the conceivable properties which the universe could have in accordance with satisfactory theory.

All three of these tasks are attacked by constructing conceptual models of the universe, in accordance with the principles of theoretical physics, and then investigating the properties and behavior of these theoretical models, for purposes of comparison with what we know or might find in the actual universe. These models are highly abstract and idealized, as contrasted with the actual universe; indeed, they have to be in order to make it possible to treat them with the mathematics at our disposal. When our main interest lies in the interpretation or extrapolation of some property of the actual universe, which has already been observed by the astronomer, we construct conceptual models which exhibit this property and of course as far as possible the other known properties of the actual universe. But when we are searching for the conceivable properties which the universe could have and which might sometime become subject to observation, we allow ourselves great latitude in the kind of models that we study. For example, even though the actual universe is known to contain matter, we may find it informing to consider the properties of a cosmological model which is completely empty, as was done by de Sitter, or one which contains nothing but radiation, as has been done by Silberstein and by myself.

THE PRINCIPLES FOR CONSTRUCTING THE MODELS

We must now inquire into the principles on which the cosmological models are to be constructed. This is a matter of the greatest importance, since the chief difference between the work of the theoretical scientist in trying to understand the properties and behavior of the universe and the fantastic speculations of amateur cranks lies in the attempt of the scientist to make his considerations conform to a satisfactory theory.

The first requirement for the construction of acceptable cosmological models is that they shall conform to the principles of mechanics. For this purpose it is not sufficient to use the classical mechanics of Newton, which has been found to give a correct description of mechanical phenomena in a limited region of space and in the absence of strong gravitational fields. It is necessary, instead, to use the relativistic mechanics of Einstein, which contains the Newtonian mechanics as an approximation, and in addition meets the three so-called crucial tests of relativity, by giving correct predictions of the motion of the perihelion of Mercury, of the bending of light in passing the sun and of the shift in the wave-length of light that originates in a strong gravitational field.

It would be too difficult and time-consuming a task to try to give a logical presentation of the axioms of relativistic mechanics and their physical interpretation. I may call your attention, however, to some of the fundamental equations.

The space-time continuum in which physical events take place is regarded as characterized by a line element of the form

$$\begin{aligned} ds^2 &= g_{11}dx_1^2 + 2g_{12}dx_1dx_2 + \cdots + g_{44}dx_4^2 \\ &= g_{\mu\nu}dx_\mu dx_\nu \qquad (g_{\mu\nu} = g_{\nu\mu}) \end{aligned} \tag{1}$$

where x_1, x_2, x_3 and x_4 are the spatial and temporal coordinates and the $g_{\mu\nu}$ are the ten gravitational potentials. The motion of light rays and material particles in this space-time is given by the equations

$$ds = 0 \qquad \text{(For light rays)} \tag{2}$$

$$\delta\int ds = 0 \qquad \text{(For particles)} \tag{3}$$

And the dependence of the gravitational potentials on the distribution of matter and energy is given by the ten equations

$$-8\pi T_{\mu\nu} = G_{\mu\nu} - \tfrac{1}{2}Gg_{\mu\nu} + \Lambda g_{\mu\nu} \tag{4}$$

where $T_{\mu\nu}$ is the energy-momentum tensor, $G_{\mu\nu}$ and G are obtained from the Riemann-Christoffel tensor by contraction, and Λ is the so-called cosmological constant. Although the precise value of the quantity Λ is unknown, we do have observational evidence that shows that it is in any case a very small quantity. Hence we should prefer to take it zero if possible.

In addition to applying relativistic mechanics in the construction of cosmological models, I have found that it is also necessary to apply the principles of relativistic thermodynamics. For this purpose an extension of thermodynamics to general relativity was needed. To obtain this[4] I have taken as the analogue of the first law of classical thermodynamics the known relativistic equation

$$\frac{d\mathfrak{T}_\mu^\nu}{dx_\nu} - \tfrac{1}{2}\mathfrak{T}^{\alpha\beta}\frac{dg_{\alpha\beta}}{dx_\mu} = 0 \tag{5}$$

which corresponds in relativistic mechanics to the classical principles of the conservation of energy and momentum. And I have taken as the analogue of the classical second law of thermodynamics an expression

$$\frac{d}{dx_\mu}\left(\varphi_0\sqrt{-g}\,\frac{dx_\mu}{ds}\right)dx_1\,dx_2\,dx_3\,dx_4 \geqq \frac{dQ_0}{T_0} \tag{6}$$

which can be shown to be a covariant generalization of the usual second law. The quantity φ_0 in this expression is the proper entropy density of the thermodynamic fluid as measured by a local observer, the quantities dx_μ/ds the components of the macroscopic "velocity" of the fluid, dQ_0 the heat as measured in proper units flowing into the region $dx_1\,dx_2\,dx_3$ in the time dx_4, and T_0 the proper temperature. The equals sign ($=$) applies to reversible processes and the sign "is greater than" ($>$) to irreversible processes.

STATIC COSMOLOGICAL MODELS

Let us now turn to the actual cosmological models that have been constructed in accordance with these principles.

The first of these was the famous static model of Einstein[5] with the line element

$$ds^2 = -\frac{dr^2}{1 - r^2/R^2} - r^2d\theta^2 - r^2\sin^2\theta\,d\varphi^2 + dt^2 \tag{7}$$

where R, the so-called "radius" of the model, is a constant. This line element corresponds to a permanent uniform distribution of matter and energy, having from a large-scale point of view the pressure p and density ρ as given by the equations

$$8\pi p = -\frac{1}{R^2} + \Lambda \tag{8}$$

$$8\pi\rho = \frac{3}{R^2} - \Lambda \tag{9}$$

Although the study of this model has been very instructive, two objections to it may be immediately mentioned. In the first place being a static model it prescribes a state of rest for the heavenly bodies and gives no account of the apparent recession of the nebulae as shown by the red-shift in the lines of their spectra. And in the second place it necessitates a positive value of the cosmological constant Λ to prevent the expression for the pressure from becoming

[4] Tolman, *Proc. Nat. Acad.*, 14, 268, 701, 1928; *Phys. Rev.*, 35, 876, 896, 1930.

[5] Einstein, "Berl. Ber.," p. 142, 1917.

negative, and as mentioned before we should be glad to set $\Lambda = 0$ if possible.

The second relativistic cosmological model to be constructed was that of de Sitter[6] with the line element

$$ds^2 = -\frac{dr^2}{1 - r^2/R^2} - r^2 d\theta^2 - r^2 \sin^2 \theta d\varphi^2 + (1 - r^2/R^2) dt^2 \quad (10)$$

where the radius R is connected with the cosmological constant by the equation

$$\frac{3}{R^2} = \Lambda \quad (11)$$

and the line element corresponds to a completely empty universe with the pressure and density everywhere both equal to zero

$$p = \rho = 0 \quad (12)$$

The most immediate objection to this model is of course its complete emptiness, which means that the matter in our actual universe would produce some distortion away from the proposed form. Furthermore, although it was found that particles introduced into such a universe would undergo complicated motions, provided Λ was not set equal to zero, a good explanation accounting both for the red-shift and the density of distribution of the nebulae was not obtained with this model.

NON-STATIC COSMOLOGICAL MODELS

The two line elements of Einstein and de Sitter are called static, since the gravitational potentials $g_{\mu\nu}$ do not depend on the time. Since neither of these models is found to agree with observational facts, it is first natural to inquire if perhaps some other static line element would give an explanation of the red-shift. Some years ago, however, I found[7] it possible to prove that the Einstein line element and the de Sitter line element are the only possible *static* ones which would correspond to a universe having homogeneous properties throughout, and suggested that the study of *non-static* line elements would be very interesting, especially since static line elements take no explicit cognizance of any general evolutionary processes that may be occurring in the actual universe.

As a matter of fact, such studies had already been initiated by a gifted Russian, Friedman,[8] who had given in 1922 an expression for the very non-static line element that is now the subject of so much investigation. The derivation which he gave for the line element is not entirely satisfying, and he considered no direct physical application of his results. He had, however, a masterful grasp of the problem and in spite of the lack of attention that has been paid to his article should receive credit as the originator of the new chapter in cosmology.

This non-static line element can be written in the form

$$ds^2 = -e^{g(t)}\left(\frac{dr^2}{1 - r^2/R^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2\right) + dt^2 \quad (13)$$

where R is a constant and the dependence of the line element on the time is given by the exponent $g(t)$. This line element corresponds to a homogeneous distribution of matter and energy having the pressure p and density ρ

$$8\pi p = -\frac{1}{R^2} e^{-g} - \frac{d^2g}{dt^2} - \frac{3}{4}\left(\frac{dg}{dt}\right)^2 + \Lambda \quad (14)$$

$$8\pi\rho = \frac{3}{R^2} e^{-g} + \frac{3}{4}\left(\frac{dg}{dt}\right)^2 - \Lambda \quad (15)$$

which will in general be changing with the time owing to their dependence on the quantity g, which is itself a function of the time. As has been specially emphasized by Einstein, this line element would not require a value of Λ differing from zero in order to prevent the pressure from becoming negative provided d^2g/dt^2 is negative.

The first application of this non-static line element to the phenomena of the actual universe was made by Abbé Lemaitre,[9] of the University of Louvain, who noted that the line element would correspond to an expanding model of the universe if g is increasing with the time, and showed that this would permit an immediate explanation of the mysterious red-shift in the light from the nebulae, since in an expanding universe the nebulae would be moving away from us and this would be expected to increase the wavelength of the light that we receive from them by what is known as the Doppler effect. His detailed treatment contained an unnecessary and improbable assumption as to the value of the cosmological constant Λ, and a specialization which would rule out the presence of radiation and the transformation of matter into radiation in the universe; but the important discovery that the red-shift in the light from the nebulae can be interpreted with the help of an expanding model of the universe is due to him.

The work of Lemaitre was followed by a very fundamental article by Professor Robertson,[10] of Princeton, giving a derivation of the non-static line element for the universe based on very general mathematical assumptions, and by a derivation of my own[11] based on the physical assumption that we can regard the material filling the universe, from a large-scale point of view, as having uniform prop-

[6] de Sitter, *Monthly Notices, R.A.S.*, 76, 77, 78, 1916–1917.
[7] Tolman, *Proc. Nat. Acad.*, 15, 297, 1929.
[8] Friedman, *Zeit. Phys.*, 10, 377, 1922.
[9] Lemaitre, *Ann. Soc. Sci. Bruxelles*, 47, Series A, 49, 1927.
[10] Robertson, *Proc. Nat. Acad.*, 15, 822, 1929.
[11] Tolman, *Proc. Nat. Acad.*, 16, 320, 1930.

erties at a given time throughout the whole of space. At the time I did not know of the previous work that had been done in this field and was led to investigate non-static models from the consideration that the transformation of matter into radiation, which appears to be taking place everywhere throughout the universe, would necessarily lead to a non-static line element. The result obtained showed that it was possible to account both for the transformation of matter and the recession of the nebulae with the help of a non-static line element.

A very powerful additional argument in favor of the non-static line element for the universe was then presented by Eddington,[12] who showed that even if we should start with a static Einstein model of the universe, this would be unstable and would change to a non-static model as the result of small disturbances. The non-static line element for the universe was then also considered by de Sitter[13] who calculated the theoretical course of expansion and contraction for a large number of different conceptual models, assuming that the transformation of matter into radiation could be neglected and assuming the general form of the non-static line element, but making different assumptions as to the unknown quantities R and Λ in the equations.

Since then there have been many further articles[14] on the non-static line element, not only because there are many special problems to be considered into which we can not go here, but also because our knowledge of the actual universe is still too limited to allow a decision between different specific possibilities which would be in accord with the general form of the non-static line element. For this latter reason it is profitable for the theoretical physicist to consider many different conceptual possibilities as to the universe, in advance, and perhaps sometimes generations in advance, of the observational possibilities available to the astronomer. As a result of all the work that has been done, nevertheless, there has grown the increasing conviction that it is an entirely sensible procedure to attempt to explain the once mysterious red-shift in the light from the extra-galactic nebulae as due to an expansion of the universe, or, at the least, of that part of the universe which we have so far observed.

12 Eddington, *Monthly Notices, R.A.S.*, 90, 668, 1930.

13 de Sitter, *Bull. Astron. Inst. Netherlands*, 5, 211, 1930; *ibid.*, 6, 141, 1931.

14 Tolman, *Proc. Nat. Acad.*, 16, 409, 1930; *ibid.*, 16, 511, 1930; *ibid.*, 16, 582, 1930; McRae and McVittie, *Monthly Notices, R.A.S.*, 91, 128, 1930; McVittie, *Monthly Notices, R.A.S.*, 91, 274, 1930; Laue, *Naturwiss.*, 19, 530, 1931; Einstein, "Berl. Ber.," p. 235, 1931; Takēuchi, Proc. Phys. Math. Soc. Japan 13, 166, 1931; Heckmann, *Nachr. Wiss. Goettingen*, II, p. 126, 1931; McRae and McVittie, *Monthly Notices, R.A.S.*, 92, 7, 1931.

APPLICATION OF RELATIVISTIC THERMODYNAMICS TO COSMOLOGICAL MODELS

The foregoing studies of cosmological models have all been based on relativistic mechanics. To complete our considerations we must also consider the thermodynamic aspects of their behavior. This has appeared specially important to me and has engaged a good deal of my attention.

In the case of the older static models no thermodynamic studies were necessary, since nothing at all ever happened in these static models. In the case of the newer non-static models, however, we must inquire into the compatibility of their behavior not only with the principles of relativistic mechanics, and hence with the relativistic form of the first law of thermodynamics, but also as to compatibility with the relativistic form of the second law of thermodynamics. By doing so we gain, I think, two important extensions in our ideas as to the kind of process which might take place in the universe, without contradicting the rules enforced by that disintegrating ogre, the second law of thermodynamics.

The first of these extensions has to do with the occurrence of thermodynamically reversible processes, that is, processes which take place without increase in entropy and without dissipation of energy. In the classical thermodynamics, disregarding purely mechanical changes, it was found that no processes could take place both at a finite rate and at the same time reversibly without increase in entropy. To make processes reversible, it was found necessary to carry them out at an infinitesimally slow rate, in order to obtain, so to speak, the perfect efficiency necessary for reversibility. In relativistic thermodynamics, however, we find a possibility for certain reversible processes to take place both at a finite rate and without increase in entropy, owing to the possibility of correlated changes in entropy density and gravitational field which were neglected in the older theory. This is a matter of some importance, since in the past we have felt that we would be obliged to interpret any thermodynamic change taking place in the universe at a finite rate as evidence that the universe was necessarily running down.

I have considered this new possibility in connection with two different conceptual non-static models of the universe,—one containing nothing but radiation,[15] and the other containing an equilibrium mixture of radiation and perfect gas.[16] It was found that both of these models could expand or contract at a finite rate and yet reversibly without increase in entropy —the first necessarily so, and the second under the additional assumption that the rates of transformation

15 Tolman, *Phys. Rev.*, 37, 1639, 1931.

16 Tolman, *Phys. Rev.*, 38, 797, 1931.

of matter into radiation and the reverse were fast enough to maintain equilibrium conditions. It was also shown that there would be a number of phenomena in these models, such, for example, as the outward flow of radiation in an expanding model, which would be interpreted by an ordinary observer familiar only with the classical thermodynamics as definite evidence for the irreversible dissipation of energy into the "cold depths" of space, in spite of the fact that from the more legitimate point of view of relativistic thermodynamics everything would really be taking place entirely reversibly.

It is also to be noted that in the case of processes which take place reversibly without increase in entropy there would be no thermodynamic hindrance to a repetition of the same process over and over again. And, indeed, in the case of the two models just mentioned I have been able to show that they could undergo a continued succession of identical expansions and contractions, without ever coming to a state of rest, provided the cosmological constant in Einstein's equations is zero or negative.[17]

The second extension provided by relativistic thermodynamics in our ideas as to the kind of changes that can take place without contradicting the second law of thermodynamics has to do with the results to be expected when irreversible processes take place. The consideration of irreversible processes is very necessary, since, although we can imitate some of the important phenomena in the actual universe with the help of models that change reversibly, it is evident that reversible processes form only a limited class of the totality of conceivable thermodynamic processes. And we can not expect that the phenomena of the actual universe are all of them really reversible.

In accordance with the classical thermodynamics, when irreversible processes take place in an isolated system, the energy of the system has to remain constant on account of its isolation, and the entropy can then only increase to the upper maximum value which is consonant with this energy. This upper limit corresponds to the most probable internal arrangement of the system attainable with the fixed amount of energy available, and when this limit is reached no further changes would be possible. Hence the classical thermodynamics has accustomed us to believe that the entropy increases taking place in the universe are leading towards a final run-down condition of maximum entropy where all change would cease—the sun and stars cold, all of creation dead and lifeless.

[17] Tolman, *Phys. Rev.*, 38, 1758, 1931. Such expansions and contractions also occur in the case of a model previously treated by Einstein (Ref. 14) which was, however, of such a simple mechanical nature as not to suggest the application of thermodynamics.

In relativistic thermodynamics, however, a different condition of affairs arises, owing to the fact that the principles of relativity do not prescribe a constant value for the total integrated proper energy of an isolated system. For this reason when irreversible processes take place in an isolated system the accompanying increase in entropy is not necessarily subject to the limit imposed in the classical mechanics by a fixed value of the energy, and the possibility arises for isolated systems in which the entropy would continue to increase without ever reaching a maximum value where all change would have to cease.

I have examined this possibility in connection with non-static models of the universe,[18] and have found it possible, if the cosmological constant is zero or negative, to construct conceptual models in which irreversible processes, such as the transformation of matter into radiation under non-equilibrium conditions, would take place, and nevertheless which would show a continued succession of expansions and contractions of increasing amplitude without ever coming to that dreadful final state of quiescence predicted by the classical thermodynamics.

Evaluation of the Results

This completes the description of cosmological models which I had in mind, and we may now ask how far has the theoretical physicist been successful in his three tasks of explanation, of extrapolation and prediction, and of investigation into the conceivable properties that the universe might theoretically have.

In his first task I think that the physicist has been reasonably successful, since he has been able to construct models in accordance with the principles of physics which agree with the findings of the astronomer in at least four ways. They contain a finite density of matter; this matter is uniformly distributed; the gradual transformation of matter into radiation is permitted; and the light from distant objects would show a red-shift approximately proportional to the distance.

As to his second task, that of extrapolation and prediction, I do not think that as yet he has been very successful. He has indeed been able to make one prediction as to a relation that should be found between the apparent diameter, luminosity and red-shift of the nebulae,[19] which can be subjected to observational test. As to the extrapolation of present findings to great distances, however, neither the astronomer nor the physicist can now say whether or not the density of matter and rate of expansion

[18] Tolman, *Phys. Rev.*, 39, 320, 1932.
[19] See Tolman, second article, ref. 14.

remain approximately the same as has been observed out to 10^8 light years, and we can not yet even tell whether the universe is finite or hyperbolic.[20] And although we can completely describe the life history of special models, nevertheless, if we desire to extrapolate in time for the actual universe, we can merely say with some confidence that the process of expansion has lasted for an enormous time in the past and will continue for a very long time in the future.

In the third task of investigating the conceivable theoretical properties of the universe, there has perhaps been somewhat more success, since the conceivable is subject to less rigorous constraints than the actual. And I think the new thermodynamic possibilities which I have described to you are of considerable interest.

POSSIBLE PHILOSOPHICAL BEARINGS

In conclusion I should like to make a few remarks which may have a certain philosophical bearing.

First, I wish to call your attention to the consideration that although the principles of the physical theory for treating such problems as the one we have described must of course in any case agree with observational facts, these principles are actually suggested to the theoretical physicist in two ways—not only as immediate generalizations of experimental findings, but also as desiderata for the inner harmony and simplicity of the theoretical structure he is attempting to build. We must admire Galileo for insisting on observational fact as the ultimate arbiter, and thus breaking away not from Aristotle but from a decadent Aristotelian tradition. And we must admire him for his power and skill in obtaining physical principles from the immediate generalization of experimental facts. But we must not let this just admiration blind us to the power and skill of those other theoretical physicists who obtain the suggestion for physical principles from the inner workings of the mind and then present their conclusions to the arbitrament of experimental test.

This method of proceeding finds a supreme illustration in Einstein's discovery of the system of relativistic mechanics which we have used in constructing our cosmological models. Einstein did not and could not have based his theory merely on attempts to generalize from the known fact that the orbit of Mercury showed an extremely slight deviation from that predicted by Newtonian mechanics. He built his theory rather on two fundamental principles, the principle of equivalence and the principle of covariance, which suggest themselves on grounds of simplicity, generality and logicality; and the theory so obtained was then found to contain not only an explanation for the orbit of Mercury but predictions as well of those other phenomena which have since been verified.

The next matter to which I wish to direct your attention once more is the highly abstract, simplified and idealized nature of the cosmological models that we construct, as compared with the actual universe. This is nothing new in physics, but illustrates a procedure always employed to a greater or less degree in theoretical considerations. It is analogous to the rigid weightless levers of simple mechanics, or the spherical, rigid and perfectly elastic molecules of the simple kinetic theory of gases. The reason for this idealization is obvious. Without such simplifications as are provided, for example, by the assumption of a uniform, homogeneous distribution of matter and radiation throughout our model, the already difficult mathematics would become extremely hard to handle. If, however, we find that our simplified models exhibit such properties as a transformation of matter into radiation and a red-shift in the light from distant objects, we can have a feeling of comfort when we discover these phenomena in the actual universe. And if we discover hitherto unsuspected thermodynamic properties in some of our models, we can at least keep our eyes open for such possibilities in the actual universe.

As a final remark that might have some philosophical interest, I should like to emphasize the special danger in the field of cosmology of the evils of autistic or wish-fulfilling thinking. In studying the problem of cosmology we are immediately aware that the future fate of man is involved in the issue, and we must hence be particularly careful to keep our judgments uninfected by the demands of religion, and unswerved by human hopes and fears. Thus, for example, what appears now to be the mathematical possibility for a highly idealized conceptual cosmological model, which would expand and contract without ever coming to a final state of rest, must not be mistaken for an assertion as to the properties of the actual universe, concerning which we still know all too little. To conclude then: Although I believe it is appropriate to approach the problems of cosmology with feelings of awe for their vastness and of exultation for the temerity of the human spirit in attempting their solution, they must also be approached at the same time with the keen, balanced, critical and skeptical objectivity of the scientist.

[20] See Heckmann, ref. 14.