SCIENCE

VOL. 75 FRIDAY, MARCH 25, 1932 No. 1943

SCIENCE: A Weekly Journal devoted to the Advancement of Science, edited by J. MCKEEN CATTELL and published every Friday by

THE SCIENCE PRESS

New York City: Grand Central Terminal

Lancaster, Pa. Garrison, N. Y.

Annual Subscription, $6.00 Single Copies, 15 Cts.

SCIENCE is the official organ of the American Association for the Advancement of Science. Information regarding membership in the Association may be secured from the office of the permanent secretary, in the Smithsonian Institution Building, Washington, D. C.

THE RÔLE OF HYPOTHESIS IN ECONOMIC THEORY[1]

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THE distinction between a natural and a theoretical science lies essentially in the presence or absence of a free spirit of making hypotheses and definitions. In a natural science facts are recognized and systematized, with a purpose purely descriptive, but as the same field of knowledge is investigated in theoretical fashion, definitions become constructive rather than denotive, and hypotheses are introduced and tried out, in order to see what sort of results may be deduced from them. If the chains of deductive reasoning are complicated the science is driven to employ a characteristic method for their simplification. The mathematical method is at this point a requirement for progress.

Mathematics itself is not always completely theoretical. In geometry, we still reason from "general" geometric figures, much in the manner described by Kant, and are content with that kind of reasoning until we encounter contradictions which force us to make a further analysis. Even when we know that a complete analysis is possible, and the nature of the system completely definable by more or less logical postulates and definitions, we are sometimes inclined, as mathematicians, to turn that analysis over to the logicians, and forget it.

The question naturally arises as to the degree to which we may speak of a theoretical economics, and the extent to which we may call it mathematical.

In the first place, we notice constructive definitions. We do not usually give such a definition of a concept like capitalism, contenting ourselves with denot-

[1] Delivered at a joint session of the Econometric Society and of Section K of the American Association for the Advancement of Science, January 1, 1932.

ing the thing as it exists in certain systems, saying America and France are capitalist countries, but Russia is not. On the other hand, when we define rate of interest, rate of exchange, price, velocity of circulation, income tax, we have definitions given entirely in terms of previously acquired concepts. Terms which are in part given by means of earlier concepts and in part by referring to illustrations of them, so that their definitions are partly constructive and partly denotive, are those like monopoly and competition.

As far as a definition is constructive, relations may be deduced from it and other such definitions by a mathematical process. In this manner, from the definitions of price and velocity of circulation we deduce the "equation of exchange"; from the definitions of price and trade indices we express this same equation in another form. We can give the "dimensions" of all quantities fixed by constructive definitions. The mathematics of finance is obtained merely by using the definition of the rate of interest, and combining it with definitions of present value or value at an arbitrary time. Strange as it may seem to the novice, one or two such definitions may thus lead to an extensive body of real knowledge.

But we do not in this way exhaust the entire body of economic theory. In fact, we may say that the main object of economic theory is to make hypotheses, to see what relations and deductions follow from such hypotheses, and finally, by testing the consequences in comparison with the facts of existing economic systems, to describe them in terms of those hypotheses.

Demand is a concept which, according to Marshall, is destined to occupy continually increasing prominence in economics. What do we mean by demand? What hypotheses are connected with it? Perhaps we may say that the demand for a commodity is the amount of it which would be bought in unit time in a given "state" of the system. But then we must add some hypothesis or definition as to what constitutes the "state" on which the demand depends, for it is not practicable to have very many variables. We may, for instance, consider the following assumptions, which are not by any means a complete list of those which have practical meaning;

(1) $y = f(p)$, p the price of the commodity,
(2) $y = f(p, \frac{dp}{dt})$
(3) $y(t) = f(p(t-T))$, T a constant,
(4) $y(t) = f(p(\tau))$, τ all values between t–T and t,
(5) $y = f(p_1, p_2, \ldots, p_k)$, the p's being prices of several commodities.

We might perhaps start with (1) and make a simple approximate study of equilibrium, for in that case several of the possible hypotheses become identical, the prices being assumed to be mere constants. But if we extend our investigations to moving prices our results do not have the same degree of generality, but depend on the particular one of the hypotheses which we assume, these hypotheses for variable prices being all different. We must then for the sake of theory separate carefully the various systems which depend on different hypotheses, and not regard as results, in one system, conclusions which follow only from the postulates of another.

A central concept in economics has been that of utility. This has sometimes been regarded as a psychic quantity, to be identified with satisfaction or pleasure. But underneath such a definition there must lie assumptions, tacit or explicit. Even though we are not willing to assume that this psychic quantity is directly measurable, if we are to use it in equations we have nevertheless to be able to add small increments of it, to compare its rates of change, etc. What, for instance, would a writer like Jevons need to assume about it?

Let S stand for this satisfaction-quantity, let χ stand for a quantity of a particular commodity, and $\dot{\chi}$ for its increase per unit time. Following Jevons[2] we write U as the intensity of sensation or degree of utility, due to increase of $\dot{\chi}$, and regard it as a new kind of psychic quantity. We are supposed to be able to compare these degrees of utility or sensations for various quantities, and we regard U therefore as having some kind of dimension of its own, not necessarily calculable in terms of the dimensions of physical quantities. Let [U] be the dimensions of U, [S] the dimensions of S. Then the pleasure V caused per unit time by a rate of consumption $\dot{\chi} = d\chi/dt$ is a quantity like $U\chi$ and has dimension

$$[V] = [U]\,[\dot{\chi}] = \frac{[U]\,[\chi]}{[t]}.$$

But the total pleasure is like $\int V\,dt$ and therefore has dimensions

$$[S] = [Vt] = [V]\,[t] = [U]\,[\chi].$$

In other words:

$$[\chi] = \frac{[S]}{[U]}.$$

Accordingly, no matter what the commodity χ, its dimensions are given by the above formula. All commodities are quantities of the same dimensions. This means that we can evaluate a unit of one commodity in terms of the unit of another, in the sense

[2] Jevons, "Theory of Political Economy," Ch. III, IV, London, 1871.

that if the unit of pork is changed to one twice as large, the unit of fountain-pens must also be changed to one twice as large, in order for the economic equations to have any meaning. How are appropriate units to be found, in the first place? To say the least, the situation is inconvenient.

Conceivably we object that the particular treatment given by Jevons is unnecessary, and that another based on similar ideas would not necessarily involve the same difficulties—the difficulties arising from the fact that the intensities U are regarded as a single kind of psychic quantity. We shall look at the situation therefore from a less restricted point of view, and consider a theory which does not require the explicit use of any "psychic" quantity.

We leave out of account the question as to whether or not utility is itself measurable, but suppose that there is a quantity associated with it which is measurable, and whose measure we may call an index of utility. If utility is measurable, the measure of utility itself may be taken as the index of utility. Otherwise, we are in the same situation as when we take temperature as an index function for warmth. For any state of the system, accordingly, we assume that there is a number, which, with reference to the individual, may be called the index of that state, and the individual's preference for one state or another is reflected in the magnitude of the corresponding index values. If it is an index of utility each individual tries to make it a maximum. We have thus arrived at a definite hypothesis, and many economists have tried to make it a basis for a general theory of economics.

We can make an estimate of the generality of such a system. In the first place, it is essentially competitive. If we add all the individual index numbers together to form a social utility function, we can not say, although many have said so, that society as a whole works to make this total function a maximum. For that is the same as saying that the maximum of a sum is necessarily the sum of the maxima of the parts—which is absurd. Hence a cooperative aspect of the system—which is so pronounced in modern society—must be introduced as an extra hypothesis. Will this make too many hypotheses for the number of independent variables?

Consider the process by means of which such an index number may be set up. Presumably two situations which are widely different can not be compared, as they are, directly, but only through a process, consisting of small steps, by means of which one step is transformed into the other. That is, we do not compare directly the situation (x_1, y_1, z_1) with the situation (x_2, y_2, z_2), but the state (x, y, z) with the neighboring state $(x+dx, y+dy, z+dz)$. That is, we write

$$(6) \qquad dI = X\,dx + Y\,dy + Z\,dz$$

X, Y, Z being quantities that depend on the state (x, y, z), and then decide whether the change (dx, dy, dz) makes dI positive or negative or zero.

But now we encounter another difficulty. If the state is given by two numbers (x, y) we know that there exists generally a function I such that dI is positive, negative or zero according as

$$X\,dx + Y\,dy > 0,\ < 0 \text{ or } = 0.$$

It is obtained by a proper choice of the integral of the differential equation

$$X\,dx + Y\,dy = 0$$

In other words, we can build up an index function by means of the curves of indifference. But if the state of the system is given by three or more numbers we also know that there does not exist in general such an index function. The expression of this fact in mathematical terms is the statement that an equation like

$$X\,dx + Y\,dy + Z\,dz = 0$$

is not "completely integrable." If we wish to have a utility function we must introduce some hypothesis on the coefficients X, Y, Z.

One simple case is that emphasized by Pareto in which X depends only on x, Y on y, Z on z; for in that case we may write I as some function of the quantity

$$\int X\,dx + Y\,dy + Z\,dz$$

The general condition for three variables, in order that I should exist is

$$X\left(\frac{\partial X}{\partial y} - \frac{\partial Y}{\partial z}\right) + Y\left(\frac{\partial X}{\partial z} - \frac{\partial Z}{\partial x}\right) + Z\left(\frac{\partial Y}{\partial x} - \frac{\partial X}{\partial y}\right) = 0,$$

which seems to have no particular economic significance.

If this condition is not satisfied, and the general integration can not be carried out, the summation of $X\,dx + Y\,dy + Z\,dz$ from one state (x, y, z) to another will depend on the order in which the various increments dx, dy, dz are taken during the process. It will thus depend on the particular process by means of which we pass from the one state to the other, but not merely on the two states themselves. It has sometimes been stated that this summation nevertheless affords a satisfactory determination of I, on the ground that the process is regulated by the customs of industry and markets. But this again is the same as saying that certain relations already hold between the variables x, y, z, . . . ; and they are no longer independent. Moreover, if, in spite of these relations,

there remain three or more independent variables, the difficulty still persists.[3] Hence we must assume that all our situations relative to a utility function must not contain more than two independent variables, or else we must introduce directly a postulate of integrability. It seems an arbitrary limitation.

Having obtained or assumed such a function, let us suppose that we make it a maximum. We are inclined to set dI equal to zero, and without further consideration to regard the problem as solved. But if the maximum occurs on the boundary of the region of variation of the variables, as is likely, for instance, in the case of an individual who is trying to corner the market, there is no need for dI to be zero. Usually, also, there are various subsidiary conditions —restraints, "obstacles," necessary relations between the variables, which have to be satisfied at the same time. And altogether, these equations and the equations given by $dI = 0$ furnish as many equations as there are unknowns. But this is purely an automatic process. If we overlook a restraint, $dI = 0$ merely furnishes us one more equation than it did previously. It is absolutely no check on the correctness of statement of the problem that the number of equations is the required number.[4]

Finally, if we are to study group phenomena and distinguish between cooperative and competitive elements in the system, we must, as we have already remarked, introduce also group utility indices or group "ophelimities," and these have no transparent relation to the individual ones. The individual can not make his individual utility a maximum at the same time that society makes the sum of all the utilities a maximum, for the maximum of a sum is not in general the sum of the maxima of the separate terms. The group utility function can not, therefore, with this interpretation of the problem, be the sum of the utility functions of its members. And from this point of view the doctrine of "laissez faire" lacks mathematical foundation.

It is apparent, from what we have said, that the use of a utility function reduces us to the study of a very special case. The special case may, of course, be worth while. But in the terms heretofore used this study is entirely abstract; moreover, if it is to be made sound, it is dependent on the introduction of a number of special conditions which are not statable by means of economic concepts. The result is merely a collection of equations which have no relation to economics, except in the names that are used. Would it not be better than to abandon the use of the utility function, and investigate situations more directly in terms of concrete concepts, like profit and money value of production, in order to take advantage of the fact that money is fundamental in most modern economics and to use the numbers which it assigns to objects?

Concrete concepts suggest concrete hypotheses.

[3] This apparently is not a unanimous opinion among economists; *Cf.* the review of the author's "Mathematical Introduction to Economics" by Professor Henry Schultz (*Journal* of the American Association, December, 1931). It is assumed that a decision is possible as to which of two situations is preferable, or better, or more satisfying, even though each depends on several variables and without regard to processes of transformation; but we notice that this is itself merely the assumption of integrability. It is warrantable only for a limited class of problems. How many individuals, for instance, can decide, without reference to process, which of the two situations he desires—peace, or justice, in China?

[4] That we have the same number of equations as unknowns is a more or less adequate check on the correctness of statement of a problem when each relation comes directly from the problem or its intuitive analysis. At best the method is not satisfying, even when restricted to geometric or algebraic relations. But economists have been tempted to carry the idea over into these problems of maxima and minima, where, for the reasons given above, it is absolutely worthless.

CRITERIA AND METHODS IN THE INVESTIGATION OF AVIAN COCCIDIOSIS

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WHILE the economic importance of coccidial infection in domestic fowls is quite generally recognized, much of the investigation in this field has yielded only conflicting results and all too little in the way of actual accomplishment. The chief difficulties appear to have been due to the failure to recognize that a number of distinct species of *Eimeria* occur in poultry, even in a single host species, and also to the failure to employ adequate controls. Thus the coccidial infections encountered in various domesticated birds have been commonly attributed to a single poorly defined species, usually alluded to as *"Eimeria avium."* While the usual text-book description of the coccidium life cycle appears to have been understood and widely utilized, certain well-established principles have been frequently ignored.

It is the purpose of this communication to review some of the methods employed by the author in previous work in this field and also to discuss certain criteria which have been found to have applica-