free T and A in a "neutralized" mixture are negligible and that x equivalents of TdA are formed at the expense of TA, the expression at equilibrium would be

$$\frac{K[1-x]}{[x]} = \frac{K'\ [x]}{[1-x]}.$$

This would, of course, be strictly true only if K is so small that essentially all of A is in the form of TA. If $K = K'$, that is, if T and Td have the same affinity for A, $x = 0.5$. In other words, one half of the toxoid added has combined with the antitoxin and half of the toxin has been liberated.

If $K = 0.5\ K'$ (Td has 0.5 the affinity of T for A),

$x = 0.41$;

if $K = 0.1\ K'$, $x = 0.24$;

if $K = 0.01\ K'$, $x = 0.09$.

Thus, in the case considered, even if T has one hundred times the affinity of Td for A, an appreciable amount of toxin would be liberated from the neutralized mixture by the addition of Td. Actually, the greater the value of K, the smaller the amount of toxin liberated, since free A would be greater and less dissociation of TA would occur. However, if one accepts the experimental results of Madsen and Schmidt, as well as the explanation herein given, K must be relatively small, since liberation of toxin is actually observed.

Interesting in this connection is the analogy to the toxin-toxoid reaction pointed out some years ago by Northrop;[4] namely, that a pepsin-albumin mixture diluted with inactivated pepsin contains more active pepsin than one diluted with buffer alone, the effect being in harmony with the hypothesis that inactivated pepsin, as well as active pepsin, combines with the peptone formed by digestion of the protein.

A simple physicochemical consideration of the conditions of equilibrium therefore suffices to account for the increase in toxicity of neutralized toxin-antitoxin mixtures to which toxoid or anatoxin has been added, and the experimentally untouched affinity relations of these as yet vaguely defined substances need not be taken into account.[5]

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A NEW LAW OF SATELLITE DISTANCES

BESIDES the celebrated Bode's (or Titius) law there have been a number of attempts to establish a law governing the distances of satellites from their central body, including two discussions of the subject in SCIENCE in 1929. My approach to this subject was made about four years ago in somewhat the same manner as that of Dr. A. E. Caswell[1] who holds "the mean distances of the planets from the sun are proportional to the squares of simple integral numbers." I added, however, to the square of the integer the integer itself, thus assuming that the terms differ from the squares of integers by a progressively changing amount. For example, adding to each of the integers 1, 2, 3, 4, 5 . . . its square, we obtain the values 2, 6, 12, 20, 30. . . . This is simplified by dividing throughout by 2, giving us the series 1, 3, 6, 10, 15. . . . Those familiar with Bernoulli's *Tabula Combinatoria*[2] will recognize the series as the ternaries of his table.

The following table shows the results for all the satellite systems, including the planets as satellites of the sun, of the solar system where there are at least

TABLE ILLUSTRATING LAWS OF SATELLITE DISTANCES

System	Satellite	Relative Distance	New Law	Bode's Law	Caswell's Law
Sun	Mercury	3.87	3	4	3.82
"	Venus	7.23	6	7	6.80
"	Earth	10.0	10	10	10.6
"	Mars	15.2	15	16	15.3
"	Ceres	27.7	28	28	27.2
"	Jupiter	52.0	55	52	51.4
"	Saturn	95.3	91	100	95.6
"	Uranus	191.0	190	196	187.0
"	Neptune	300.0	300	.....	310.0
"	"Planet X"	400.0	406	388	408.0
		430.0	435	388	435.0
Mars	Phobos	1.00	1	.....	1.70(?)
"	Deimos	3.22	3	4	3.82
Jupiter	V	2.71	3	4	3.82
"	I (Io)	6.27	6	7	6.80
"	II (Europa)	10.0	10	10	10.6
"	III (Ganymede)	15.8	15	16	15.3
"	IV (Callisto)	27.9	28	28	27.2
"	VI	169.4	171	.....	170.0
"	VII	173.0	171	196	170.0
"	VIII	348.5	351	.....	357.0
"	IX	371.0	378	388	382.0
Saturn	Mimas	10.0	10	10	10.6
"	Enceladus	12.8	.....	.....	.....
"	Tethys	15.8	15	16	15.3
"	Dione	20.3	21	.....	20.8
"	Rhea	28.0	28	28	27.2
"	Titan	66.0	66	52	61.2
"	Themis	78.1	78	.....	83.1
"	Hyperion	79.0	78	100	83.1
"	Iapetus	190.0	190	196	187.0
"	Phoebe	698.0	703	772	712.0
Uranus	Ariel	10.0	10	10	10.6
"	Umbriel	14.1	15	16	15.3
"	Titania	22.8	21	.....	20.8
"	Oberon	30.4	28	28	27.2

[1] A. E. Caswell, "A Relation between the Mean Distances of the Planets from the Sun," SCIENCE, n.s., 69: 384, 1929.

[2] D. E. Smith, "Source Book in Mathematics," p. 273, 1929.

[4] J. H. Northrop, *J. Gen. Physiol.*, 2: 482, 1919–20.

[5] The authors of this paper are working under the Harkness Research Fund of Presbyterian Hospital.

two known satellites on which to base comparisons. The base term for the relative distances in each satellite system is 10 except in the case of Mars where it is one. Distances under Bode's law (represented by the series 4, 7, 10, 16, 28, 52, 100, 196, 388 and 772) are included not only for comparison of the results with those of the new law but also because that law gives very good values for some of the planetary satellites as well as for those of the sun. Likewise the distances under Dr. Caswell's law are included. These are obtained as explained in his article in SCIENCE.[1]

The new "Planet X" has been officially reported in a circular from the Lowell Observatory as having a distance from the sun of 40 to 43 astronomical units. Therefore double values have been included in the table until a more accurate determination of the distance is made. Enceladus, the second satellite of Saturn, does not, apparently, belong to any of the three series, but it is here included for completeness. The orbit of Themis is not accurately determined, but it seems that there may be a faint satellite at about the distance given. If in the satellite system of Mars 10 be taken as the relative distance for Deimos, better values will be obtained for the Bode and Caswell series, but the value of one was adopted for Phobos instead because of the near correspondence of the distance of that satellite to that set by Roche's limit. Ceres, the largest of the asteroids, is included in the table although it may belong to a system of harmonics rather than to the fundamental series. Referring to the table, it is seen, of course, that not all of the successive terms of the new law are represented in any one satellite system nor are the same terms represented in different systems.

It is important to notice that Bode's law gives fairly good values for at least five of Jupiter's satellites, for four of those of Saturn and for three of those of Uranus. This fact seems to be overlooked in nearly all modern popular discussions although it was referred to in some articles fifty years ago. Sir James Jeans,[3] like many others, has said ". . . it seems more than likely that it [Bode's law] is a mere coincidence with no underlying rational explanation." Yet Bode's law is an approximation for several terms both of Dr. Caswell's law and of my own, and Dr. Caswell[1] believes that his series "suggests the possibility that the orbits of the planets may be 'quantized' somewhat after the manner of the electronic orbits in the Bohr atom." The quantum principle is important in wave-theory, and my own solution resulted from studies in that field. The new series can be derived by taking the fiction, employed in the mathematics of a vibrating sphere, of a double source of wave-action of suitable strength at the center of the sphere and by modifying this concept to that of two sources of wave-action whose distance apart is relatively small in comparison with the length of the waves set up in the surrounding medium. Then, by disregarding the distance apart of the two sources, it is possible to develop the series by superposition. This assumes that the solar system can be treated somewhat on the order of the Schrödinger[4] atom rather than that of Bohr. A French writer, Lieutenant-Colonel Delaunay,[5] also believes wave-action is important in this problem. Finally, Victor Goldschmidt, of Heidelberg, according to Dr. Malisoff,[6] has shown that "a mathematical treatment strictly analogous to the phenomena of *standing waves in sound,* the distribution of lines in spectra, the progress of crystallization and similar phenomena gives the same law of harmonic relations of distances not only for the planets but also for satellites." Referring to Goldschmidt's[7] original article, I find that he has represented the distances of the planets as follows: Mercury 3.90; Venus 7.10; Earth 10.0; Mars 16.7; Jupiter 50.0; Saturn 100; Uranus 200; Neptune 300, and (following his method) "Planet X" 400. Dr. Caswell[1] has given it as his opinion that "on the whole the agreement is good, and can scarcely be accidental." After considering all this evidence independently arrived at by different investigators I am disposed to agree with him and offer as my opinion that Bode's law, as a first approximation, may have its origin in actual causal phenomena.

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THE NATIONAL ACADEMY OF SCIENCES

AT the annual meeting of the National Academy of Sciences, held in Washington on April 28, 29 and 30, the following papers were presented:

The structural basis of the integration of behavior: G. E. COGHILL (introduced by C. Judson Herrick). The development of behavior in a lower vertebrate, in which

[3] Sir James Jeans, "The Universe around Us," p. 20, 1929.

[4] Arthur Haas, "Materiewellen und Quantummechanik" (p. 81 in Eng. trans. by L. W. Cobb, "Wave Mechanics and the New Quantum Theory," London, 1928).

[5] Lieutenant-Colonel Delaunay, "Problèmes Astronomiques," Paris, 1920.

[6] William Marias Malisoff, "Some New Laws for the Solar System," SCIENCE, n.s., 70: 328–329, 1929.

[7] Victor Goldschmidt, "Über Harmonie in Weltraum, ein Beitrag zur Kosmogonie," *Annalen der Naturphilosophie,* 5: 51–118, Leipzig, 1906.