UNIVERSITY AND EDUCATIONAL NOTES

CYRUS H. K. CURTIS, Philadelphia publisher, has given Ursinus College $200,000 to start a fund for a new natural science building which will cost $450,000.

DR. GEORGE H. CLAPP recently made a gift of $15,000 to the endowment fund of the Carnegie Institute, Pittsburgh. This is the second donation to the institute by Dr. Clapp to be duplicated by the corporation. His previous gift amounted to $25,000.

DR. DANIEL J. MCCARTHY has been appointed director of the newly established neurological foundation of Temple University, Philadelphia.

PROFESSOR H. E. CLIFFORD has been appointed acting dean of the engineering school of Harvard University during the absence of Professor Hector J. Hughes, who has leave of absence for the second half of the academic year.

THE appointment of Dr. Alfred E. Emerson as associate professor in the department of zoology is announced by the University of Chicago. Dr. Emerson, who went to Chicago from the University of Pittsburgh, will have charge of developing the work in the general biological aspects of entomology.

DR. JOHN WYLLIE, of Glasgow, has been appointed to the new Elliot chair of public health and preventive medicine at Queens Medical College at Kingston, Canada, established through the gift of $50,000 from Samuel Insull, of Chicago.

G. G. MOE, associate professor of agronomy at the University of British Columbia since 1922, has been promoted to a professorship and appointed head of the department.

PROFESSOR F. E. WEISS will retire at the end of the present session from the George Harrison chair of botany and the directorship of the botanical laboratory of the University of Manchester. He has held these appointments since 1892.

DISCUSSION

M MU VERSUS MU MU

IN this journal for November 8, 1929, may be found on page 453 a note by Mr. John P. Camp entitled "The Micrometric Muddle." It includes the following sentences:

> Certainly the system should be definitely put in order. What to do about it is not so easy to decide; for though it might seem proper to insist on strict adherence to the present authorization of the Bureau of Standards it can be argued that their system is that of the physicists who are a minority and that the biologists and chemists besides being greatly in the majority use the terms and symbols most widely distributed in scientific literature.

In this journal for March 4, 1927, on page 233, I devoted twenty-one lines to indicating that mμ and μμ should mean, respectively, the 10^{-9} and 10^{-12} parts of the meter. In that note the suggestion contained in the next quotation was made to account for the origin of the error involved in taking μμ to represent 10^{-9} meter, *i.e.*, 10^{-6} mm. "Probably this error arose from the following sequence of folly: 1 μ equals 0.001 mm, hence the symbol μ denotes the multiplier 10^{-3}. Therefore μμ must mean (10^{-3}) (10^{-3}) mm or 10^{-6} mm." Doubtless the use of the word "probably" weakened the main argument to such an extent as to prevent the earlier note from receiving the serious attention of many open-minded readers.

In the present note it will be shown beyond peradventure that the only consistent and logical definitions of mμ and μμ are 10^{-9} meter and 10^{-12} meter, respectively. In order to prove my point it will be necessary to review the history of the origin of the symbol μμ. This history should not be without interest for the reasons that it does not seem to be sufficiently well known and that it throws helpful sidelights on the question of the alleged "muddle."

In the year 1883, J. Springer published a book, entitled "Lehrbuch der Spektralanalyse," from the pen of a very prominent physicist, Professor Heinrich Kayser, to whom experimental spectroscopy owes an incalculable debt. The following quotation is a true translation of the footnote occurring on page 11:

> Here, as well as throughout the entire book, the wave lengths shall always be given in millionths of a mm. Since there still exists no simple notation for this quantity and as it is the most suitable for all wave measurements, I have introduced for it the new notation μμ which is derived from the symbol μ, for a thousandth of a mm, just as mm is obtained from m. Accordingly we may have the lengths: m, mm, μ, μμ each of which is the one one-thousandth part of the preceding unit.

This idea is faulty and unjustifiable in at least two respects. In the first place it is tacitly admitted by Kayser that the left-hand m in mm denotes one thousandth, and that m had this signification prior to his knighting the Greek μ to the Order of the Thousandth. Hence, even in the same line, he writes two different symbols having the same operational power, that is, 10^{-3}. This procedure is unscientific since it adds a superfluous quantity to the terminology and thus violates a canon of beauty demanded of scientific presentation—brevity consistent with clarity and generality.

Again Kayser's premise ". . . the symbol μ, for a thousandth of a mm, . . ." is false. The unit of length in the metric (metre-ic) system was, and still is, the meter. In spite of all the inconsistencies that have been introduced into non-mathematical literature and into the English dictionaries and encyclopedias by the fluent confusion makers, μ has escaped contamination as yet in the Encyclopaedia Britannica. On page 739 of Vol. 27 of the thirteenth edition of this incomparable (though not infallible) work may be found: "Another relatively minute unit is the 'micron,' denoted by μ, and equal to one millionth of a metre; . . ." In the fourteenth edition, Vol. 17, page 877, is written: ". . . or again the 'micron,' denoted by μ and equal to one millionth of a metre is employed." The micron is numerically equivalent to each of the following measures—10^{-3} millimeter, 10^{-4} centimeter, 10^{-5} decimeter, 10^{-7} decameter, 10^{-8} hectometer, 10^{-9} kilometer and 10^{-10} myriameter. The micron can not be *defined* legitimately in eight different ways and Kayser chose arbitrarily one of the seven possible unaccepted and unacceptable ways of defining μ in order to arrive at the unfertile hybrid notation μμ. Even in the c. g. s. system of units the millimeter does not occur as the independent or definitional unit. There is no m (millimeter) g. s. system.

Kayser's argument is not based on fact to a greater extent than the following specious reasoning. The symbol mm is a special case of a general rule which states that when a symbol is repeated once the first or left-hand letter acts as an operator signifying the one one-thousandth part of the second or right letter which denotes the unit (operand) involved. It follows at once that the abbreviations for milliampere, milligram, milliliter, millivolt, millimicron, etc., should be respectively the pretty self-conjugate pairs aa, gg, ll, vv, μμ, etc.

The disillusionment with regard to μμ as 10^{-9} meter is far from being of recent origin. In the *Archiv der Pharmacie* published by the Deutschen Apotheker-Verein may be found (in Vol. 232, pp. 3–36) an article by a chemist, Dr. Hugo Erdmann, which was received by the journal on December 10, 1893. The translation of the title is: "The Salts of Rubidium and their Importance for Pharmacy." Erdmann consistently expresses his wave-lengths in terms of mμ and he makes the following sententious remarks in a footnote on page 10. A faithful translation of this footnote constitutes the next paragraph:

> From the universally accepted notation μ for the micron or one millionth of a meter it follows quite naturally that by μμ is to be understood a millionth of a micron, while one has to write one thousandth of a micron or millimicron mμ, precisely as one writes a thousandth of a meter mm. Hence the following measures for small linear magnitudes result:
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> | 1 m = 10^0 m | 1 μ = 10^{-6} m | 1 μμ = 10^{-12} m |
> | 1 mm = 10^{-3} m | 1 mμ = 10^{-9} m | 1 mμμ = 10^{-15} m |
>
> I emphasize this [matter] in this place too for the reason that nowadays great confusion still obtains in the notation for this small measure. Thus Kayser and Runge as it suits them (*e.g.*, page 12) make use of the millimicron as unit which however they denote by μμ instead of by mμ and then again, without further explanation, they go over to another measure, to the "tenth-meter" which is in general use in England and is also called the "Ångström unit." Furthermore reduction is simple because a tenth-meter = 10^{-10} m is the tenth part of a millimicron mμ. (*Cf.* H. Erdmann, on orders of magnitude, *Zeitschr. f. Naturwissenschaften,* 1893, 66, 73).

There remains only one conclusion, that mμ and μμ should denote the 10^{-9} and 10^{-12} parts of the meter, respectively. Incidentally, I now desire to retract the use of the word "probably" in my earlier note.

It is certain therefore that many years ago a *physicist* started the symbol μμ with its inconsistent signification and that a *chemist* showed clearly the unsatisfactory character of the physicist's definition. The apparently general adoption of μμ as 10^{-6} mm was probably due largely to the following causes: (a) The scientific authority of Professor Kayser which retarded the day of emancipation in a manner analogous to the obstacle placed unintentionally by Sir Isaac Newton in the way of the designing of achromatic lenses. (b) The extremely frequent occurrence of μμ after the numerics of optical wave-lengths in numerous valuable papers by Kayser and his students advertised the unit in question very extensively. (c) The intensive prosecution of experimental research in a special field requires so much time, energy and concentration that it often gives rise to potential indifference as to the credentials of units borrowed from another field so long as they seem to be properly sponsored and to be of noble birth and ancestry.

With regard to Mr. Camp's unguarded use of the epithets "minority" and "majority" with respect to physicists on the one hand and to biologists and chemists on the other hand, and to his implications in general, I should like to call attention very briefly, but in the best of humor, to a few considerations.

As an immediate consequence of the quantitative nature of physics, as well as of the mathematical and experimental technique of this science, the need for permanent representative units was first recognized by physicists. For this reason (and not because of any inherent superiority) the definition, design, construction, calibration and preservation of prototype standards of length, mass, etc., have been entrusted to them.

Again, if mere numbers of human beings count for anything in the realm of the natural sciences then radio waves are propagated by the atmosphere (so-and-so is "on the air"), John the Baptist subsisted partly on cicadas (the "man-on-the-street" calls them locusts), steam is visible (white), lightning follows zigzag paths with sharp bends, mammals of the order Cetacea (porpoises, dolphins, whales, etc.) are fishes, the tongue of a snake is its "stinger," moist air is heavier than dry air, etc., etc.

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MATHEMATICS AND THE TRUTH

IT is frequently said that the modern mathematician does not ask whether a certain result which he regards as established is true but that he is interested only in proving that it can be derived from the system of postulates which he has formulated. While it would be difficult to determine the amount of mathematical work which is now being done in such a philosophical frame of mind it would doubtless be safe to say that this amount is relatively very small. If one listens to papers presented at the meetings of mathematicians one can not fail to notice that there is a remarkable degree of confidence in the truth, and hence permanent character, of the new results which are being communicated. It is seldom that any reference is made to the system of postulates which are ultimately involved and it is quite likely that many of the most successful investigators would find it difficult to exhibit these postulates if they should be asked to do so.

As far as we know now the ancient Greeks were the first to realize the fact that mathematical reasoning must ultimately be based on postulates, and hence that the results with which mathematicians deal can be regarded as true only if these postulates are true. Recent discoveries have established the fact that the pre-Grecian mathematical developments were much more extensive than had been previously assumed, but these discoveries have not yet exhibited any system of postulates which antedates those of the Greeks. It is a very interesting fact in the history of scientific ideas that all known evidences support the view that before the ancient Greek civilization mathematical results were regarded as truths which were not ultimately dependent upon systems of postulates. The Greeks seem to have originated the philosophical frame of mind as regards mathematical results, and they fortunately also greatly extended these results so as to provide ample material for the activities of those who accept as true much that they themselves have not traced back to the ultimate postulates.

The popular orator who seeks to clinch a statement by saying that it is mathematically true conveys thereby a more useful view of mathematics than the critical student who observes that nothing can be really proved in mathematics since it is necessary to assume some things before you can reason about any question. Both those views are in order under appropriate circumstances, and they supplement each other. To exhibit the Greek view as regards the necessity of postulates in the development of mathematics we quote the following from Aristotle's Posterior Analytics:

> By first principles in each genus I mean those the truth of which it is not possible to prove. What is denoted by the first [terms] and those derived from them is assumed; but, as regards their existence, this must be assumed for the principles but proved for the rest. Thus what a unit is, what the straight [line] is, or what a triangle is [must be assumed].

What is perhaps of more importance in this connection is the fact that Aristotle not only knew that some of the postulates of mathematics can not be proved but he also saw that they do not necessarily appear self-evident to the beginner. This is shown in the following statement found in the work to which we referred in the preceding paragraph:

> Now anything that the teacher assumes though it is a matter of proof is a hypothesis if the thing assumed is believed by the learner, and it is, moreover, a hypothesis, not absolutely but relatively to the particular pupil; but, if the same thing is assumed when the learner either has no opinion on the subject or is of a contrary opinion, it is a postulate.

Hence it appears that at least some of the ancient Greeks looked at mathematics and the truth in about the same way as we do now. This is a very important fact in the history of the development of mathematical ideas, especially since during some of the intermediate centuries the postulates of mathematics seem to have been regarded as self-evident truths. The development of non-Euclidean geometry exerted a powerful influence towards making the function of the postulates in elementary geometry more widely known.

Some of the Greek writers called attention to what appeared to them as different properties of postulates and axioms, and many of the modern writers have followed them in this regard. On the other hand, there are those who see no essential differences between the concepts represented by these terms. The relation between mathematics and the truth is, however, not affected thereby. If at least one of the two terms axiom and postulate was used by the Greeks to represent a concept which was not regarded as self-evident they must have realized the philosophical difficulties involved in regarding mathematics as true in the sense that it is possible to establish a contradictory system based upon another set of postulates. O. Neugebauer recently directed attention to the fact that an important feature of pre-Grecian mathematics is that it excludes the concept of irrationality which plays such a fundamental rôle in the mathematics of