SPECIAL ARTICLES

THE LAWS OF DISTRIBUTION OF PARTICLES IN SUSPENSION

THE motion and distribution of fine particles suspended in a fluid are subject to laws which have been derived by statistical methods by Einstein and extensively studied experimentally by Perrin. Their results are well known and frequently cited. I wish here to point out a very much simpler derivation than Einstein's of a slightly more general law than Perrin's. The reasoning is thermodynamic rather than statistical.

Throughout any aggregate of particles in thermal equilibrium, the kinetic energy of thermal agitation is everywhere RT/M calories per gram (R gas constant, T absolute temperature, M molecular weight), otherwise there would be a net flow of heat which is contrary to the assumption of uniform temperature. The size or kind of particle is assumed to be immaterial and it may be either molecular or microscopic. The pressure corresponding to this energy, which we shall call the kinetic pressure

(1) $$\varkappa = CJRT/M \text{ dynes/cm}^2$$

is the integrated kick of all the molecules or particles of a given class (defined by M and concentration C) or of all classes. For microscopic particles it is the pressure due to Brownian movement which is caused by and is in equilibrium with the thermal agitation of the molecules of the suspending fluid.

The weight of such an aggregate of particles is

(2) $$W = Ckg \text{ dynes/cc}$$

where C is concentration in grams/cc, $g = 980$ dynes/gram and $k = 1 - \varrho_1/\varrho_g$, ϱ_g being the density of the grains and ϱ_1 that of the suspending fluid.

The pressure gradient $d\varkappa/dz$ depends upon the variation of the ratio CT/M with depth. If this ratio is the same at all depths, the kinetic pressure must be uniform and the gradient zero. In a true solution C and M are both constant if the temperature is uniform and CT = constant (Soret effect) if it is not. Even in a suspension of fine particles CT/M may be constant if, at various levels, the concentration is proportional to size of particle, *i.e.*, if the number of particles in unit volume is constant. In general, an aggregate of particles will be in equilibrium distribution if the pressure gradient upward just equals the weight of suspended material in unit volume

(3) $$d\varkappa/dz = dw$$

or, by substitution from (1) and (2), if

(4) $$\frac{d \log(CT/M)}{dz} = \frac{Mkg}{JRT}$$

which is the most general equilibrium condition. In the special case of uniform mass of particle M and uniform temperature T, (4) reduces to $d \log c/dz = Mkg/JRT$ which may be integrated giving

(5) $$\log (C/C_0) = Mkgz/JRT$$

the integration constant yielding C_0, the concentration at the surface where $z = 0$. (5) is the equivalent of Perrin's equation.

If the pressure gradient in a suspension is greater than its immersed weight then it will diffuse upward to the surface; if less than the weight it will settle according to the generalized Stokes' law with the velocity

(6) $$v = \frac{2}{9}\frac{r^2\varrho_g}{\eta C}\left(Ckg - \frac{JRT}{M}\frac{dC}{dz}\right).$$

The expression bracketed is the resultant force acting on all the particles in unit volume. When the concentration gradient $dC/dz = 0$, (6) reduces to the ordinary form of Stokes' law. This and other equations above may be put in different forms by using the substitutions $C = mN$, $M = V\varrho_g = M$ times the mass of the hydrogen atom, 1.66×10^{-24} gram.

From the form of (6) it is evident that velocity of fall is very sensitive to size of particle. It may readily be zero or even negative (upward) for the smaller particles. As each size of particles falls, smaller particles are not only left behind but tend to diffuse upward.

The kinetic pressure theory of suspensions here developed assumes clean reflecting walls for the containing vessel. A soft or muddy bottom, for example, quite actively assists in pulling down a suspension. Light pressure also effects settling, fine suspensions depositing on the far side wall of a tube exposed to strong horizontal illumination.

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THE OCCURRENCE OF VIABLE COTTON ROOT-ROT SCLEROTIA IN NATURE

A TRUE sclerotial stage of the cotton root-rot fungus, *Phymatotrichum omnivorum* (Shear) Duggar, was first observed in Arizona by C. J. King and H. F. Loomis in September, 1928, in laboratory cultures.[1] These observations were later confirmed by the writer in Texas, and such sclerotia have also been discovered in the soil, first in northern Texas at the U. S. Cotton Breeding Field Station near Greenville, in Hunt County, and later in southern Texas at the U. S. Field Station near San Antonio. As the sclerotia formed readily and abundantly in culture jars containing layers of sterile soil, sand and cotton roots,

[1] In press in the *Journal of Agricultural Research.*