

this fact and the difficulty of securing specimens from other museums it will be impossible to describe all the known fossil teeth. Therefore it is hoped that others who may have supplementary material available will be sufficiently interested in the work to communicate with the writer.

The study requires that microscopic sections of the teeth be made, since their interior structure is very minute. At least three sections must be made of each tooth, and, if the tooth is large, enough more to show all its characteristics. It is also necessary that these sections be made so that they correspond in position with those of other teeth in order to establish a uniform basis of comparison. The three standard sections comprise: one of the grinding or biting surface (called the crown section), one taken parallel to the crown section through the tooth at a point about half way down the side (called the medial section) and one taken vertically from root to biting surface (called the vertical section). In order to facilitate comparison of the teeth, microphotographs are made of each section.

The description of the teeth on the basis of microscopic structure will require a new system of nomenclature. The problem resembles very much that of the description and classification of the bryozoans which was so admirably accomplished by Ulrich and Bassler. The introduction of descriptive terms new to the science necessitates adequate definition of them as an introduction to the study. Familiar terms such as enamel, root, pulp, etc., will be used whenever possible.

The work of sectioning and photographing the teeth is almost complete for the Psammodont and Petalodont families of the Walker Museum collection. When the other families are finished the work of reduction of species and the correction of classifications will be undertaken. It is possible that enough teeth of a single species may be present in the collection to warrant the reconstruction of the dentition of some forms.

It is the writer's hope that as a result of this study the geologist engaged in field work of either paleontological or stratigraphical nature, or both, will regard fossil shark teeth as valuable additions to his collection, since it will no longer be necessary to compare them with hundreds of others in books or in collections to make doubtful determinations. Under the new plan the geologist will make sections of the teeth as indicated above, and after carefully noting the characteristics by microscopic examinations he will be able to make positive determinations by either the trial and error method or by elimination. It is only when such positive identifications are possible that fossils become of stratigraphic value. In this connection the stratigraphic range of each of the species studied will be established wherever possible. Such information in table form will be an important part of the paper.

This work is being done under the supervision of Dr. Carey Croneis and Dr. Alfred S. Romer, of the department of geology (paleontology division) of the University of Chicago.

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SPECIAL ARTICLES

THE INTERNAL TEMPERATURE OF THE EARTH'S CRUST

RECENT investigations in the utilization of the earth's internal heat led the author to consider the possibility of obtaining an equation which would represent the average temperature gradient of the earth's crust. The use of the linear gradient of 1° F. in 55 ft. is not satisfactory because it leads to large errors at even shallow depths.

Kelvin's equation (Thomson and Tait, "Treatise on Natural Philosophy," vol. 2, p. 458, 1883) which yields the solution,

$$\Theta = \frac{2\Theta_0}{\sqrt{\pi}} \int_0^\beta e^{-\beta^2} d\beta \text{ and } \frac{d\Theta}{dx} = \frac{\Theta_0}{\sqrt{2\pi kt}} e^{-\frac{x^2}{4kt}} \quad (1)$$

where $\beta = \frac{x}{2\sqrt{kt}}$ and in which Θ is the temperature,

x is the depth, k is the coefficient of thermal diffusivity, and t is the time since the earth was at the initial temperature of Θ_0 , is unsatisfactory, because it neces-

sitates assumptions which are not in accord with the facts. Consequently, this equation is more of hypothetical than of practical value. Kelvin's equation has considerable theoretical background, being the solution of the well-known Fourier equation for assumed limiting initial conditions. The equation, however, takes no account of internal heating which may arise from causes other than the original molten condition, such for example as those of radioactivity, chemical activity, and the like, which have been amply demonstrated as effects which can not be neglected. Furthermore, even when using a value for the earth's age which gives a geothermal gradient equal to known measurements, the computed temperatures have an almost linear relation to the depth for the first few miles, which is within the measured limits and the discussion in this note.

Butavand (Butavand, *Le Génie Civil*, May 10, 1919) and Lees (Lees, *Proc. Royal Soc.*, 83 A: 339, 1909) have proposed equations for the geothermal

temperature, but they were principally concerned with topographical configurations.

It is quite apparent that the ratio of the measured depths of the earth's crust to the earth's diameter is so small as to preclude any consideration of curvature. The sole question is one of the linear propagation of thermal disturbances in a heat-generating body, and the problem is concerned in the method of generation only to the extent of determining whether or not the heat liberated is a function of the temperature.

It is very easy to show that the rigorous differential equation for the linear propagation of thermal disturbances within a heat-generating body is given by

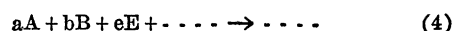
$$k \frac{d^2\theta}{dx^2} = c\rho \frac{d\theta}{dt} + Q \quad (2)$$

where k is the thermal conductivity, c is the specific heat capacity, and ρ is the density of the material.

In the steady state in which $\frac{d\theta}{dt} = 0$, this becomes

$$k \frac{d^2\theta}{dx^2} = Q \quad (3)$$

Consider now a complex exothermic chemical reaction of the order



in which case the velocity of the reaction is

$$\frac{dc}{dt} = K(C_a - C)^a(C_b - C)^b(C_e - C)^e \dots \quad (5)$$

where C is the concentration and K is the specific reaction rate. Since a differential change in concentration, dC , gives rise to the liberation of the differential quantity of heat, dH , consequently

$$\frac{dH}{dt} = Q = n \frac{dC}{dt} = nK(C_a - C)^a(C_b - C)^b \dots \quad (6)$$

If the reaction is essentially one of constant velocity, then

$$Q = mK \quad (7)$$

where both m and n are proportionality constants.

Harcourt (Harcourt, *Phil. Trans. Roy. Soc.*, 212: 187, 1913) has shown that for a variety of chemical reactions the following relation holds between the specific reaction rate and the temperature:

$$K = p\theta^v \quad (8)$$

where p is a proportionality constant and v is an exponent to be determined experimentally. Hence we have in the steady state

$$k \frac{d^2\theta}{dx^2} = mp\theta^v \quad (9)$$

and finally, placing $u = \frac{mp}{k}$,

$$\frac{d^2\theta}{dx^2} = u\theta^v \quad (10)$$

This differential equation expresses the temperature at any point perpendicular between two plane surfaces in chemically exothermic media after the steady state has been attained. By the use of the integrat-

ing factor, $2 \frac{d\theta}{dx} dx$, it can be reduced to the form

$$x = \frac{1}{\sqrt{v+1}} \int \frac{d\theta}{\sqrt{C_1(v+1) + 2u\theta^{v+1}}} + C_2 \quad (11)$$

but, as far as the author is aware, no general solution of this integral can be given. It can, however, be integrated for certain values of v , such as $v=1$ and $v=3$ (Pierce, "A Short Table of Integrals," 1910, pp. 20, 34), but the value of v is an unknown quantity. When $v=1$, the exponential equation

$$\theta = \theta_s e^{ax} \quad (12)$$

where θ_s is the surface temperature, is a solution. When $v=3$, the solution is a transcendental quadratic in θ which can not be simply solved to give θ as a function of x . The form of the solution, however, suggests that it would lead to a complicated exponential function. If $v=0$, as would be the case if radioactive disintegration were the cause of heat liberation, since it has been fully demonstrated by Fajans (Fajans, "Radioactivity," 1922, p. 13), by Bronson (Bronson, *Proc. Roy. Soc.*, 78 A: 494, 1906), by Schmidt (Schmidt, *Phys. Zeit.*, 9: 113, 1908), by Curie and Onnes (Curie and Onnes, *Le Radium*, 10: 181, 1913), and by others that the phenomenon is independent of all physical conditions, high temperature included, the solution becomes a simple quadratic (Pierce, *op. cit.*, p. 16)

$$\theta = \theta_s + Ax + Bx^2 \quad (13)$$

From measurements on the temperatures in deep wells made by Van Orstrand, Hallock and others (Darton, U. S. Geol. Survey Bul. 701, 1920) the constants of equation (12) and equation (13) can be computed as $a = 0.000146$, $A = 0.00828$, and $B = 0.000000816$.

Now it is very easy to show by equating coefficients that

$$B = \frac{Q}{2k} = 0.000000816. \quad (14)$$

According to Joly (Joly, "Surface History of the Earth," 1925, p. 90) the average radioactive content of deep-seated basalts which have poured out of the continental surface amounts to about 1.0×10^{-12} grams of radium and 0.8×10^{-5} grams of thorium per gram of basalt. Assuming a mean density of 3.0, the average evolution of heat by radioactivity within the earth's crust amounts to approximately 0.33×10^{-12} gram calories per second per cubic centimeter of rock. In English units this is equivalent to $Q = 0.371 \times 10^{-10}$

B.t.u. per sec. per cu. ft., so that finally we have for the thermal conductivity of basalt, $k = 0.0000227$ B.t.u. per ft. cubed per deg. F. per sec., or in metric units $k = 0.00549$ gram cal. per cm cubed per deg. C. per sec., which agrees with the values of specific conductivity of granite, continental rock, basalt and sandstone, as quoted by Joly (*op cit.*, p. 72). That is to say, radioactivity alone can account for the known temperature of the earth's crust down to 8,000 feet.

Calculations of temperature in the earth's crust made with the two foregoing equations check exceedingly well, on the whole, with the temperature measurements that have been made by Van Orstrand, Hallock and others (Darton, *op. cit.*) on the world's deepest wells. Care must be exercised in applying the formulae to oil and Artesian wells, unless the depth of the source of flow be accurately known, for, if the seepage is from a greater depth than the bottom of the well, the temperature of the discharge will be far higher than the computed quantity. On the other hand, if the well taps a fissure short of the bottom, the temperature of discharge will be less than the computed value. Departures from the computed values may be attributed also to variations in the thermal conductivity of the rock. Of course the constants may be suitably modified for local conditions.

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A BLUE MUTATION IN THE RAT (*MUS NORVEGICUS*)

IN a genetic study of hypotrichosis or so-called hairlessness in rats two young in a litter of nine were found to be colored unlike any that had appeared before among more than 2,500 individuals. They were light yellowish or reddish gray in color, produced apparently by a dilution of the black.

All our stock went back on the paternal side to one "hairless" male captured at Farmington, Illinois, and on the maternal side to albino females from the Wistar Institute. The immediate father was unknown, though it was one of six males which together with the mother had been used by the division of physiological chemistry in some experiments with cystine and after being returned were held as stock animals. Only the one litter was produced, though the female was kept for twelve months after producing the litter with the unusually colored young.

Both these young were females and one died at five weeks of age. The other female was mated to both intense colored and albinos, intense colored progeny from both kinds of matings being produced as indicated in Table I.

TABLE I
MATINGS AND PROGENY OF DILUTE FEMALE

Mated to	Number of young	Color of young
Intense	8	Intense
Albinos	12	Intense

That this character is not in the color series is shown by the production of intense from matings with albinos.

Subsequent matings of heterozygous intense inter se, heterozygous intense with dilute, and dilute with dilute were made with results as given in Table II.

TABLE II
VARIOUS MATINGS INVOLVING DILUTION

Mating	Number of progeny	Color	
		Intense	Dilute
Dd* × Dd	52	36	16
Dd × dd	128	63	65
dd × dd	5		5

* d = dilution, D = allelomorph (intensity).

Agouti, non-agouti, hooded and non-hooded were other factors involved in these crosses, but results of these will be left for consideration after linkage studies are complete.

When agouti is absent the color of the mutant is very similar to the dilute black or "blue" of mice, rabbits, cats, dogs and *Mus rattus*. Dr. W. E. Castle mated a dilute gray male which I sent him to red-eyed yellow females of the formula $rrCcHhaa$. From these matings the following kinds were produced:

Gray self
Gray hooded
Black self
Black hooded
Albinos

The male was probably of the formula $RRCcHhAa$ and homozygous for the new factor which tentatively is designated d. The red-eyed females would be homozygous for D.

Since this color had not previously appeared among some 2,500 descendants of the original stock, it appears probable that this is a recent mutation, though being a recessive it may have been carried some time before appearing.

In the light of the fact that it has not been reported, as far as I can discover, it is probably a new mutation in the rat, which causes a dilution of the black producing blues analogous to the so-called blues found in some other mammals.

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