nated eggs was given and after three weeks the experiment was terminated. Examination of the flushed intestinal contents of the fifty-seven chickens gave 133 nematodes, whereas in the scrapings of the intestinal mucosa no worms were found in any case. The technique worked admirably with both small and larger worms, the range in length being from 3.2 mm to 95.2 mm. Numbers of worms likewise caused no difficulty, for as few as one and as many as twenty-five worms were present in a chicken. Thirty-three per cent. of the birds were infested at autopsy.

In the second experiment, thirty chickens were parasitized at the age of nine weeks by giving to each bird fifty embryonated eggs of the nematode. Two weeks later the experiment was terminated with results similar to those of the first experiment, *viz.*, that from the flushed intestinal contents 186 worms were isolated, while in the scrapings of the mucosa of the same intestines not a worm was found. The percentage of infested birds in this experiment was 92; the range of individual infestations was from one to thirty-three worms, and the lengths of the worms varied from 2.1 mm to 11.5 mm. The results of these experiments give evidence that the technique is highly efficient in the removal of roundworms from the intestines of chickens.

The temperature of the water for flushing the intestine may vary many degrees and still be effective. Temperatures above 60° C. and below 35° C. caused contractions of the muscles of the intestine and thus interfered with distention and free flushing.

While the technique is especially valuable for small worms, it works equally well with larger ones, and should be readily adapted to studies on the various larval and adult nematodes, living free in the small intestine of birds and reptiles and of small and medium-sized mammals. Apparatus such as shown in Fig. 1, while desirable, is not necessary for the application of this technique. The flushing cone (Fig. 2) can be used on any hot-water faucet to which a hose couple can be attached. It was made by threading a small brass cone on a hose couple.

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PARAMECIUM BURSARIA AS A LABORATORY DEMONSTRATION OF CYCLOSIS

THE use of *Paramecium bursaria* for demonstration of cyclosis in laboratory classes in zoology has several possible advantages, as compared with the customary use of Nitella. In the first place, it obviates the necessity for drawing on the plant kingdom for illustrative material. Furthermore, this ciliate may be maintained easily in laboratory cultures at any season of the year, and in addition it furnishes, with its contained Chlorella, an excellent example of symbiosis.

Cyclosis is unusually rapid in this species of Paramecium, and is readily followed under a 4 mm objective. The writers have found that especially interesting preparations may be made by staining vitally with neutral red. Clean slides, after being warmed slightly over a flame to eliminate excess moisture, are filmed with a solution of neutral red (1:1500, or more dilute) in absolute alcohol. After the film has dried a drop of culture material is added, and a cover-slip sealed in place with melted vaseline. Numerous small scattered globules are stained with neutral red, and these add to the clearness with which cyclosis may be observed. In addition, this method affords a good laboratory demonstration of the effects of vital dyes on a protozoon, while the neutral red also serves as an indicator of the pH of the inclusions. If the dye solution is dilute enough, the organisms should live for twenty-four hours or more; hence, if several laboratory sections are to be supplied, the same preparations may be used in successive laboratory periods.

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SPECIAL ARTICLES

MICHIGAN PAPYRUS 620; THE INTRODUCTION OF ALGEBRAIC EQUATIONS IN GREECE

THE Egyptians some two thousand years before the Christian era set up equations of a purely algebraic type. In one type of these problems, as given in the Ahmes Papyrus,[1] an unknown number with some fractional part of it is set equal to a known number and the solution is effected by the so-called method of false position. In other problems[2] two unknowns appear as the sides of a rectangle with known area, and their ratio being given, the two unknowns are determined. The brief translation of portions of the Moscow papyrus given by Touraeff[3] also indicate that the analogous problems which appeared in Euclid's Data had their beginnings in ancient Egypt. The eagerly awaited complete trans-

[1] T. Eric Peet, "The Rhind Mathematical Papyrus," Liverpool, 1923.

[2] H. Schack-Schackenburg, "Der Berliner Papyrus 6619," *Zeitschrift für Ägyptische Sprache und Altertumskunde*, Vol. XXXVIII (1900) and Vol. XL (1902).

[3] B. Touraeff, "The Volume of the Truncated Pyramid in Egyptian Mathematics," "Ancient Egypt," 1917, pp. 100–102; L. C. Karpinski, "An Egyptian Mathematical Papyrus in Moscow," SCIENCE, 57 (1923): 528–529. In the article in SCIENCE I pointed out the importance of the other problems to which Touraeff refers. L. C. K.

lation of the Moscow papyrus will doubtless throw further light on these questions.

In Greece the geometrical version of the linear equation in one unknown appears in the first book of Euclid with the problem to construct a parallelogram (or rectangle) given the area and one side. This is given as the beginning of the theory of the application of areas and is elaborated in the sixth book of Euclid, as well as in Euclid's Data, leading in effect to the geometrical solution of different types of quadratic equations in one unknown as well as to the construction of the pentagon. In the second book of Euclid the theorem on the division of a line into equal and unequal parts leads also, in effect, to the solution of a quadratic in one unknown. Notably by this method undoubtedly the Pythagoreans achieved the division of a line into extreme and mean ratio and the solution of the construction with ruler and compasses of the pentagon.

There is, then, indeed, a great amount of algebraic material in Euclid, and in the works of Archimedes and Apollonius. Books seven to ten of Euclid's Elements are entirely arithmetical and algebraical, clothed to some extent in geometrical form. The work of Diophantus of Alexandria appears in the third century of the Christian era with an immense amount of strictly algebraical material, free from any geometrical implications. This material and the algebraic problems of the Greek Anthology[4] are so different from the algebraical material as found in the works of the classical Greek authors that the transition must have involved the activity of many Greek students of mathematics from the time of Archimedes to the time of Diophantus.

The late Professor Francis W. Kelsey acquired in Egypt in 1921 a Greco-Egyptian papyrus which is confidently dated as not later than the second century of the Christian era. There are in this papyrus three algebraical problems which in their formulation, solution and notation mark a notable advance over the problems of ancient Egypt of which these problems are the continuation.

Writers on the history of algebra have long known that material of the type given by Diophantus must have developed in the period between that writer and Euclid. A series of algebraic problems in one unknown, or easily reducible to one unknown, is found in the Greek Anthology. These problems were largely preserved in the writings of the grammarian Metrodorus, who flourished five centuries after the beginning of the Christian era. Several of these problems relate to the distribution of apples and nuts, leading to first degree equations. Thus, the twelfth problem[5] leads to the equation

$$\frac{x}{5}+\frac{x}{4}+\frac{x}{19}+\frac{x+\frac{x}{20}}{10}+12+120=x.$$

Now Plato refers to problems connected with bowls and apples, from which Heath[6] concludes that problems of this type go back to the fifth century B. C. It is worthy of note that in the commentary on the Charmides discussing the Greek logistic reference is made to the Egyptian method of multiplication. Elsewhere Plato[7] refers to the fact that the Egyptians teach their children arithmetic by means of games such as those involving the distribution of apples.

A system of solving a series of n simultaneous linear equations in n unknowns is described by Iamblichus[8] in his Commentary on the arithmetic of Nicomachus. This rule is called the "flower" or "bloom" of Thymaridas,[9] an ancient Pythagorean.

A further series of simultaneous equations is associated with the name of Heron.[10] These involve the perimeter and areas of rectangles and further rational right triangles in which the sum of the area and the perimeter is given. So far as the rectangles are concerned it is interesting to note that here again we have ancient Egyptian problems of a similar nature, as noted above. These problems also are not dated and it is on internal evidence that Heath concludes[11] that the original formulation falls in the period between Euclid and Diophantus.

The Greek symbol for the unknown quantity which is used in the Michigan papyrus appears also in manuscripts of Diophantus. Concerning this symbol there has been much speculation.[12] Heath has made the suggestion that "this is the contraction for the two initial letters of ἀριθμός and that Diophantus may not have made the contraction himself." This agrees with the symbol as found and used in the Michigan papyrus, although it must be said that here the symbol seems to be used not simply as a "tachygraphic abbreviation" but somewhat as an algebraic symbol like our x.[13]

The Greek text is to appear shortly in *Classical Philology*.

4 See G. Wertheim, ''Die Arithmetik und die Schrift über die Polygonalzahlen der Diophantus von Alexandria,'' (Leipzig, 1890), Appendix III, ''Die arithmetischen Epigramme der griechischen Anthologie.''

5 G. Wertheim, ''Die Arithmetik und die Schrift über die Polygonalzahlen der Diophantus von Alexandria,'' Leipzig, 1890, p. 334.

6 ''A History of Greek Mathematics,'' Oxford, 1921, Vol. I, p. 14; Vol. II, p. 442.

7 Laws 819 A-C.

8 Iamblichus, In Nicomachum, p. 62. 18.

9 See Heath, ''History,'' Vol. I, pp. 94–96.

10 Heiberg and Zeuthen, *Bibliotheca Mathematica*, VIII 3, 1907–1908, pp. 118–134.

11 ''History,'' Vol. II, p. 447.

12 Heath, ''Diophantus of Alexandria,'' second edition (Cambridge, 1910), pp. 32–37.

13 See also Cajori, ''A History of Mathematical Notations'' (Chicago, 1928), Vol. I, pp. 71–72.

The papyrus is a roughly rectangular piece, about 210 mm broad and 125 mm from top to bottom in its greatest dimensions. The top margin is preserved, and some fifteen lines of the text and accompanying calculations of two columns of mathematical problems; the full breadth of the left-hand column is there, but only the left half of the right-hand column. The text itself clearly shows that each of the problems consisted of at least three parts, the hypothesis in which the conditions of the problem were stated, the solution, and finally the check (*apodeixis*), in which it was shown that the values derived in the solution would satisfy the conditions laid down in the hypothesis. After each problem there were appended the calculations, in ordinary Greek numerical notation, involved in the solution. What we have left in the preserved fragment includes, in Column i, the very end of the solution of one problem, the check complete and the calculations very nearly complete; in Column ii, the last two and a half lines of the solution of a second problem, its check and a good share of the calculations, followed by the hypothesis and the first line or two of the solution of a third. In explanation of the translation which follows attention should be drawn to the use of a symbol very unusual, perhaps unique, in the papyri, which, however, as stated above, appears in the manuscripts of Diophantus to stand for x, the algebraic unknown.[14] In the form it closely resembles both the abbreviation for "drachma," which occurs frequently in the Greco-Egyptian papyri, and the Greek numeral 6, but it can have neither of these significances in P. Mich. 620. It is supposed to be ultimately an abbreviation of the Greek word *arithmos*, "number," and in this papyrus can apparently mean both "number," "quantity," and "*the* number," that is, the number we are seeking, the unknown. The contents of the papyrus are as follows:[15]

COLUMN i

Four numbers: their sum is 9900; let the second exceed the first by one seventh of the first; let the third exceed the sum of the first two by 300, and let the fourth exceed the sum of the first three by 300; to find the numbers[16]. . . . *To get the value* of the fourth number, again *take* 150 *30 times; it gives* 4500; and the 600[17] in its assigned value make 5100; this is the fourth number. Add the *four* numbers; 1050 + 1200 + 2550 + 5100 = *9900*. Check. Since it says, "Let the second number exceed the first by one seventh," take one seventh of the first, 1050; it is 150; *add this* and 1050; this gives 1200, which is the second number. Again, since it says, "Let the third exceed the first two by 300,"[17] add the first and *second; it gives* 2250; and add the 300[17] of the excess; it gives 2550, which is the third. And since it says, "Let the fourth exceed the first three by 300,"[17] add the three; it gives 4800; and the 300[17] of the excess; this makes 5100, which is the fourth number.

1/7		300[17]	300[17]	9900[17]
7x	8x	15x + 300	30x + 600[17]	
1050	= 1200	2550	*5100*	
150				

COLUMN ii

. . . Since the second number is *four times* the first, *multiply* 4 x 42; it gives 168; *and add the 12 of the excess, which gives 180;* this is the second number. Check. *Take one sixth of the second number;* it is 30; but add 12; it is 42, which is the first number; *and multiply 42 by 4,* which is 168; then add 12; it gives 180, *which is the second number.*

1/6(?)	+ 12[18]	4	12[18]
x		$4x$	
1/3 = 14[18]		2 × 42	= 2 . . .
42		168 + 12	= 180

Three numbers. The sum of the three is 5300. Let *the sum of the first and second* be 24 times the third, *and let the second*[19] *be* 5 times the first. *To find the three numbers.* . . . Inasmuch as the first and the second *are 24 times the third,* therefore the sum of the three is *25 times the third. Divide 5300 by* 25 and it gives 212, *which is the third number.* . . .

In the form of solution of these algebraic problems we have a remarkable approach to modern algebraic symbolism. The problems themselves are strictly algebraic and in conception logical continuations of such a problem as that of the Ahmes papyrus, "a quantity and its seventh, it makes nineteen." These problems connect also in idea with the problems found in the Liber Abbaci of Leonard of Pisa (A. D. 1202) and current in Italian arithmetics from his time on for centuries.

Among the Arabs Al-Karkhi gives the following problem:[20]

If the first of four men receives from the second one dirhem he will have the double of what the second has

[14] Heath, "Greek Mathematics," II, 456–457.

[15] The italics indicate words, numbers and phrases supplied to fill lacunae. In the calculations, the signs of addition and multiplication are supplied; the papyrus has the sign of equality.

[16] This is the hypothesis of the first problem, entirely missing from the papyrus, but quite easy to restore from the quotations from it made in connection with the check, and from comparison with the third problem.

[17] In each of these cases the numeral is preceded by the symbol mentioned above used in the manuscripts of Diophantus to indicate the unknown term. Here it seems to mean simply "the quantity" and may be disregarded in translation. The same symbol occurs in the second line of the calculations, however, as the algebraic x.

[18] The numerals thus marked are preceded by an abbreviation which probably stands for *monades*, "units."

[19] "Let the second and third," etc., from the point of view of the Greek, is perhaps an easier and better restoration. This, however, would necessitate a fractional solution, which seems to be avoided in the other two problems.

[20] F. Woepcke, "Extrait du Fakhrī, Traité d'Algĕbre par Abou Bekr Mohammed ben Alhaçan Alkarkhi" (Paris, 1853), pp. 139–141.

remaining; if the second receives from the third two dirhems, he will have triple what the third has remaining; if the third receives from the fourth three dirhems he will have four times as much as remains with the fourth; and finally if the fourth receives from the first four dirhems he will have five times as much as remains with the first. How much does each one have?

For several centuries algebra was taught in Europe employing such problems.

By this important document a noteworthy link is placed in the chain which connects the mathematics of early Egypt with Greece and with the algebra of the Hindus, of the Arabs and of our European predecessors in this field.

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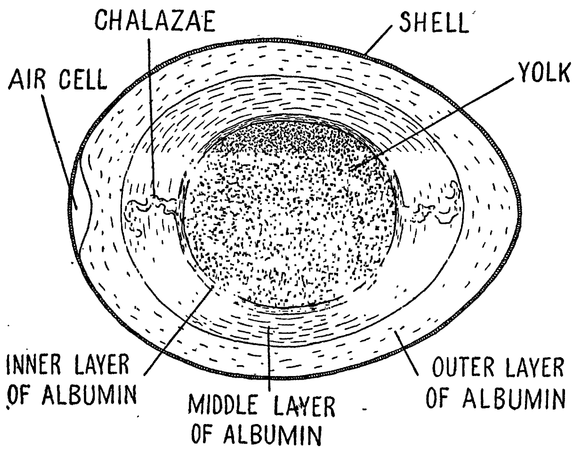
THE DRY MATTER IN DIFFERENT LAYERS OF EGG ALBUMIN

IF a fresh hen's egg is opened in a careful manner, there can be distinctly observed with the naked eye the three different layers of albumin: outer, middle and inner, as shown on the diagram. The albumin of



the egg represents the secretion of the oviductal glands. It is a complex mixture of organic and inorganic matter. The histology of the albumin is not exactly known, but the very probable theory has been advanced that it consists of a network of fibers containing fluids in their meshes.

The process of formation of albumin is in an intimate relation with the yolk of the egg—to complete the female reproductive cell. When the yolk of the egg or ova leaves the ovary it has a slow revolving movement which is controlled by the peristaltic contraction of the oviduct of the hen. In that long passage-way the yolk of the egg gets its threefold covering of albumin from certain portions of the oviduct. In the upper glandular part of the oviduct is secreted the innermost layer of albumin, which is especially dense and forms a sort of membrane over the yolk. It also extends from each side of it as a twisted cord, the chalazae (see diagram). Later in the passage the second layer or fluid albumin is secreted, which lies next to the dense one. And finally, when the yolk is almost at the end of the long journey—in the area of partially formed shell—the third, watery layer of albumin is formed.

The quantity of the different layers of albumin described above has been studied, and the following table gives an illustration of the relative amounts of fresh albumin of all the three layers in grams and in percentages:

	Outer	Middle	Inner	Total
Amount in grams:	12.81	18.43	.97	32.21
Amount in percentages:	39.8	57.2	3.0	100.0

Their physical characteristics suggest that the water content is very likely to be different.

A little experiment has been carried out in our laboratory of experimental embryology to determine quantitatively the dry matter content in the different layers of albumin. A simple method was employed for the determinations. The egg was broken into a saucer, and each layer of albumin was pipetted into the crucible and dried to a constant weight in Freas electric vacuum oven at 80° C. To prevent frothing of the albumin care was taken to start the vacuum gradually and slowly increase it up to 63.5 cm (25 inches). The table below, on five eggs as an example, gives the data in percentages for the content of dry matter in the three layers of albumin:

Egg Number	Percentage of Dry Matter		
	Outer	Middle	Inner
1	12.55	12.87	15.14
2	11.07	11.98	13.61
3	12.13	12.96	14.66
4	11.09	12.24	14.00
5	11.12	12.23	15.07
Average	11.59	12.45	14.55

Such an experiment can be performed only with fresh eggs or eggs well preserved. The albumin of old or incubated eggs loses the distinctive physical appearance of the three layers and does not give the variable results upon the analysis.

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