We shall define the uncertainty in the value of A associated with this state by the equation,

$$(\Delta A)^2 = \int \bar{\varphi} A^2 \varphi d\tau - \left(\int \bar{\varphi} A \varphi d\tau \right)^2 \qquad (2)$$

where ΔA is written for the uncertainty in A. This evidently corresponds with the classical definition of the uncertainty as the square root of the mean of the square of the deviation from the mean. One observes that if φ is such that $A\varphi = a\varphi$, where a is an ordinary arithmetical number, then ΔA vanishes. In such a case the value of A is precisely a. For example, if A is really the Hamiltonian function and φ is one of the solutions of Schrödinger's equation for the system, one has the proof that the Schrödinger wave functions correspond to states of the system in which the total energy has precisely one of the allowed values of the energy.[2]

The examples illustrating propositions (*a*), (*b*) and (*c*) are afforded by considering the different components of angular momentum of a particle about the origin. These will be denoted by M_x, M_y and M_z. As is well known, the operators corresponding to these three quantities do not commute with each other, the operators being,

$$M_x = \frac{h}{2\pi i}\left(y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y} \right) \qquad (3)$$

the other two being given by cyclic permutation of x, y and z. These operators satisfy the following commutation rules,

$$M_x M_y - M_y M_x = \frac{h}{2\pi i} M_z, \qquad (4)$$

and two others given by cyclic permutation of x, y and z.

As to (*a*), we observe that the operator for M^2, where

$$M^2 = M^2_x + M^2_y + M^2_z,$$

commutes with the operator for any component, M_x, M_y or M_z. If we consider in particular states where φ is of the form

$$\varphi = R(r) e^{im\phi} P^{m}_{l}(\cos\theta), \qquad (5)$$

where R(r) is any function of r, and the z axis is the pole of the spherical polar coordinate system, it may be readily verified that such states correspond to precise values for M^2 and M_z given by

$$M^2 = l(l+1)\left(\frac{h}{2\pi}\right)^2 \text{ and } M_z = m\frac{h}{2\pi}$$

If one compute the value of ΔM_x or ΔM_y for such states it turns out to be equal to

$$(\Delta M_x)^2 = (\Delta M_y)^2 = \tfrac{1}{2}[l(l+1) - m^2]. \qquad (6)$$

[2] This formulation corresponds to that of Weyl, "Gruppentheorie und Quantenmechanik," Leipzig, 1928, p. 67.

This is finite for finite values of l and m. Therefore we have here a class of states of the particle in which, since ΔM_z is zero, the product of the uncertainties in ΔM_x and ΔM_z is zero, in spite of the fact that the operators M_x and M_z do not commute. Hence the truth of (*a*).

As to (*b*), we notice that if

$$\varphi = R(r) \qquad \text{(independent of } \theta \text{ and } \varphi\text{)}$$

this corresponds to the precise values,

$$M_x = 0, \quad M_y = 0, \quad M_z = 0,$$

with no uncertainty in any of them. Hence the truth of (*b*).

As to (*c*), we observe that if we deal with the class of states in which M_z is known to have the value zero precisely, then from the commutation rule (4), applied to any φ of this class, the operator $M_x M_y$ gives the same result as $M_y M_x$. But nevertheless the uncertainty in neither M_x nor M_y is zero in such a state. The φ functions for this class of states are evidently of the type of (5) with m set equal to zero, so the preceding calculation of ΔM_x and ΔM_y given in (6) applies here. Hence the truth of (*c*).

It would appear, therefore, that a general uncertainty principle is not simply to be formulated in terms of the commutativity or lack of commutativity of the operators associated with A and B, as is usually implied in discussions on this subject. What the exact criteria may be, we are not prepared to state.

Although largely of a negative character, these remarks have considerably clarified the situation for me. In working them out, I have derived much benefit from stimulating conversations with my colleagues, especially Drs. J. E. Mack, E. C. G. Stueckelberg, G. P. Harnwell and H. P. Robertson.

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HONORARY DEGREES AND A SUGGESTED OPPORTUNITY

BEFORE many moons in American universities will come the harvesting time of the four years' crop of A.B.'s, B.S.'s and other varieties of B's less well known. At the same time will come to maturity the smaller crop of Ph.D.'s and D.S.'s which have had a somewhat longer ripening period under the watchful care of the academic gardeners. These exhibits are deservedly of much local interest, but it is the exceptional specimens, the men with honorary degrees, who as the big pumpkins of the harvest festival attract most attention and gain the front page of our newspapers. It may not be amiss, in the interest of

academic gardening and crop improvement, to discuss for a moment the blue ribbons, in the form of honorary degrees, which along with marriage licenses reach their peak in the month of June.

The character of the doctorate taken in course is fairly well defined. It is like the "Sunkist" trademark stamped on oranges and is presumed to denote a certain superiority in quality and at the same time suggests uniformity in grade. It is given as an indication of the recipient's ability to think and carry on constructive work independently, as judged largely by his thesis work under guidance. The test is not a perfect one since research under guidance may consist merely in working out the ideas suggested by the guider. The best proof, as of a pudding, would be an actual test of the finished product, *i.e.*, the performance record shown by independent work after weaning from alma mater. Such an ideal test is obviously impractical as a uniform procedure under present methods of mass production.

The honorary degree, in distinction from the doctorate in course, appears to have a more varied function than the stamp on the orange. Judging from appearances, honorary degrees are given in no small measure to lend honor and dignity to commencement exercises. When the presiding officer drapes the doctor's silken hood around the neck of a president of the United States and says, "Sir, in bestowing upon you this honorary degree, the university honors herself more than she honors you," he probably tells the truth. The university has not discovered the recipient's scholastic attainments and constructive service to humanity and, by bestowal of an honorary degree, brought them from relative obscurity to public recognition. Rather, in the instance mentioned, public recognition has come first and the university finally concludes that the public is right and that the president of the United States is a great man and should be designated as such by an honorary degree—if he can be induced to take the time to come and get it. Of course the example suggested is an extreme one. We have only one president of the United States at a time, and it would be physically impossible for him to receive personally in the single month of June all the degrees that universities would be willing to bestow upon him even if he made use of air transportation. And if he did, would we think any more of him or would it become any easier for him to obtain increased opportunities for the exercise of his peculiar talents in service to mankind?

In contrast to presidents we may take a hypothetical example from the opposite extreme. The recipient of the honorary degree is an assistant professor in a small institution. Despite his load of teaching he has managed to carry on research of a high order and has to his credit publications which in quality and quantity are above the average of those from one obtaining a doctorate in course. Among men of his particular profession his standing as an investigator is secure, but in his own institution his research activities have not materially heightened his standing with the administration. For some reason, he never went through the doctorate mill, and without the brand of D.S. or Ph.D. he has found advancement difficult in his own college. Negotiations were once well under way regarding a position in another institution where greater opportunities for research would have been possible, but the negotiations abruptly ceased when it was learned that the man did not have a doctor's degree. He was the type wanted for the position but unfortunately a blanket ruling called for a man with a doctor's degree. Realizing the handicap under which he was laboring, he had about decided to discontinue his investigations for a year or for so long as would be necessary to satisfy the residence and course requirements of a reputable university where he could matriculate and work for his doctorate. He had hesitated, however, on account of the expense and the serious interruption to his research which such a change would involve. Finally a specialist in his particular line of work who had known of the circumstances just related happened to be acting president of a university during the spring semester and smuggled in our hypothetical friend among the national celebrities who were honoring and being honored with honorary degrees.

The two examples given, while admittedly extremes and representative of only small groups, are based on actual instances probably familiar to all. They may serve as an introduction to a brief discussion of some of the functions of awards and honors. Formal recognition of constructive accomplishment rightly affords the recipient the satisfaction of knowing that his efforts have been successful. Honorary degrees to those who have made eminent contributions to knowledge and the welfare of humanity are especially fitting in a democracy in which orders of nobility can not be distributed by the crown. There is probably less cause for criticism of our system than of nobility. Some might wish that honorary degrees, in cases in which gratitude on the part of the university for material benefits received was a major cause for the award, might be distinguished by a special designation such as D.Dn. (Doctor of Donations). In the rare instances, however, in which such criticisms might be justified, the donations are effective in promoting education and research rather than being a price paid to the treasure chest of the political party in power, as is said at times to be the condition in monarchies.

Another aspect of the problem of honors and awards is their relation to the present campaign throughout the country for increased research activity. It seems probable that research output can be fostered more by early recognition and advancement of exceptional individuals than by attempts to raise the general average. While honors are a fitting recognition of a life spent in original and constructive contributions to knowledge, such recognition fails of its greatest value if delayed until the recipient has already reached an age or position in which such recognition no longer tends to enhance his opportunities for greater employment of his talents. Recognition of any kind from the outside, including especially calls from other institutions, is likely to be of direct advantage to a younger man by securing for him increased time and facilities for research. It may have the same effect indirectly through an increase in salary which lessens the financial pressure that may have necessitated his engaging in scientifically nonproductive work during vacations or spare moments. The discovery of exceptional talent and the devising of methods for shortening the reaction time between its discovery and increased opportunities for its exercise seem worthy problems for institutions of higher learning.

It would be out of place in the present article to enter into a full discussion of these problems. It will suffice to offer only one specific suggestion regarding a single phase which to some may perhaps rightly appear of but minor importance. We refer to the desirability of less infrequently conferring honorary degrees upon men of recognized scientific standing who have not taken their doctorate in course. No lowering of standards would need be involved if the precept were followed: "By their fruits ye shall know them." On the contrary, proper safeguards should guarantee a distinct raising of standards. Were it not for the difficulties of administration, one might feel inclined to advocate raising the master's degree to greater dignity and conferring the doctorate only for work carried on after graduation from professorial guidance. To withhold the advantages of a doctorate from one who, instead of following the conventional educational system, has devised a successful "case system" of his own seems unduly conservative.

The different divisions of the National Research Council afford specialists in the various groups of science capable of advising universities regarding qualifications of possible candidates. It is not our desire to suggest whether universities through the departments involved, the National Research Council or individuals should take the initiative regarding the honorary degrees in question. Our purpose will have been served if, in calling attention to a possible opportunity, we have raised a problem for discussion.

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HIGH TEMPERATURE DEATHS AMONG EXPERIMENTAL RATS

IN the light of the report in SCIENCE for December 7, regarding thyroid feeding and high room temperature, an experience of the author may be of interest. In this laboratory, young albino rats are being kept on Sherman's A-free diet, in a study of some of the underlying factors in the incidence of respiratory and middle ear infections in this deficiency. Controls are kept on the same diet plus vitamin A in the form of cod-liver oil. Room temperature for these rats, as for the breeding rats, is kept between 70 and 80 degrees F. Before temperature control was established, the room temperature rose one night to a maximum of 96 degrees F., with the death of three animals. Of these, two were on the A-free diet, although not yet depleted, while the third was on a diet normal in every respect. Autopsy revealed no gross pathology except congestion of the lungs.

A few weeks later, the same thing happened in another room, with the loss of two male and two female rats from the breeding stock, kept on Sherman's whole wheat-whole milk diet. Since there have been no other deaths either before or since this incident, among the experimental or breeding animals, the rise in temperature to 96 degrees F. seems to be entirely responsible for these deaths, and one must assume that this temperature is sufficiently high to cause death in normal animals as well as in those suffering from vitamin A deficiency.

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NATURALISTS OF THE FRONTIER

DR. SAMUEL WOOD GEISER, of the Southern Methodist University at Dallas, Texas, has begun the publication of a series of papers entitled "Naturalists of the Frontier." No. 1, recently published, gives the story of Jacob Boll. Another number will soon be issued, and this will relate to the well-known collector of insects and other natural history material, G. W. Belfrage, a man whose name is well known to the older generation of entomologists and other naturalists. These papers are appearing in the *Southwest Review*. Dr. Geiser will be glad to send separata to those especially interested, but it will be necessary for persons who wish the Belfrage paper to write him in advance so that he may order the necessary number of reprints.

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