research, with an infinite capacity for taking pains. Temporary failure has only spurred him to renewed activity. Few men have possessed to such a striking degree the blending of the imagination of the dreamer with the practical, driving force of the doer. In the record of his inventions and improvements rests the unimpeachable testimony that he has brought things to pass.

I have been interested in his account of a visit to the White House in 1878 to exhibit his newly developed phonograph. He relates that he came at 11 o'clock in the evening upon the invitation of President Hayes, who, with Mrs. Hayes and their guests, became so engrossed in the marvelous device that the inventor did not get away until 3:30 A. M. But we know that Edison has never made a practice of retiring early.

The field of electricity will be most closely associated in future years with the name of Edison. It has been asserted somewhere that there is scarcely an electrical process or instrument of to-day which does not reflect in some way changes wrought by his researches. Steinmetz, who should be an authority, said Edison had done more than any other man to promote the art and science of electrical engineering. In his invention of the incandescent lamp and in the perfection of means for developing and distributing electrical energy he literally brought light to the dark places of the earth. Through these and other products of his genius old industrial processes have been revolutionized, new ones developed, and our daily lives have been made easier, our homes pleasanter and more comfortable.

Although Edison belongs to the world, the United States takes pride in the thought that his rise from humble beginnings and his unceasing struggle to overcome the obstacles on the road to success well illustrate the spirit of our country. We are happy to share his achievements as our contribution to progress. He represents the finest traditions of our citizenship. At the request of the Secretary of the Navy in 1915 he became president of the Naval Consulting Board, which looked into inventions and devices designed to aid us in preparedness and later in our participation in the world war. From 1917 to 1919 his entire time was at the disposal of the government. Not only by his own discoveries, but by training in his laboratories men who have gone out to important places in the scientific and industrial world and by encouraging countless others to renewed efforts in applied science and invention, he has made a notable contribution to education.

This is my message to Mr. Edison: Noble, kindly servant of the United States and benefactor of mankind, may you long be spared to continue your work and to inspire those who will carry forward your torch.

THE HISTORY OF ELEMENTARY MATHEMATICS IN THE PLIMPTON LIBRARY[1]

WHEN Christopher Plantin and his successor, John Moretus, sought to develop their great printing establishment at Antwerp in the sixteenth century, they collected a large number of specimens of the best medieval and Renaissance manuscripts and of representative products of the leading presses of Germany, France and Italy. This collection was increased by their successors and now forms one of the best libraries extant for the study of the history of bookmaking. Plantin felt that in order to be a master printer he must know thoroughly the history of his art. He wished to make the best books possible, and to do this he must know the best that had already been accomplished.

It was a similar circumstance that led me, fifty years ago, to begin collecting books for my own library. I am a publisher of text-books; my firm (Ginn and Company, of Boston, New York and London) seeks to make the best text-books possible, and I felt that, to do this, it was necessary to know thoroughly the historical development of books of this nature. I, therefore, began to collect such material, both in the manuscript form used in the Middle Ages and in the printed form beginning in the Renaissance period. As a result, my library covers the entire field of education, the mathematical text-books being merely one division, although one of the most important.

It is impossible, in the few minutes at my disposal, to do more than refer to a few of the important features of the section devoted to the early history of elementary mathematics. The subject was treated more than twenty years ago by Professor David Eugene Smith in his "Rara Arithmetica," and various references to my later acquisitions are mentioned in his "History of Mathematics" (1923, 1925). When the "Rara Arithmetica" was written he found that while DeMorgan was able to examine less than one hundred arithmetics printed before 1601, and while Boncampagni secured for his own great library less than three hundred, and Libri a still smaller number, my own collection was even then in excess of Boncampagni's, and in the last few years it has been very much enlarged.

Of the mathematical manuscripts in the library, something over one hundred, only a few of the most

[1] Address before the International Mathematical Association, at Bologna, Italy, on September 6, 1928.

important can be mentioned at this time. They are as follows:

(1) In Euclid's "Elements" the library is rich. It has the translation of Euclid by Campanus, directly from the Arabic manuscript, which Campanus presented to the Patriarch of Jerusalem, who afterwards became Pope Urban IV. He was Pope from 1261–1264. This is supposed to be the earliest known Latin manuscript, and it is written on vellum. The translation of the theorems is thought to be the same as that made by Adelard of Bath about 1120. He is supposed to have made a translation, but no copy is known.

(2) Another very interesting manuscript is dated 1294 and happens to be bound with the manuscript of Boethius. It seems to be complete and is fully illustrated.

(3) Euclid, fragment of the "Elements." Vellum, c. 1350.

(4) Euclid, the first five books of the "Elements." Paper, c. 1375.

(5) Euclid, Book I. Paper, c. 1460. German hand.

(6) Euclid, Books I–III. Manuscript, vellum, c. 1500, being Zamberto's translation from the Greek, manuscript of Theon. The translation was printed in 1505 (Venice).

(7) Gerbert (Sylvester II). Manuscript copy on paper, c. 1600. It includes his geometry.

(8) Boethius, "Arithmetica." Manuscript, vellum, tenth century.

(9) Boethius, "Arithmetica." Manuscript, vellum, c. 1294.

(10) Boethius, "Arithmetica." Manuscript, vellum, c. 1300.

(11) Boethius, fragment of "De Musica." Vellum, c. 1300.

Arithmetic based largely on Nicomachus who lived 100 A.D. He gives elaborate theory of ratios and devotes considerable attention to figure numbers such as the triangular, square pantagnal and cubic. It was the standard of church schools throughout the Middle Ages.

(12) Mohammed ibn Musa, Algebra. Manuscript, paper, 1456. So far as is known this manuscript has never been published. It differs in many particulars from the Robert of Chester Manuscript.

(13) Venerable Bede (673–735). Manuscript, vellum, written in 1129 and stated by the scribe to have been copied from Bede's own manuscript. An unusually well-written and carefully preserved manuscript. Treats of the division of time, finding of Easter, division of months and their names in Anglo-Saxon. Burke calls him the father of English learning.

There is also an early fifteenth century English language manuscript on arithmetic that has the four subjects of addition, subtraction, multiplication and division.

A rather interesting anonymous manuscript of about 1565 was written right here in Bologna. It opens with a set of column multiplication tables.

(14) Isidorus of Seville, born 570; died 636, "Etymologies." Manuscript on vellum, twelfth century. The library also has the early printed editions of the work. The third one of twenty books is on mathematics. The manuscript is a sort of encyclopedia. The Council of Toledo designated him as the "most learned man of the ages."

(15) Anianus, "Computus Manualis." Manuscript on paper, bearing the date 1384 and written about that time. Important as being apparently the oldest manuscript of this work that can be rather definitely dated. The library contains a considerable number of the early printed editions, including what seems to be a unique copy of the Poitiers edition of 1527–1528, as shown in Professor Smith's recent (1928) bibliography of this notable work.

(16) Paolo Dagomari. Paolo dell'Abaco, Paolo Astrologo, Paolo Geometra, Paolo Arismetra, Paul of the Abacus. He was born in Prato in 1281 and died at Florence in 1374. The manuscript was written about 1339. It is primarily a treatise on arithmetic. Part of this manuscript is an ordinary commercial arithmetic such as the Florentine teacher used in the fourteenth and fifteenth centuries. It is interesting as showing some tendency towards the use of percentage.

(17) This manuscript of Albertus Magnus was written about 1350. He was called *Doctor Universalis*. The first folio contains part of the calendar, and a few random memoranda, including an old price mark of three ducats.

(18) Giovanni, son of Luca of Florence, "Trattato di Arithmetica." Manuscript, paper, 1492. One of the best sources for the history of commercial arithmetic of the fifteenth century.

(19) Benedict of Florence. "Trattato d'Arimetricha." Manuscript on vellum, c. 1460. Also an excellent source for the study of arithmetic at that period. Here are the problems, grain of wheat on chessboard; hare and hound; jealous husband; testament of the dying man.

(20) In my collection there is a manuscript by Rollandus, who was a native of Lisbon and canon of Sainte-Chapelle, Paris, in 1425. The date of this manuscript is 1424. It was prepared at the command of John of Lancaster, Duke of Bedford, son of Henry IV of England, at one time Protector of England and Regent of France. Rollandus dedicated this treatise

to him. It is a long dedication and sets forth Lancaster's interest in France and the status of mathematics at that time. In his manuscript Rollandus covers all the theoretical arithmetic then known, but takes up no practical problems. He treats of irrational numbers, a topic which is now considered algebra. It is doubtful whether there is a manuscript extant which throws more light upon the nature of French university mathematics at the time this was written.

In addition to the manuscripts already mentioned there are many others of equal importance. These relate to arithmetic, algebra, geometry and astronomy and include works of such men as Joannes de Gmunden, Sacrobosco, Bradwardin, Raymond Lullius, Vergerius, Johannes Ross, Leonardo of Pisa (extracts), Nicolo de Orbelli, Canacci and Leonardus Maynardus.

Now when we come to the printed books that deal with these subjects, the library has the first arithmetic printed at a small town called Treviso, in Italy, in the year 1478. There are in all, perfect and imperfect, 6 or 7 copies known. This work is commercial in character—the fundamental processes being taken up in common order and followed by the rule of three. It has the calendar for church purposes. The book contains the mathematics necessary for business requirements and uses Hindu-Arabic numerals.

There is also the first printed edition of Isidorus of Seville, supposed to have been printed about 1469. The date is uncertain. It is a book of etymologies. We have already spoken about his manuscript. This is the first authority we have on the learning of that period. We do not call this the first printed arithmetic because it has other chapters besides arithmetic; in fact, as I have said before, it is a sort of encyclopedia of that time.

In addition to this one, I have another very interesting first edition of the Tractatus Proportionum by Albert of Saxony, which was printed about 1478.

The mathematical activity in Italy during this period between 1472 and 1480 was very considerable. During this time there were thirty-eight mathematical works printed in the country. In the next decade there were sixty-two and in the next one hundred, with thirteen of uncertain date between 1472 and 1500, making a total of two hundred and thirteen appearing in a period of less than thirty years.

In 1480 an anonymous work was issued from the Caxton press in London, entitled "The Mirrour of the World or Thymage of the Same." This has the first chapter on arithmetic.

Here is the first commercial arithmetic ever printed. It is by Giorgio Chiarini, 1481. While this is not strictly speaking an arithmetic, it is the first printed book to give the customs relating to exchange in use among the Florentine merchants at the close of the fifteenth century. It is the source from which several later writers drew their material and is particularly valuable in showing the nature of the practical problems of the time. Copies of this first edition are extremely rare.

Prosdocimo de Beldamandi, and Liverius. First edition, 1483. Prosdocimo de Beldamandi was born in Padua about 1370 and died about 1428. He was educated at the University of Padua and also taught there. He wrote on arithmetic, music and astronomy. This book was written for the Latin schools and is a good example, the first to appear in print, of the non-commercial algorisms of the fifteenth century.

Pietro Borghi, first edition, 1484. Borghi, a Venetian arithmetician, died about 1494. This is the first edition which I have, and it is really the second treatise on commercial arithmetic to be printed in Italy. This is much more elaborate than the Treviso Arithmetic. Moreover, it set the standard for arithmetic in the early centuries, and none of the early text-books deserves more careful study. He pays considerable attention to the rule of three, which was developed many centuries earlier by the Oriental mathematicians, and was not improbably learned by the Venetian traders through their contact with the East. Partnership was to the fifteenth what the corporation is to the twentieth century. The subject of barter is treated more or less and was common in our own American text-books until about fifty years ago. The library is especially rich in different editions of this.

I have an interesting early printed book on "Ars Numerandi." Many have questioned whether it was not printed by Fust and Schoeffer, about 1470. This is a treatise on grammatical usage as applied to numbers, and a considerable portion of the text is devoted to the distinction between ordinals and cardinals and the methods of using them.

There is also the first edition of the Boethius Arithmetic, printed at Augsburg in 1488. The library is particularly rich in editions of the Boethius books.

It also has the first printed edition of Anianus and Johannes Sacrobosco, Strasburg, 1488. Anianus was a fifteenth century astronomer and poet of Strasburg, and Sacrobosco was born in Halifax, Yorkshire, and died in Paris in 1244 or 1256. Sacrobosco wrote on astronomy and algorisms. In the book of Anianus appears for the first time in print: "Thirty days hath September," etc. This probably is also the first book on mathematics printed in Strasburg. There are a great many different editions of Anianus as well as Sacrobosco.

The library possesses also the first book printed in Germany on arithmetic by Johann Widman. Widman was born in Bohemia, in 1460, and was educated at Leipzig. The interesting feature about this book, the first printed work on calculation by the aid of counters, is that it contains the device of Martin of Würzburg and was probably printed by him about 1488. After a brief introduction on the use of counters, he takes up the subjects of division, subtraction, multiplication, progression, etc. There are many different editions of Widman in my library.

Another rare and interesting book is that of Philippi Calandri, which was printed in Florence in 1491. This is the first arithmetic with illustrations. My library happens to have two copies of this. I found this on a book-stall in Rome and paid one lira for it.

One of the rarest of the early printed books is that of Francesco Pellos or Pellizzati, Turin, 1492. Pellos was a native of Nice. Pellos first considers the fundamental operations with integers, following this by a treatment of proportion, square root and cube root. He then discusses the subject of fractions in much the same order, the rule of three, certain rules relating to weights, time, money and other measures and such topics as partnerships, barter, interest, alloys and the rule of false position, single and double. He closes the work with a chapter on mensuration, or as he calls it, "De la art de jeumentria," and gives a number of interesting woodcuts. The chief interest of the book attaches, however, to the fact that Pellos came very near the invention of decimal fractions, and that he actually used the decimal point. It can not be said, however, that he had any conception of the real value of the decimal fraction as such; Pellos simply used the decimal point to indicate division by some power of ten, writing a common fraction in the quotient. Thus, to divide 425 by 70, Pellos would divide 42.5 by 7, writing the result 6 5/70. Professor David Eugene Smith first discovered this book, and always takes a great deal of interest in saying that I paid for it as many dollars as there are days in the year.

The library has also the first edition of Luca Paciuolo, "De Borgo San Sepolcro." This has arithmetic, algebra and geometry, being treated largely from the scientific rather than from the practical standpoint. Here is included an interesting chapter on finger symbolism. The arithmetic, for example, gives the various methods in multiplication and division, instead of emphasizing the one or two most prominent in business circles. In the same way Paciuolo's treatment of the rule of three, the rule of false position (el cataym), partnership, pasturage, barter, exchange and interest, while nominally practical, was too elaborate for the mercantile schools. His was the first printed work to illustrate the finger symbolism of number.

Thomas Bradwardin, first edition, 1495. Bradwardin, Archbishop of Canterbury, was a professor of theology at Oxford. He was the first really great mathematician after Beda and wrote four different works on arithmetic, largely based on the theory of numbers.

Martianus Mineus Felix Capella, first edition, 1499. Capella was born at Carthage about 1475 and lived in Rome. This work is more or less an encyclopedia and treats of the various classes of numbers, such as plane and solid. It is one of the great text-books of the Middle Ages.

Gregorius Reisch, "Margarita Philosophica," first edition, 1503. Reisch was born in Württemberg, and died in Freiburg, 1523. He was a Carthusian monk and was prior of the cloister at Freiburg. This is a sort of encyclopedia, also. It contains a compendium of the trivium, the quadrivium and the natural and moral sciences. It is made up of twelve books. The arithmetic closes with a treatment of line reckoning, giving the four fundamental operations and the rule of three.

The first edition of Juan de Ortega (1512), a Spanish priest of the Dominican order in Aragon. This arithmetic was one of the most celebrated written in Spain in the sixteenth century. It is a purely commercial text, beginning with notation, taking up the four processes with integers, the progressions, the roots and the checks on operations and the same operations in the same order with fractions, and then discussing the business rules. There are several editions of it in my library.

An interesting arithmetic is that by Jacob Köbel, first edition, 1514. Köbel was born at Heidelberg in 1470 and died in 1533. He studied at Cracow, and one of his fellow pupils was Copernicus. This book appeared under very many different titles. It was a celebrated work and became a very famous arithmetic.

I have a number of arithmetics by Girolamo and Giannantonio Tagliente, the first edition of which was issued in 1515. The work opens with a brief statement of notation and finger symbols, followed by the multiplication table, the proof of sevens, various methods of multiplication, division by the galley method, addition chiefly by denominate numbers, subtraction, the operations with fractions in the same order, exchange, rule of three and applied problems. There are very few arithmetics that have exerted the influence that this did in the shaping of subsequent arithmetics.

I have a very interesting collection of various authors, bound in one volume—Joannes De Muris (born in Normandy, 1310; died in 1360); Thomas Bradwardin, about whom I have already spoken; Nicolaus Horem (born at Caen, 1323; died in 1382); Georg Von Peurbach (born, 1423; died, 1461), and Johannes de Gmunden (born, 1380; died in 1442)—dated 1515. The work consists of five parts.

The library contains the first edition of Gaspar-Lax (1515), who was born in Spain in 1487 and died at Saragossa in 1560. He was a famous teacher in Paris and Saragossa. His book is largely theoretical arithmetic, based on Boethius and his medieval successors.

Henricus Grammateus. First edition in 1518. This book is really a mercantile arithmetic, and the operations are both according to the abacus and by Hindu-Arabic numerals. It has a chapter on bookkeeping, and gives a good deal of consideration to the theory of numbers, the rules of the Coss (algebra), music, bookkeeping and gauging. The signs "plus" and "minus" are here first found in the rule of false position. He differs from Widman in that he uses them for a different purpose.

Estienne de la Roche. First edition, 1520. This is really the best of all the early French arithmetics, and is semi-mercantile in character. Here is a very complete treatment of the operations with integers, fractions and compound numbers, and a large number of business applications. Much of his work, however, is taken from his teacher, Chuquet. I have several editions of this.

I have the first arithmetic printed in England by Cuthbert Tonstall, 1522. Tonstall was born at Hackforth, Yorkshire, in 1474, and died in 1559. He was educated at Oxford, Cambridge and Padua. He was bishop of London, and later of Durham. His object in writing the book, he said, was to check certain goldsmiths whose accounts he suspected were incorrect. It is in Latin, but is based largely on Italian models, and includes many business applications such as partnership, profit and loss and exchange. He dedicates the book to his friend, Sir Thomas More. More speaks of Tonstall in the opening lines of his "Utopia":

> I was colleague and companion to that incomparable man Cuthbert Tonstall, whom the king with such universal applause lately made Master of the Rolls: but of whom I will say nothing; not because I fear that the testimony of a friend will be suspected, but rather because his learning and virtues are too great for me to do them justice, and so well known, that they need not my commendation unless I would, according to the proverb, "Show the sun with a lanthorn."

The "Utopia" was first printed in 1516, so this sonorous praise was written some years before Tonstall's arithmetic appeared.

There was also the arithmetic of Adam Riese, first edition, 1522. Riese was born in 1489, and died in 1559. He was one of the most celebrated of the mathematical teachers of the sixteenth century. So firmly did he impress himself upon the schools that *nach Adam Riese* is a common expression in Germany to-day. His books were to Germany what Borghi's book was to Italy, and what Recorde's was to England. The library has most of the many editions of Adam Riese.

It has also the first edition of *Orontius Finaeus*, 1530–1532. He was supposed to be the most pretentious French mathematician of his time. The library has a great many different editions of this.

I have also several editions of Michael Psellus, 1532. He was one of the last Greek writers on arithmetic, and his work was largely devoted to the theory of numbers. It covers the medieval Quadrivium—arithmetic, music, geometry and astronomy.

Then I have the first edition of Nicomachus, 1538, which is the most celebrated of the few Greek treatises upon the subject. Nicomachus was born 100 A.D. He tried to do for the Greek theory of numbers what Euclid did for geometry.

Gemma Frisius, first edition, 1540. I have many different editions of this, which was one of the most popular arithmetics of Germany in the sixteenth century. It combined the older science of numbers with commercial arithmetic, so that it appealed remarkably to the teachers of that time.

The library has numerous editions of Robert Recorde, the first noteworthy arithmetic printed in the English language, 1542. Recorde was born about 1510 and probably died about 1558. He was educated at Oxford and Cambridge. The arithmetic was in the form of a dialogue between master and student. The student thinks the study of number is useless and vain, that people can get along just as well without it and that hence it is a waste of time. The master replies:

> If number were so vile a thing as you did esteem it, then need it not be used so much in men's communication. Exclude number, and answer to this question, "How many years old are you?"
>
> "Mum," the student replies.
>
> "How many days in a week?" "How many weeks in a year?" "What land has your father?" "How many men doth he keep?" "How long is it since you came from him to me?"
>
> "Mum," answers the student.
>
> "So that if number want, you answer all by mummes."

Michael Stifel, "Arithmetic," 1544. Stifel was reformer and one of the most skilful arithmeticians of his time. The library has several editions of his books.

Johann Scheubel, first edition, 1545. Scheubel was born in 1494 and died in 1570. He was professor of mathematics at Tübingen. He wrote on arithmetic and algebra, and edited part of Euclid.

I have also the first edition of the rare Spanish arithmetic by Gaspard de Texeda, 1546. Here the fundamental processes with integers, fractions and denominate numbers are given.

Nicolo Tartaglia, "General Trattato," first edition, 1556. Tartaglia was born at Brescia in 1506 and died at Venice in 1559. He was one of the best mathematicians of his time, and his connection with the general solution of the cubic equation is well known.

Iean Trenchant, first edition, 1566. One of the best of the sixteenth century text-book makers of commercial arithmetics in France. The book was divided into three parts, the first dealing with the fundamental operations with integers and fractions; the second treats of the rule of three in its various forms; and the third treats of the properties of numbers, including figurate roots and progressions, and has some work on discount.

The second important arithmetic printed in the English language was by Humphrey Baker, 1568. It is called "The Well Spring of Sciences, which teacheth the perfect worke and practise of Arithmeticke, both in whole Numbers and Fractions." Baker's work was a rival to Robert Recorde's "Ground of Artes." He has chapters on merchandise, fellowship, barter, alligation, false position and the like.

A rather interesting edition of Petrus Ramus's Arithmetic, which has on the title page in the handwriting of the author the inscription, giving it to his pupil Johannes Sturm, the famous German scholar.

Another interesting arithmetic is that by Leonard and Thomas Digges, first edition, 1572. There are only about twenty pages that are arithmetic. Then follows a brief treatment of algebra, after which are certain matters relating to military affairs. Leonard Digges was born at Barham, Kent, and studied at Oxford. He died in 1571. Thomas was a son of Leonard, and was also born in Kent and educated at Oxford. He died in 1595.

There was a very interesting arithmetic by Thomas Masterson, London, 1592. This book, however, does not make any particular contribution to the subject of arithmetic. Book I is on the fundamental operations with integers and fractions; Book II is a collection of practical problems representing the mercantile activities of London; and Book III refers chiefly to irrational numbers, which would now be considered algebra.

Frisius, Cardan, Trenchant and Bombelli, of the seventeenth and eighteenth century writers, the most important ones, judged from the standpoint of their text-books, are generally included. There are first editions of most of their works, but the list is too long to be given at this time.

The library has always been open freely to scholars, and for many years it has been used by them in the preparation of theses and of books.

I am often asked what my plans are for the future of this library, and the matter is so important for students of the history of mathematics that the question deserves a reply. I do not believe that it is for the best interests of the world, at least for the United States, that such libraries as are now in my country should be dispersed. Collectors and booksellers may wish them to go to the auction rooms, but it seems to me that a library like mine should be kept intact for the use of scholars. This I propose shall be done in this case, provision being made for its preservation and growth in one of our large universities.

GEORGE A. PLIMPTON

NEW YORK, N. Y.

ANDREW HENRY PATTERSON
1870–1928

As a result of the death on September 9 of Andrew Henry Patterson, dean of the University of North Carolina's School of Applied Science and professor of physics, the South has lost one of its ablest educators and men of science.

Dr. Patterson was born September 28, 1870, at Winston-Salem, North Carolina. He received a liberal education in the arts and sciences, taking his Ph.B. and a B.E. at the University of North Carolina in 1891. In order to broaden his scientific education he entered Harvard, at which institution he took his A.B. in 1892, remaining there the following year for his M.A., with a thesis on "High Tension Phenomena."

The next year found him as a student engineer and electrician with the Consolidated Traction Co., of New Jersey, whence he joined the faculty of the University of Georgia as instructor of physics, thus reverting to his original plans for which his education had best fitted him. His success as a teacher was at once recognized by his promotion to an adjunct professorship in 1897 and to professor of physics in 1898.

His first sabbatical leave in 1905 was spent at Berlin and at the Charlottenburg Technische Hochschule, where he devoted himself to the study of electrical engineering.