DISCUSSION AND CORRESPONDENCE

NEWTON'S LAW OF GRAVITATION—A DANGER SIGNAL

THE writer is fully aware of the fact that it is very much easier to criticize adversely a book on physics than it is to write a reliable text in this field. On the other hand, he has also observed that it is almost impossible to eradicate an error when it has once obtained a start through the prestige of the name of a noted scientist. A brief history of a case of this kind is recorded on page 127 of the second edition of "The Principles and Methods of Geometrical Optics" by James P. C. Southall. But, in my opinion, errors are not as serious in advanced works as in text-books written for college students, since investigators think independently, whereas the undergraduate usually considers anything printed in a book adopted by his instructor to be absolutely unimpeachable, quite regardless of whether the author is a novice in the art of exact expression or an old, seasoned writer. Among the relatively large number of text-books on physics submitted to me recently for examination two contain an altogether gratuitous inexactness of statement of Newton's law of gravitation which should not be allowed to pass unnoticed. It is especially desirable to call attention to this matter because it is highly probable that these two volumes will influence a large number of students.

"Any two bodies attract each other with a force proportional to the product of their masses (*i.e.*, to the amount of matter they contain) and inversely proportional to the square of the distance between them." ". . . when the distance between them is *d*." Again: "Any two bodies in the universe attract each other with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between them." ". . . *d* the distance between their centers, . . ."

The book from which the first quotations were made is extremely misleading with respect to the meaning of the word "distance" and the second text adds confusion by referring to an undefined "center." For illustration, what is *the* center of a plane triangular lamina of homogeneous material? Is it the center of mass, or the center of the inscribed circle, or of the circumscribed circle, or of an escribed circle, or the intersection of the altitude lines, etc.? Taking the law as stated and implied, what would be the value found by a student for the force between a homogeneous sphere and an exactly concentric enclosing homogeneous spherical shell of finite thickness of wall? If the distance (zero) between the centers is taken the result will be infinite; if the radial distance between the outer surface of the solid sphere and the inner surface of the enveloping shell is taken the result will be variable. In marked contrast with the above citations is the presentation in article 6, page 139, of another very recent book, the one by A. A. Knowlton.

It would be helpful to many students if they were afforded the opportunity of mastering the following extremely simple case. Let a mass M (or the center of a homogeneous sphere) coincide with the geometrical center of an arc of a circle (or wire of negligible cross-section) along which a mass m is uniformly distributed. Let the radius of the arc and the angle subtended at M by the extremities of the arc be respectively a and 2ϑ. The force along the bisector of the angle 2ϑ is given correctly by

$$F = (GmM \sin\vartheta)/(a^2\vartheta).$$

If the mass m were concentrated at a point on this bisector at a distance x_1 from M, the same force would be exerted when

$$x_1 = a\vartheta^{\frac{1}{2}}(\sin\vartheta)^{\frac{1}{2}}.$$

The center of mass of the arc is situated at

$$\bar{x} = (a \sin\vartheta)/\vartheta.$$

If "center" in the second quotation means the middle point of the arc the force will be

$$F' = (GMm)/a^2 = (\vartheta F)/\sin\vartheta.$$

If "center" signifies the center of mass the force will be

$$F'' = (GMm\vartheta^2)/(a \sin\vartheta)^2 = (\vartheta^3 F)/(\sin\vartheta)^3.$$

The errors in per cent. may be read from the following table.

ϑ	$100\,F'/F$	$100F''/F$
0	100.	100.
$\pi/6$	$100\pi/3 \;= 104.72$	$100\pi^3/27 \;= 114.84$
$\pi/4$	$100\pi/2^{3/2} = 111.07$	$100\pi^3/2^{9/2} = 137.03$
$\pi/3$	$200\pi/3^{3/2} = 120.92$	$800\pi^3/3^{9/2} = 176.80$
$\pi/2$	$100\pi/2 \;= 157.08$	$100\pi^3/8 \;= 387.58$

Also, when

$$\vartheta = \pi/4,\ x_1 = 1.0539a \text{ and } \bar{x} = 0.9003a.$$

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THE PRAIRIES AGAIN

THE explanation of the cause of the treelessness of the prairies, offered by Professor Jones in SCIENCE for October 7, 1927, represents one of those cases, especially numerous in this field, in which broad generalizations are based on rather limited observations. It is therein assumed that rapid drainage of postglacial waters was responsible for the establishment of the prairie flora.