To imply that the theory of evolution is in the least endangered by this discovery of a mistake about a single fossil is as though a bridge builder abandoned his bridge and helped to destroy it because a single girder, not yet built into the structure, was found defective on the testing floor. Certainly no evolutionist believes that the theory is impaired. Had the tooth proved really to belong to some man-like or ape-like creature, that might have meant something about the history of the New World monkeys. It would neither have strengthened nor weakened materially the idea that evolution is a fact.

To have the public interested in science is a great advantage to both parties, but not entirely free from danger. Scientific research going on in a show window might mislead watchers who tarry too short a time to understand what it is all about. When first found the Nebraska tooth was heralded popularly beyond its real importance. Doubtless its fall will be too widely hailed now as another "mistake" of the scientists. In truth it is but a trivial incident in the slow rise of the edifice of science. The theory of evolution is far too hardy a creation to be ruined by losing one tooth.—*New York Herald-Tribune.*

SCIENTIFIC BOOKS

The Abilities of Man, their Nature and Measurement. By C. SPEARMAN. New York, The Macmillan Co., 1927. vi + 415 + xxxii pp.

THE Grote professor of philosophy of mind at the University of London has written an important book. It could not be otherwise when the book represents the cumulation of intellectual endeavor for a period of a quarter century by such as he. It may well be that he does not know exactly what his theories and facts signify; it is certain that I do not. The work has been supported during its progress by the collaboration of a multitude of Spearman's pupils and by others, it has drawn widely upon the investigations of other schools, it has also had constant opposition and the book has been severely criticized in a review in *Nature* (August 6, 1927, p. 180) which has led to an interchange of views between author and reviewer (*Nature,* November 12, 1927, p. 690). Into this difference I will not enter except to say that whether the book is mathematically complete or not does not interest me; this is unimportant. Science advances not so much by the completeness or elegance of its mathematics as by the significance of its facts. You can not upset the findings of the "Origin of Species" either by the contraposition of your religious convictions or by observing that Darwin's statistical technique was not up to standard. Science goes forward upon "evidence beyond reasonable doubt"; to that evidence incomplete mathematics may contribute valuable elements.

Spearman's chief thesis is that when a group of persons x, y, z, . . . are given a test *a,* say of arithmetic or spelling or literary interpretation, the marks m_{ax}, m_{ay}, m_{az}, . . . which they score represent in part their respective general intelligences g_x, g_y, g_z, . . . and in part their special abilities in the subject, s_{ax}, s_{ay}, s_{az}, . . . This would seem incontrovertible provided we mean by ability in the subject, ability to get scores in the test. The necessity for this proviso may be illustrated as follows. I have some general intelligence; I have some mathematical ability; yet if an examiner should set me a mathematical test in Yiddish, which might be "easy meat" for a lot of candidates for admission to our colleges, I should miserably fail. It may further be remarked that the scores m_{ax}, m_{ay}, . . . may depend on the manner of scoring used by the examiner or his clerk. For example, if the test be of the simple sort where a large number of questions are answered yes or no, one method of scoring is to count the number of right answers, R_x, R_y, . . .; another method is to take the difference between the numbers right and wrong $(R-W)_x$, $(R-W)_y$, ... If all the N questions are answered, the scores are equivalent since $W = N - R$ and the series of scores R_x, R_y, . . . and $2R_x - N$, $2R_y - N$, . . . are in the same order, will give the same correlations with other tests, etc. But if some of the questions are unanswered (U), the second series becomes $2R_x - U_x - N$, $2R_y - U_y - N$, . . . which need not be equivalent to R_x, R_y, ... How are we to compare the answers of two persons to 50 questions if one answers 40 all correctly and the other answers all 50 with 45 right and 5 wrong?

The next thesis is that when a battery of tests *a, b,* . . . are sufficiently different, so that the scores may be assumed to have in common only the general intelligence we may write for the nk marks of the n individual x, y, z, . . . on the k tests *a, b,* . . .

$$m_{ax} = c_a g_x + s'_{ax} \qquad m_{bx} = c_b g_x + s'_{bx}$$
$$m_{ay} = c_a g_y + s'_{ay} \qquad m_{by} = c_b g_x + s'_{by} \qquad (1)$$

in such a manner that the general intelligence g and the special abilities s′, are uncorrelated, *i.e.,*

$$\Sigma g_x s'_{ax} = 0, \qquad \Sigma g_x s'_{bx} = 0, \ldots \qquad (2)$$
$$\Sigma s'_{ax} s'_{bx} = 0, \qquad \Sigma s'_{ax} s'_{cx} = 0, \ldots \qquad (3)$$

when the summation runs over the individuals x, y, z, ... This leads to some correlation algebra to prove both that such a resolution of the marks is possible and that it is unique. I have read the proofs with care (including the references to the literature, not all of which has been reproduced in the book) and have found no errors in the mathematics. Yet I am not entirely happy, satisfied. I should like to have found at least one example worked out in detail—one set of nk scores for n individuals on k tests worked through to the determination of the n values $g_x, g_y, \ldots$ of the general intelligences of those individuals and of the nk values $s'_{ax}, s'_{ay}, \ldots; s'_{bx}, s'_{by}, \ldots; \ldots$ of their special abilities on each of the tests. Theorems which prove the existence of some possibility do not satisfy the practical applied mathematician—we do not so much want to know that there is a solution to the problem as to know what the solution is! I will work an example below.

What solution does the author offer us? (First he adopts scales which reduce the scores on each test so that they have the same dispersion about their means, we may take it as unity, which is also the dispersion of g.) If r_{ag} be the correlation coefficient between g and the test a, he shows that the solution is

$$g_x = r_{ag} m_{ax}$$

with a probable error of $.6745\ (1 - r^2_{ag})^{1/2}$. Note that the answer is a regression equation. We do not know the individual values $g_x, g_y, \ldots$; we could write

$$g_x = r_{ag} m_{ax} + e_{ax}$$
$$g_y = r_{ag} m_{ay} + e_{ay}$$

where $e^2_{ax} + e^2_{ay} + \ldots = n(1 - r^2_{ag})$. If the author desires to prove that testing does not determine the general intelligence of the persons tested he has succeeded. Why did he pick on test *a* to determine g? Evidently one could equally well write

$$g_x = r_{bg} m_{bx}, \text{ etc.}$$

with a probable error $.6745\ (1 - r^2_{bg})^{1/2}$, etc. Practically we might choose that test as *a* which has the highest correlation with g. Better, he shows how to weight the different tests so as to get a combined score *t* which best determines g. In the example this best score gives $r_{tg} = .75$ so that the probable error in g_x is $.6745 \times .6614 = .446$. When we recall that the scale of g is such that the standard deviation of g is unity or that one half of the n values of g lie between $-.67$ and $+.67$, two thirds of them between -1.0 and $+1.0$, we can appreciate the significance of a probable error of .45. The solution for the special ability is likewise

$$s_{ax} = (1 - r^2_{ag})^{1/2} m_{ax}$$

with the probable error $.6745\ r_{ag}$. The better the test *a* estimates g the worse it estimates the special ability. Spearman's comment is: "We are faced by the fact that the current measurements of specific abilities—upon which have come to hang the weal or woe of countless individuals in industry and otherwise—are little more than the blind leading the blind." Rather pessimistic I call it, possibly unjustifiably so in view of such success as persons like O'Connor (West Lynn Works, General Electric Company) have in their placement work.

Spearman gives a long discussion of the attempts that have been made to define general intelligence. He does not define it, he computes it, and at that only by a regression equation, he does not measure it any more than he would weigh a person by computing his weight from his height through a regression equation of weight on height. He sets forth a hypothesis that the general intelligence is energy, the special abilities are engines, with apparently the will as engineer. This is allegory. If intelligence were energy it should be measured in ergs—but again he calls it a force (p. 414), so perhaps he thinks of measuring it in dynes. Or perchance the whole is mere logomachy. It would be interesting to enquire which of the technical physical terms is most like g, the general intelligence. Perhaps it might be efficiency! It would also be interesting to know just what he or Maxwell Garnett (a competent applied mathematician) means by the word unique in the proof that the resolution into g's and s's is unique. He can hardly mean that the regression equation $g_x : \sigma_g = r_{ag} m_{ax} : \sigma_a$ is unique since there is one such for each test and they give different results. If he means that given the nk grades $m_{ax}, m_{bx}, \ldots, m_{ay} \ldots$ we can determine the actual values of $g_x, g_y, \ldots$, why are we given the regression?

Example (preamble). If we can assign the k quantities $c_a, c_b, \ldots$ and the n values $g_x, g_y, \ldots$ equations (1) will determine the nk special abilities s′ from the nk grades m. Equations (2) if the n values $g_x, g_y, \ldots$ are known will determine the k values $c_a, c_b, \ldots$ as $c_a = \sigma_a r_{ag} / \sigma_g$. To have the intelligence g on a uniform scale we shall assume $\sigma_g = 1$ which gives one quadratic equation between the n values $g_x, g_y, \ldots$

$$g^2_x + g^2_y + g^2_z + \ldots + g^2_n = n \qquad (4)$$

We must find the k coefficients r_{ag}. Equations (3) when expressed in terms of the m's and g's give the equations

$$r_{ag} r_{bg} = r_{ab}, \qquad r_{ag} r_{cg} = r_{ac} \qquad (3)$$

and if there be three or more tests enable us to solve for r_{ag}, etc., as

$$r_{ag} = \sqrt{\frac{r_{ab} r_{ac},}{r_{bc}}} \text{etc.} \qquad (5)$$

This requires that the values r_{ag} should be fractional or (if all the correlations r_{ab}, r_{ac}, r_{bc}, . . . between tests be positive, as is usually the case) that

$$r_{bc} \gtreqless r_{ab} r_{ac} \qquad (6)$$

or that the partial coefficients $r_{bc.a}$ shall all be positive, and, if there are more than three tests, that the so-called tetrad relations vanish, *i.e.*,

$$r_{ac} r_{bd} - r_{ad} r_{bc} = 0. \qquad (7)$$

These relations (6) and (7) are verified within the experimental error with respect to a large variety of intelligence tests. There is the equation

$$g_x + g_y + g_z \ . \ . \ . + g_n = 0 \qquad (8)$$

introduced to simplify the analysis and refer all g's to their mean. If the m's are also thus expressed, as is most convenient, the s's will be relative to their means. The k equations (5) are linear in the g's, *viz.*,

$$m_{ax} g_x + . \ . \ . + m_{an} g_n = n \ r_{ag} \sigma_a \qquad (5')$$

We have, therefore, in (5′), (8) and (4) the number $k+1$ of linear equations and one quadratic equation in the n quantities g. It would seem as though the n values g could be found with $n-k-2$ degrees of freedom, *i.e.*, that, as n is generally much larger than $k+2$, the solution should be indeterminate rather than unique.

Example (*solution*). Try a case. Let the marks of 6 students on 3 tests be (the first columns give the actual marks, the second columns give the differences from the means)

	a		b		c	
1	10	5	8	3	1	2
2	8	3	5	0	9	4
3	6	1	9	4	4	−1
4	4	−1	7	2	8	3
5	2	−3	0	−5	1	−4
6	0	−5	1	−4	1	−4
$6\sigma^2$		70		70		62

$$r_{bc} = .66, \ r_{ac} = .73 \ r_{ab} = .74$$

The equations to be solved are

$$5g_1 + 3g_2 + g_3 - g_4 - 3g_5 - 5g_6 = 18.5$$
$$3g_1 + 0g_2 + 4g_3 + 2g_4 - 5g_5 - 4g_6 = 16.7$$
$$2g_1 + 4g_2 - g_3 + 3g_4 - 4g_5 - 4g_6 = 16.3$$
$$g_1 + g_2 + g_3 + g_4 + g_5 + g_6 = 0$$
$$g^2_1 + g^2_2 + g^2_3 + g^2_4 + g^2_5 + g^2_6 = 6$$

The result of solving the first four for g_1, g_2, g_5, g_6 in terms of g_3 and g_4 and substituting in the last is

$$g_3 = .49 - .7g_4 \pm .89_4 \sqrt{(-g^2_4 + .40g_4 + .069)}$$

The radical is positive only if g_4 lies between −.14 and +.52 and any value of g_4 between these limits is possible. For the two limits the solutions for the g's are

		diff.
$g_1 = .91$	$g_1 = 1.43$	−.5
$g_2 = 1.21$	$g_2 = .42$	+.8
$g_3 = .59$	$g_3 = .12$	+.5
$g_4 = -.14$	$g_4 = .52$	−.7
$g_5 = -1.47$	$g_5 = -.80$	−.7
$g_6 = -1.10$	$g_6 = -1.69$	+.6

Notice that the ranges of possible intelligence for the six are different; we have a better line on 1 and 3 than on 6 and know least about 2.

Let us next compute the special abilities so standardized that their standard deviations are unity. The equations given by Spearman are like

$$m_{a1}/\sigma_a = r_{ag} g_1 + \sqrt{1 - r^2_{ag}} \, s_{a1}$$
$$m_{a1}/3.4 = .905 \, g_1 + .42 \, s_{a1}$$
$$s_{a1} = .7 \, m_{a1} - 2.1 \, g_1$$

On the basis of the extreme alternative solutions given above we have

		diff.
$s_{a1} = 1.6$	$s_{a1} = .4$	+1.2
$s_{a2} = -.4$	$s_{a2} = 1.2$	−1.6
$s_{a3} = -.5$	$s_{a3} = .5$	−1.0
$s_{a4} = -.4$	$s_{a4} = -1.8$	+1.4
$s_{a5} = +1.0$	$s_{a5} = -.4$	+1.4
$s_{a6} = -1.2$	$s_{a6} = +.1$	−1.3

Similarly we could compute for tests *b* and *c* the limits of specific ability. (The calculations given above have been carried to so few places that a positive check can not be expected, either for the zero mean or the unit standard deviation.) What we have shown is that the complete solution can be obtained but is indeterminate. We have had no need of any harder mathematics than the solution of a set of $k+1$ linear equations and 1 quadratic equation. We do not need the generalized Bravais distribution (as used by Garnett) and in view of Yule's wise comments on mental measurements (*Brit. J. Psych.*, vol. 12, p. 100.) to all of which I hereby subscribe, it would seem quite superfluous to introduce this higher mathematics, involving a probability theory which probably does not apply anyhow, to make determinate (if it does) that which without it seems indeterminate.

Do g_x, g_y, . . . whether determined or undeterminable represent the intelligence of x, y, . . .? The author advances a deal of argument and of statistics to show that they do. This is for psychologists, not for me, to assess. I believe he does not adequately emphasize the fact that they represent the intelligence only relative to the set-up of the tests. That this is so is evident from general considerations of the transformation theory of correlation algebra; but as even the term "transformation theory of correlation algebra" seemed unknown and unintelligible to a large group of persons professionally interested in statistics and in education when I recently mentioned it to them, I take the space, in a review already too long, to expound the obvious. 1°, If we have nk marks of n individuals x, y, z, . . . on k tests *a, b, c,* . . . we may combine these marks into new sets of scores, call them *a′, b′, c′,* . . . , in a linear fashion as

$$m'_{ax} = c_{11}\, m_{ax} + c_{12} m_{bx} + \ldots$$
$$m'_{ag} = c_{11}\, m_{ay} + c_{12} m_{by} + \ldots$$

$$m'_{bx} = c_{21}\, m_{ax} + c_{22} m_{by} + \ldots$$

$$m'_{cx} = c_{31}\, m_{ax} + c_{32} m_{by} + \ldots$$

with k^2 constants c_{ij}. These new scores m′ contain all the information of the old scores because they can be solved for the old scores m, but the information is differently assembled. It may be that these scores do not measure any particular ability such as spelling or literary interpretation or mathematical judgment, but they do represent scores involving certain weighted combinations of such abilities and with my limited knowledge of intelligence testing seem to represent some sorts of ability. 2°, Irrespective of whether the tetrad relations (7) are fulfilled, we can choose the constants c_{ij} in infinitely many ways so that the new scores are all uncorrelated, *i.e.,* $r'_{ab} = r'_{ac} = r'_{bc} = \ldots = 0$. In this case the tetrad equations for the new correlation coefficients must vanish. Now if $g_x, g_y, \ldots$ be the general intelligence of the persons tested the equations (3′) can no longer be solved as in (5) for $r'_{ag}, r'_{bg}, \ldots$ because (5) become indeterminate; but inspection of the equations

$$r'_{ag}\, r'_{ag} = r_{ab} = 0$$
$$r'_{ag}\, r'_{cg} = r_{ac} = 0$$

shows that all of the correlations of g to the new scores must vanish except at most one. Which one? As the equations defining *a′, b′, c′,* . . . are largely arbitrary, the symmetrical and natural conclusion would be that none of them are correlated with g. Or we might so form one of them say *a′* as to agree that it represents the intelligence g with $r'_{ag} = 1$ and the others represent special abilities independent of g. Next, 3°, to be more specific we may take as one simple definition of *a′, b′, c′,* . . .

$$m'_{ax} = m_{ax}$$
$$m'_{bx} = m_{bx} - \sigma_b r_{ab} m_{ax}/\sigma_a$$
$$m'_{cx} = m_{cx} + \beta m_{bx} + \alpha m_a$$

and determine β, α, so that $r'_{ac} = 0$, $r'_{bc} = 0$, and so on. Now as we know r_{ag} by (5) as other than 0, it follows that $r'_{ag} = r_{ag} \neq 0$ and that the remaining values r'_{bg}, r'_{cg}, . . . all vanish. But from the definition of *b′*

$$0 = \sigma'_b\, r'_{bg} = \sigma_b (r_{bg} - r_{ab} r_{ag})$$

or

$$r_{bg} - r_{ab} r_{ag} = 0$$

This last equation is, however, impossible since we know that $r_{ag} r_{bg} = r_{ab}$. Hence, 4°, any set of values $g_x, g_y, \ldots$ for the general intelligence of x, y, . . . which will go with the set-up of tests *a, b, c,* . . . can not possibly go with the set-up *a′, b′, c′,* . . . but must be replaced by new values $g'_x, g'_y, \ldots$ approximate to that set-up. Yet the information we had in the nk scores of x, y, . . . on *a, b,* . . . is all contained in the nk scores assigned to x, y, . . . on *a′, b′,* . . .; the persons x, y, . . . are the same but their intelligences have changed—the old values whether indeterminate or unique will no longer do. What does this leave of the concept of the intelligence of an individual x as measured by g_x? Apparently only that it is relative to the set-up, which is the obvious proposition that I set out to prove.

The intelligence tester may object that the scores on *a′, b′* . . . mean nothing, are mere artificialities, whereas those on *a, b,* . . . are real things and mean something. I would not deny the objection. Although hypothetical unrealities may illuminate the significance of realities, it is the realities that make science. All I was trying to do was to supplement Spearman's discussion of the universality of g with a little contribution on the relativity of g—as might be expected of an erstwhile physicist! It seems to be an undeniable statistical fact that batteries of intelligence tests as given and as scored tend to be what has been termed hierarchical in that they tend to satisfy the tetrad relations (7). This fact means something, it needs to be explained, Spearman has offered an explanation. Possibly the explanation should have laid more emphasis on the tests and less on the general intelligence —I do not know—but in Spearman's system we have a method of examining our data, of discussing its implications, of organizing it into a philosophical system, just as we have in Einstein's, and at least for the immediate future the system propounded in "The Abilities of Man" can not be ignored by those working in its field. That is why I said that the Grote professor of philosophy of mind at the University of London has written an important book. Moreover, it is clearly, spiritedly. suggestively, in places even provocatively, written; intelligible and entertaining even to the general reader. The mathematics has been put out of the way in a highly compressed appendix. I have chosen to take the risk of misrepresenting the character of the book by writing a very lop-sided review with its emphasis chiefly on the appendix because I know that this has offered difficulties to some very intelligent readers, because it appears logically fundamental to the whole system, and because some of its important logical implications seem not to have been expressed by the author in the main text.

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SPECIAL ARTICLES

TELESCOPIC OBSERVATION OF CATHODE AND ANODE POINTS

1. *Bright and dark sparks.* While the behavior of the anode in case of a mucronate electrode has been summarized,[1] further consideration of the cathode point is desirable. In this case the needle is to be critically set, so that the convection current is just about to pass into the spark condition. The set may often be made more sensitive, by waiting for some time until a convection current incidentally strikes and this may often be hastened by drawing sparks out of the cathode with a metal bridging the spark gap.

Since the U-tube interferometer is rather cumbersome for general observation, it may often be replaced by a suitable ear trumpet listening for the frequency of the crackle of sparks; or still better by a short range telescope (objective 6 or 8 inches off) focussed on the cathode point. Whenever the convection current passes, a bright oval cathode glow is seen in the telescope like the nucleus of a comet, while in the dark even the convection current itself may be seen looking much like a cometary tail. As soon as any spark transfer takes place, this oval glow is extinguished.

The spark successions which follow the extinction of the cathode glow are not, however, of the same kind. There are two distinct types, bright and dark. The first consists of diffuse bright purple spark filaments, passing with marked crackling between many favorable points of the electrode plates, the needle point being ignored. Being relatively luminous, they are the most desirable, but they often refuse to appear or are not sustained.

The second type which I shall have to call by a misnomer dark sparks, show no spark lines at all, but consist of a faint violet surface glow at the edges of the anode plate. The cathode glow is none the less extinguished. They are apt to be the more usual (and undesirable) occurrence, particularly after long observation. There is no appreciable crackling heard in the ear trumpet. I have therefore (without certain evidence) regarded the bright sparks, since they nearly always appear when the cathode is earthed, as resulting from a promiscuous issue of positive electrons from preferred parts of anode plate (since it here has no needle point), whereas the so-called dark sparks may be the corresponding convection discharge of positive ions from such parts of the anode plate.

2. *Negatively charged body.* The most available instrument for extinguishing the cathode glow is the charged hard rubber rod. If this is highly charged and the critical set sensitive, the rod may be passed normally along the arc of a vertical circle even 50 cm in radius around the spark gap, from right to left, always keeping the glow dark (Fig. 1). As soon as the rod passes outside of the circle by a few centimeters, the glow at once relights. This may be repeated indefinitely. A brass ball, 8–10 cm in diameter on an insulated handle and charged at the cathode of the machine is also convenient, though it acts from a smaller radius (15 cm) from the spark gap. A proof plane is still weaker. Now if the negative rod at a radius of 50 cm (say) evokes a shower of bright sparks persistently, then it usually happens that at a smaller radius of say 20 cm these bright sparks disappear, to reappear at the radius of 50 cm again. This was at first a very puzzling observation; but as the cathode glow is kept extinguished, it is a passage of the bright type of spark into the dark equivalent

[1] *Proceedings National Academy of Sciences,* February, 1928.

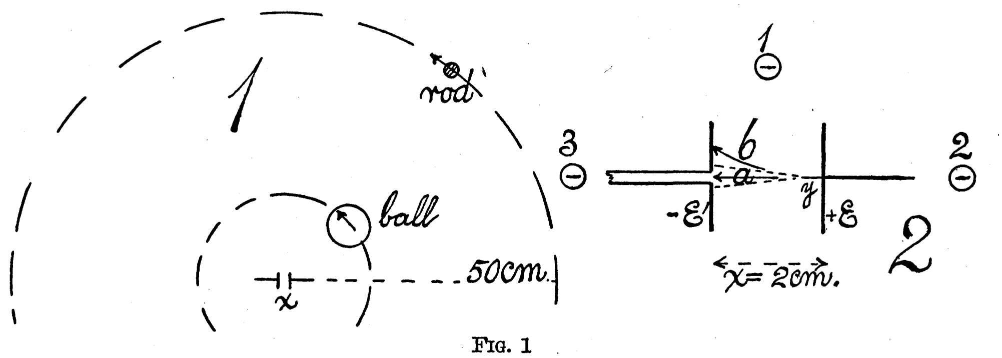

Fig. 1