of all the samples will be distributed in a perfectly definite way, depending on the nature of the total population.

Thus, it has been proved that the mean of the set of means will coincide with the mean, m, of the total population; the standard deviation of the set of means will be equal to $\sigma/\sqrt{n}$, where σ is the standard deviation of the total population; and the other moments of the set of means can be computed in terms of the corresponding moments of the total population. That is, if we assume any hypothetical values for the parameters of the total population, we can theoretically compute the parameters of the distribution of the means. Then by a subtle analysis, we can make a comparison between the distribution of the means and the observed properties of the given sample, and thus construct a test of the validity of our assumed values.

The result of such a test is commonly recorded in this form: the required mean, m, is equal to the observed mean, a, of the measured sample, plus or minus a "probable error" r. This indicates merely that if we had the totality of the means of all possible samples of n before us, 50 per cent. of these means would lie between $a+r$ and $a-r$. This use of the term "probable error" is unsatisfactory, however, since the distributions involved in the analysis are usually not symmetrical; the "standard deviation" is the more useful concept. Moreover, there is no special sanctity attached to the arbitrary choice of "50 per cent"; other ranges are often needed.

Moreover, the formulas commonly given for computing the probable error of the various parameters are only approximations which are not valid unless the original distribution is normal, and the size of the sample is large. The serious study of this whole question, for the general case of skew distributions and small samples is a product of the last two decades—one might almost say of the last two years. Some of the names associated with this study are Karl Pearson, R. A. Fisher, Tchouproff, and especially a learned British scholar who conceals his identity behind the modest pen-name of "Student." Exciting new developments are constantly appearing in *Biometrika* and similar journals; the most modern tools that mathematics can supply as, for example, the theory of integral equations, are called into play; and the very latest results are immediately put to use by practical statisticians of the Bell Telephone System and other great industrial concerns. The work is by no means completed, and even the exact nature of the answer that may be hoped for is not yet entirely clear.[2]

[2] For further information the reader is referred to H. L. Rietz's Monograph on Mathematical Statistics (Open Court Publishing Company, 1927).

A splendid field for research is opening up, the fruits of which are sure to be not only of the greatest theoretical interest but also of the highest practical utility.

EDWARD V. HUNTINGTON

HARVARD UNIVERSITY

THE GENERAL RADIATION[1]

THE impacts of electrons against atoms produce two different kinds of radiation, (a) the line spectra and (b) the general radiation, sometimes called the continuous, or white, spectrum. The general radiation usually carries a far greater amount of energy than the line spectra—hot body radiation, for instance. This is true of the X-ray region of the spectrum as well as of other regions. Although X-ray spectrum lines are often strongly marked and sharply defined, the general radiation contains more energy than the lines, for it covers a much greater range of wave-lengths. In the evolution of recent thought, however, less attention has been paid to the general radiation than to the line spectra, partly because the line spectra have important bearings on our ideas as to atomic energy levels. In this address, I wish to present to you the more important characteristics of the general radiation, as they have been discovered by about twenty men, carrying on researches in different parts of the world. Time will not permit a detailed account of the subject. These details may be found in the text-books, which contain numerous references to the original articles published by the investigators.

On account of the fact that homogeneous beams of high-speed electrons can be produced, accurately controlled and measured, and because each electron has a relatively large amount of energy, the X-ray region of the spectrum provides us with a better field for investigating general radiation than do other regions.

The curve representing the distribution of energy in the general X-radiation spectrum as a function of the wave-length resembles that for the spectrum of black body radiation. There is one important difference between the two, however, namely, the general radiation spectrum has a sharply defined short wave-length limit. The quantum theory explains this limit quantitatively and qualitatively; for the electrons striking the atoms of the X-ray tube's target can not have kinetic energies greater than the product of the electron's charge into the difference of potential through which it has fallen (namely, Ve). Therefore, the $h\nu$ value of the quanta of radiation produced can not be greater than Ve. Strictly speaking, if we apply the laws of the conservation of energy and momentum to the impact of an electron against an atom, we find that the value of

[1] Address of the vice-president and chairman of Section B (Physics), American Association for the Advancement of Science, Nashville, Tennessee, December, 1928.

hv can not be quite equal to that of Ve, as required by the quantum equation; for the electron must, in general, transfer some of its momentum to the atom, and the atom will, therefore, retain at least a small amount of the electron's kinetic energy. The corresponding correction term that must be subtracted from Ve in the quantum equation contains the ratio of the mass of the electron to that of the atom as a factor. It is, therefore, very small, so small indeed that it can not be detected experimentally—at least in the case of heavy atoms. If it turns out that the impacts of protons against atoms, or of atoms against atoms, also produce general radiation with short wave-length limits, it may be possible to detect and verify the correction term.

The short wave-length limit of the general radiation does not appear to depend upon the angle between the direction in which the X-rays travel and that of the stream of electrons which produced them. The limit is also independent of the substance composing the target of the X-ray tube. It depends only upon the maximum voltage through which the electrons fall.

By measuring the voltage applied to an X-ray tube and the frequency of the short wave-length limit of the general radiation, we get an experimental value for the ratio of h to e. Using the accepted value of e, 4.774×10^{-10} we find for the value of h, 6.556×10^{-27}. Some recent determinations of h by means of spectroscopic analysis, using Bohr's formula for the Rydberg constant, give values that differ from the above by only a very small fraction of one per cent. The accuracy of the two methods is about the same.

The general radiation extends from the short wave-length limit toward longer wave-lengths, reaching a maximum of intensity at a certain point, the exact position of which depends upon experimental conditions. After making corrections for the absorption by the matter through which the X-rays pass and for the reflecting power of the crystal grating, it has been found that, at right angles to the electron stream, the maximum intensity of a beam, unaltered by transmission through matter, occurs at a wave-length approximately fifty per cent. longer than the short wave-length limit. The position of the maximum, however, depends slightly upon the angle between the X-rays and the electron stream. The maximum point shifts towards the short wave-length limit as this angle decreases.

The total intensity radiated in the general radiation spectrum has been measured both by means of its ionizing effect and also by means of its heating effect. It has been found that, other conditions remaining the same, the total intensity increases almost exactly as the square of the voltage applied to the tube. Since the energy of the electrons increases as the first power of the voltage it appears that the *efficiency* of production of X-rays, also, increases as the first power of the voltage, in other words, as the first power of the energy of the electrons. It is interesting to note that this is an energy law, for the measurements have been carried up to such high voltages that the relativity correction for the kinetic energy of the electron becomes important. At the highest voltages used the kinetic energy of an electron exceeds the value of $\frac{1}{2}mv^2$ by as much as twenty per cent.

The total intensity of the general radiation measured by ionization methods increases with the atomic number of the chemical element composing the target. Where the order of atomic weights of the chemical elements differs from the order of their atomic numbers, the intensity of the radiation follows the order of the atomic numbers. The intensity is nearly proportional to the first power of the atomic number, there being a small, positive correction term proportional approximately to its second power. This means that the probability that the impact of an electron against an atom will produce general radiation does not depend upon the atom's *mass*, but upon the nuclear charge or the number of electrons in it.

General radiation is partially polarized. In terms of the electro-magnetic theory of radiation, the electric vector at a point is a maximum in the plane containing the stream of electrons that produce the radiation. The amount of polarization appears to increase as we approach the short wave-length limit of the general radiation spectrum. This agrees with the ideas contained in the classical theory of radiation, and with the idea that, in impacts of electrons against the solid target, the radiation near the short wave-length limit is produced by electrons that have not lost much energy, or had their directions of motion changed much, before they actually produce the radiation.

The way in which the intensity of a small portion of the general radiation spectrum of breadth $\delta\lambda$ varies with the difference of potential through which the electrons fall has been examined. The relation between the intensity and the applied voltage seems to obey very well a certain, somewhat complicated, equation. From the experiments performed in solving this problem, it has been deduced theoretically that the general radiation from an indefinitely thin target should have a maximum at the short wave-length limit and should fall off beyond this limit (toward longer wave-lengths) in the inverse ratio of the square of the wave-length.

Until recently experiments on the general radiation have been performed with solid targets only. Although the line spectrum of a liquid mercury target has been observed, there do not seem to be any observations on the general radiation from a liquid target.

Presumably such general radiation would have the characteristics of the radiation from a solid target.

Experiments have recently been made on the radiation coming from the impacts of electrons against gas atoms at low pressures. In these experiments, a stream of mercury vapor in a mercury pump passed down through a metal anode joined to earth and electrons, after falling through a constant difference of potential, passed through a hole in the anode and struck the mercury atoms. The radiation coming from these impacts through suitable openings has been examined. It has been found that the radiation at right angles to the stream of electrons has an average, or effective wave-length only a few per cent. longer than the short wave-length limit of the radiation as calculated from the quantum theory. In these experiments, the voltage applied to the tube was not sufficient to produce the L series lines of the mercury. This effective wave-length lies nearer the short wave-length limit than would be expected, if the intensity of the radiation fell off from the limit as the inverse square of the wave-length.

A number of interesting theories have been proposed in order to explain the characteristics of general radiation. Soon after the general radiation spectrum had been analyzed by means of crystal spectrometers, various assumptions were made as to the probability that an electron's impact against an atom would produce radiation and how much energy would be radiated. It was found that, by using the quantum theory and proper assumptions, a theoretical curve could be drawn which represents, at least roughly, the distribution of energy actually observed in the spectrum.

Another theory has been based on the application of Bohr's correspondence principle to the radiation, calculated on the classical theory for an electron approaching a nucleus. It gives the distribution of energy in the general radiation spectrum coming from an indefinitely thin target. This theory does not explain the short wave-length limit of the spectrum. It is arbitrarily assumed that the spectrum will be cut off at the short wave-length limit determined by the quantum theory. The theory predicts that beyond the limit, toward longer wave-lengths, the intensity of the general radiation will fall off approximately as the inverse square of the wave-length. The theory contains, also, an estimate of the total energy radiated, which appears to be of the right order of magnitude.

No solution of this problem by means of the theory of wave mechanics has been published. When such a solution is published, it will be interesting to see whether it explains the short wave-length limit of the spectrum and gives the same value for it as that deduced from the laws of energy and momentum applied to the production of a light quantum, and, further, whether it gives the correct distribution of energy in the spectrum of radiation from solid and gas targets.

At present, there does not seem to be a complete theory of the fundamental radiation problem, namely, that, if we allow an electron having a certain kinetic energy to impinge against an atom, shortly afterwards something of the same order of magnitude happens in a neighboring atom. From the point of view of theories, therefore, we are obliged to content ourselves with the application of the laws of energy and of momentum to the production and absorption of light quanta. These laws, as applied to the individual impacts of light quanta against electrons, have had extraordinary success in predicting and explaining that great discovery, made in America, which we call the "Compton Effect," and which we owe to the ability of our present chairman.

I can not allow the annual address of the retiring chairman of Section B to be delivered this year without commenting, also, upon a second great discovery, the selective reflection of electrons by crystals, the details of which Dr. Davisson will describe to us in a few minutes. Some years ago, in order to explain the reflection of light quanta by gratings and similar phenomena, a theory was proposed according to which the corpuscles transfer momentum to the grating in quanta. The magnitude of these quanta equals h divided by the grating space. It was shown that the law of quantum transfer of momentum accounts for those phenomena which we class under the heading of Fraunhofer diffraction.[2] If we introduce into the theory a quantity, λ, defined by the relation that λ equals h divided by the momentum of a radiation corpuscle, the equations of the theory take precisely the forms of those derived from the theory of the interference of waves, in which λ is the wave-length. Soon after this theory was proposed, discussions arose as to whether other forms of corpuscular radiation might not obey somewhat the same laws as those governing the diffraction of light quanta. It was suggested that radiation consisting of moving electrons, moving protons and even moving atoms ought to be reflected from gratings at least approximately in accordance with the laws of grating reflection. The ideas contained in the theory of wave mechanics seemed to offer an explanation for such reflections. There is, however, one possible difference between the theory of the transfer of momentum in quanta and the theory of wave mechanics. According to the theory of the transfer of momentum in quanta, the corpuscle should lose at least a small amount of its momentum and energy on reflection. It might lose much more on account of some electrical disturbance it produced, the ionization of some atom, for instance. The equations for the reflection of a corpuscle by a crystal grating that represent the

[2] The explanation of Fresnel diffraction seems to require further assumptions.

quantum transfer of momentum in three rectangular directions may be written:

$$mv\alpha - mv^1\alpha^1 = n_1 \frac{h}{d_1}$$
$$mv\beta - mv^1\beta^1 = n_2 \frac{h}{d_2}$$
$$mv\gamma - mv^1\gamma^1 = n_3 \frac{h}{d_3},$$

The d's represent grating spaces, the Greek letters direction cosines and the n's whole numbers, in which v^1, the velocity of the corpuscle after reflection, may differ from v, its velocity before reflection. If v^1 is equal to v and if we put $\lambda = h/mv$, the above equations reduce to those derived from the theory of wave motion. If we look at the phenomenon of reflection of corpuscles from the point of view of the elementary wave theory, we may suppose the corpuscle to be replaced by a series of plain waves having a definite wave-length and we may suppose that these waves excite oscillations in the atoms of the grating which send out secondary waves. The interference of these secondary waves produces the reflected or diffracted beam. On this theory we would expect the frequency of vibration of the diffracted beam to be the same as that of the primary beam. We would expect, therefore, that the diffracted corpuscle would have the same energy, momentum and velocity as the corpuscle had before diffraction. It has been found, however, in the experiments which Davisson and Germer described in the December *Physical Review* that the reflected electron in general has less energy after reflection than before and that the loss of energy may amount to as much as twenty-five per cent. The fact that they observed electrons with such losses of energy appears to be explained by the above momentum equations as due to the sizes of the slits in their measuring apparatus.

No general solution of the equations representing the wave mechanics as applied to this problem has as yet been found. It may be that a general solution of the equations would indicate some loss of energy and momentum on reflection. If so, it will be interesting to see whether the angles at which reflection takes place are the same as predicted by the above momentum equations. According to these momentum equations, if the corpuscle loses a certain definite amount of energy, it must be reflected at certain definite angles from the crystal grating.

Although no completely satisfactory theory has been proposed for the radiation problem in general, it may be that we are gradually approaching a solution of it. A number of interesting physical theories have been proposed in recent years. A physical theory, however, does not represent what we might call real truth. A physical theory is a collection of fundamental hypotheses and general laws, which may be used to deduce particular laws that can be applied to concrete facts. Physical theories are useful, if they explain a large number of facts in simple ways, and if they furnish definitions of terms and a nomenclature to be used in describing phenomena. Physical theories are tools and not creeds, but one is at liberty to believe they represent reality, if one wants to. The belief in a physical theory, however, is a similar process of thought to the belief in religious tenets. The greater the number of useful physical theories that are proposed, the greater the number of good tools we shall have at our disposal, to use in discovering the real truth about the way in which nature acts; for it is the way in which nature acts that is the prime object of physical research. The multiplicity of theories in physics to-day really represents a healthy growth.

WILLIAM DUANE
HARVARD UNIVERSITY

FUNDAMENTAL SCIENCE AND WAR

MUCH has been said and written about war's effect on civilization; much has been said and written about war's effect on applied science and modern invention. Indeed the two are almost inseparable for the "degree of civilization of a people is commensurate with the extent to which they accumulate, correlate and utilize knowledge."[1] It is now universally realized that applied science progresses only after the foundation stones of pure science have been firmly laid. The process of laying this foundation consists in searching out, correlating and classifying knowledge. It is of this process that the layman is hardly aware, except that he knows it is carried on to a great extent in the academic world, in the laboratories of our colleges and universities. What would happen to civilization if this process were to cease? Is this process a continuous one? Is it affected by political influences? What is the effect of war on this apparently endless task?

It is the purpose of this paper to discuss the effect of the great war on one of the fundamental sciences —chemistry. In America we feel that chemistry is making great strides. We agree, and rightly, with Calvin Coolidge, who, in addressing the American Chemical Society on the White House lawn on April 24, 1924, said in part: "Wherever we look the work of the chemist has raised the level of our civilization, and has increased the productive capacity of the nation." We feel that the war caused an awakening in chemistry in this country. What its effect has been

[1] J. Alexander, Preface of "Colloid Chemistry" (1926).