Returning now to the general theory, we state this fundamental and easily proved theorem:

As the direction or slope m varies at a given point of the first plane, the corresponding point γ moves on the derivative circle in the third plane so that its angular rate is always twice that of m and in the opposite sense.

Therefore the complete picture of the derivative $\frac{dw}{dz}$ is not a congruence of circles but a *congruence of clocks*. Here I use the word *clock* to denote a circle with a particular distinguished radius vector. We select this to correspond to the direction $m = 0$ at the point in the $z-$ plane. Thus a clock is completely determined by two vectors, namely the central vector $H + iK$ and the phase vector $h + ik$.

From the above theorem it follows that there are just *three* directions m which are parallel to the corresponding radii of the derivative circle, and that these radii are spaced at intervals of 120°. Since this is true at any point, we obtain by integration a triple family of curves (which we call the *equiangular family*) in the first and third planes.

We next define the *mean derivative* of a polygenic function as the mean value

$$\frac{1}{2\pi}\int_{}^{2\pi} \frac{dw}{dz}\, d\vartheta \text{ where } \tan\vartheta = m.$$

The result is found to coincide with the center of the derivative circle. Hence, using the symbol $\mathfrak{M}$ for mean differentiation, we have this fundamental formula

$$\mathfrak{M}(\varphi + i\psi) = H + iK = \frac{\varphi_x + \psi_y}{2} + i\,\frac{-\varphi_y + \psi_x}{2}$$

We verify the symbolic equation

$$\mathfrak{M} = \frac{D_x - iD_y}{2}$$

where D_x and D_y denote partial differentiation. We thus obtain easily positive and negative powers of this operator.

The mean derivative of a monogenic function is of course a monogenic function. The converse however is not true.

The mean derivative of a polygenic function is sometimes monogenic. This occurs when and only when φ and ψ obey Laplace's equation, that is, when φ and ψ are any harmonic functions.

For this type of harmonic polygenic function, the transformation from the point $x+iy$ to the point $H+iK$, which we call the *induced center transformation* and denote by T', is conformal (direct), though the transformation T from $x+iy$ to $u+iv$ is in general not conformal. We shall call T in this case a *general harmonic transformation*. This class of transformations, which does not form a group, includes the total conformal group (made up of direct and reverse conformal transformations) as a special case.

Further developments of the general theory will be published in the Proceedings of the National Academy of Sciences, the Comptes Rendus, and the Transactions of the American Mathematical Society.

EDWARD KASNER

COLUMBIA UNIVERSITY,
NEW YORK

SCIENTIFIC EVENTS

REPORT OF THE PRESIDENT OF THE CARNEGIE INSTITUTION

THE trustees of Carnegie Institution of Washington met in annual session on December 9, Elihu Root presiding. In recognition of the fact that the institution is completing a quarter century of activity, President Merriam, in his formal report covering the work of the institution for 1926–27, briefly characterized the policies of the quarter century.

He said that in the first years the institution's grants were commonly made for specific projects to run for limited periods. These covered a relatively wide range of subjects, affording an important stimulus to many types of agencies. In later years the tendency developed to center upon major projects which required sustained effort and concentration of funds. This tendency resulted in the development of departments in the institution's organization, each devoted to its specific subject and under leadership of an investigator of exceptional vision and ability. Although the practice of giving minor grants to distinguished individuals for special projects was continued, in many cases advantage was found in relating such problems to that department of the institution best fitted to cooperate. Still more recently a relation between departments has developed comparable to that which had developed in some instances between departments and individual investigators.

President Merriam summed up his observations on the institution's policy as it has evolved during the quarter century by saying:

> The institution to-day contains all the elements that have arisen in the course of study of its problem. There are still widely distributed special grants. The greater departmental activities still represent concentrated effort in specific fields. The increasing mutual support has not diminished initiative of the individual or of the group, but it has added an element which with the passing of time becomes more and more valuable, both in effort to concentrate upon special projects and in keeping that view of the larger field so desirable in long-continued researches.

Plans involving an expanded program of activities in the fields of early American cultures and of plant biology were also referred to by the president.

In respect to the first of these he said that the specific investigations heretofore undertaken by the institution in American archeology had been initiated in the hope that they might lead to some suggestion of laws which have governed in the development of the varying types of early peoples and cultures in America. He pointed out that the time had now come in the work of the Carnegie Institution in this field when the results should be interpreted in the light of what has been learned about the American problem in other regions and by other agencies.

The president also stated that Dr. Alfred V. Kidder, who has worked with distinction in early American history and is deeply interested in the wider aspects of the matter, has accepted leadership for the institution in its broadened activities in this field of research. This change implies no lessening of interest in the investigation of the Maya culture of Yucatan, for the results which the institution has obtained, he assured the trustees, amply justify adherence to its fundamental plan of furthering historical studies in Middle America.

In referring to proposed changes in the plant biology program the president said that during the past two decades the institution had attempted to advance the boundaries of knowledge by intensive effort at several critical points along the margin of the field. Such, he suggested, have been the researches in problems of life process in plant physiology, on the relation of life development to special types of conditions as in work at the Desert Laboratory, on the relation of life progress to environment, on problems of plant heredity, on questions touching relation of classification to heredity and to influence of environment and on problems presented by the history of plant life during the ages.

In 1926 a small committee of the institution's leading investigators in this field was asked to formulate a program for future guidance after full consideration of the matter. The report of this committee, the trustees were informed, indicates that a greater unity of attack would be profitable to all, without real loss or hardship and that a larger measure of unity in administration would facilitate the entire plant biology program.

President Merriam also spoke of the opportunity afforded the institution's investigators during the year of participating in international meetings. He believes that such meetings and the contacts with workers in related fields which they provide have distinct value. On this point he said:

> Through these relationships there has developed both the accumulation of materials arising from studies by other institutions and the contribution from our own researches, which are thus subjected to constructive review by experts in related fields. It is believed that such extension of our relationships is one of the important means to be used in finding how our own course should be steered in the unknown fields toward which we are always moving.

Dr. Merriam presented to the trustees a report covering in detail the progress of research carried on by the institution during 1926–27. This will soon be available for distribution in the form of *Year Book*, No. 26.

The trustees upon completing the business of the day spent the afternoon with their friends in viewing the exhibition prepared by members of the institution staff. This exhibition, which is set up at this time every year at the Administration Building, Washington, D. C., is designed to show the progress made in significant research activities. For three days following the annual meeting of the board of trustees, December 10, 11 and 12, the public generally was invited to view the exhibits.

EXPLORATIONS IN ALASKA BY THE U. S. GEOLOGICAL SURVEY

THE U. S. Interior Department announces the completion of the field work of another exploratory expedition in Alaska by the Geological Survey and the bringing back of maps and information regarding a tract of more than 2,000 square miles in the Alaska Range and adjacent country on the west side of Cook Inlet, in the environs of Mount Spurr, that has hitherto been shown as a blank area on all authoritative maps. This exploration is one of the series that the Geological Survey has been making throughout the last thirty years. The party consisted of S. R. Capps, geologist in charge; R. H. Sargent, topographic engineer, and four camp men. Transportation in the field of the necessary provisions, supplies and equipment for 100 days was furnished by a pack train of fifteen horses. From the time when the party landed at Trading Bay, on the west side of Cook Inlet, about the middle of June, until it returned to that place at the end of the field season, about the middle of September, the members were entirely out of communication with the rest of the world.

Among the many items reported are the discovery and mapping of a large river, numerous lakes, glaciers and mountains and an active volcano. The newly discovered large river is the Chakachatna, whose drainage basin covers an area of more than 1,100 square miles. This stream is a roaring torrent far too swift and too deep to be forded even with horses; in fact, measure-