peropithecus, in others the crown is much less worn and directly comparable with the relatively unworn premolar crowns of *Prosthennops serus* (a well-preserved palate of which had been discovered in an earlier expedition), while still others reveal more or less intermediate conditions. Moreover, the lower teeth which are apparently associated with these upper premolars are unquestionably the same or nearly the same as the corresponding lower teeth of *Prosthennops.* The still weak link in the chain of evidence consists in the fact that in *Prosthennops* the premolars that approach the type tooth of *Hesperopithecus haroldcookii* have two inner roots, whereas the type tooth has a single broad root.

This apparent difficulty may perhaps be met by the hypothesis that the type specimen is a second upper premolar, a tooth which in *Prosthennops serus* has only a single root; on the other hand, the type is far larger than any known *Prosthennops.* This much may be said: Nearly every conspicuous character of the type can be matched in one or another of the *Prosthennops* teeth. Thus, the concave wearing surface of the type is closely approximated in a certain worn upper molar of *Prosthennops*; the sharp ending of the enamel on the neck is seen also in the same specimen; the form and direction of the roots are closely paralleled in a third. Another upper molar (found by Professor Abel) and identified by him as *Hesperopithecus,* in the light of later finds is demonstrably *Prosthennops.*

It is hoped that further exploration this summer (1927) will secure sufficient material to remove all doubt in this matter.

POSTSCRIPT

Last summer (1927) Mr. Thomson made further excavations in the exact locality where the type of *Hesperopithecus haroldcookii* was discovered. A number of scattered upper and lower premolar and molar teeth were found in different spots, but every one of them appears to me to pertain to *Prosthennops,* and some of these also resemble the type of *Hesperopithecus,* except that the crown is less worn.

Thus it seems to me far more probable that we were formerly deceived by the resemblances of the much worn type to equally worn chimpanzee molars than that the type is really a unique token of the presence of anthropoids in North America.

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A NEW THEORY OF POLYGENIC (OR NON-MONOGENIC) FUNCTIONS

IF we consider an independent complex variable

$$z = x + iy$$

and a dependent complex function

$$w = \varphi(x, y) + i\psi(x, y)$$

then in general the limit of the increment-ratio $\frac{\Delta w}{\Delta z}$ depends not only on the point (x, y) but also on the direction or slope m. The function is called *monogenic* in the classic case where the limit is independent of m, so that it has only *one* value at a point. I have proposed recently (in my lectures at Columbia University, and in communications to the National Academy and to the American Mathematical Society) the new term *polygenic* to describe the case where the limit has *many* values at a point, one for each slope. Thus for a polygenic function the derivative is not a function of (x, y) or z, but of x, y, m. We write therefore the derivative in the form

$$\frac{dw}{dz} = \gamma = \alpha + i\beta = F(x, y, m).$$

We plot $z = x + iy$ in a first plane, $w = u + iv$ in a second plane, and $\gamma = \alpha + i\beta$ in a third plane.

To each point in the first plane corresponds one point of the second plane, but ∞^1 of the third plane (which we also call the derivative plane).

The locus of these points is always a circle. This is true for any polygenic function. The equation of the circle is

$$(\alpha - H)^2 + (\beta - K)^2 = h^2 + k^2 = R^2$$

where

$$2H = \varphi_x + \psi_y, \qquad 2K = -\varphi_y + \psi_x,$$
$$2h = \varphi_x - \psi_y, \qquad 2k = \varphi_y + \psi_x.$$

(In the special case where the function w is monogenic the circles of course all shrink to points, since in virtue of the Cauchy-Riemann equations h and k vanish so that the radius R is zero.)

To the ∞^2 points of the first plane correspond ∞^2 circles (in general distinct), that is, a congruence of circles. We call this the *derivative circular congruence* of the given polygenic function.

Thus while the transformation from the first plane to the second plane is a point transformation, the passage from the first to the third plane gives rise to a contact transformation.

Many noteworthy classes of polygenic functions are obtained by specializing the congruence. Thus if the congruence degenerates into the ∞^1 circles with the center at the origin, the function is of the form

$$w = f(x - iy)$$

that is an analytic function (power series) of the conjugate complex variable. If the circles all go through the origin, the components φ and ψ are dependent, that is the Jacobian must vanish. If the centers all lie on the axis of α, then we obtain the special form

$$w = W_x + iW_y,$$

where W is an arbitrary function of x and y. And so on.

Returning now to the general theory, we state this fundamental and easily proved theorem:

As the direction or slope m varies at a given point of the first plane, the corresponding point γ *moves on the derivative circle in the third plane so that its angular rate is always twice that of m and in the opposite sense.*

Therefore the complete picture of the derivative $\frac{dw}{dz}$ is not a congruence of circles but a *congruence of clocks*. Here I use the word *clock* to denote a circle with a particular distinguished radius vector. We select this to correspond to the direction $m = 0$ at the point in the $z-$ plane. Thus a clock is completely determined by two vectors, namely the central vector $H + iK$ and the phase vector $h + ik$.

From the above theorem it follows that there are just *three* directions m which are parallel to the corresponding radii of the derivative circle, and that these radii are spaced at intervals of 120°. Since this is true at any point, we obtain by integration a triple family of curves (which we call the *equiangular family*) in the first and third planes.

We next define the *mean derivative* of a polygenic function as the mean value

$$\frac{1}{2\pi}\int_{0}^{2\pi} \frac{dw}{dz}\, d\vartheta \text{ where } \tan\vartheta = m.$$

The result is found to coincide with the center of the derivative circle. Hence, using the symbol $\mathfrak{D}$ for mean differentiation, we have this fundamental formula

$$\mathfrak{D}(\varphi + i\psi) = H + iK = \frac{\varphi_x + \psi_y}{2} + i\,\frac{-\varphi_y + \psi_x}{2}$$

We verify the symbolic equation

$$\mathfrak{D} = \frac{D_x - iD_y}{2}$$

where D_x and D_y denote partial differentiation. We thus obtain easily positive and negative powers of this operator.

The mean derivative of a monogenic function is of course a monogenic function. The converse however is not true.

The mean derivative of a polygenic function is sometimes monogenic. This occurs when and only when φ *and* ψ *obey Laplace's equation, that is, when* φ and ψ *are any harmonic functions.*

For this type of harmonic polygenic function, the transformation from the point $x + iy$ to the point $H + iK$, which we call the *induced center transformation* and denote by T', is conformal (direct), though the transformation T from $x + iy$ to $u + iv$ is in general not conformal. We shall call T in this case a *general harmonic transformation*. This class of transformations, which does not form a group, includes the total conformal group (made up of direct and reverse conformal transformations) as a special case.

Further developments of the general theory will be published in the Proceedings of the National Academy of Sciences, the Comptes Rendus, and the Transactions of the American Mathematical Society.

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SCIENTIFIC EVENTS

REPORT OF THE PRESIDENT OF THE CARNEGIE INSTITUTION

THE trustees of Carnegie Institution of Washington met in annual session on December 9, Elihu Root presiding. In recognition of the fact that the institution is completing a quarter century of activity, President Merriam, in his formal report covering the work of the institution for 1926–27, briefly characterized the policies of the quarter century.

He said that in the first years the institution's grants were commonly made for specific projects to run for limited periods. These covered a relatively wide range of subjects, affording an important stimulus to many types of agencies. In later years the tendency developed to center upon major projects which required sustained effort and concentration of funds. This tendency resulted in the development of departments in the institution's organization, each devoted to its specific subject and under leadership of an investigator of exceptional vision and ability. Although the practice of giving minor grants to distinguished individuals for special projects was continued, in many cases advantage was found in relating such problems to that department of the institution best fitted to cooperate. Still more recently a relation between departments has developed comparable to that which had developed in some instances between departments and individual investigators.

President Merriam summed up his observations on the institution's policy as it has evolved during the quarter century by saying:

> The institution to-day contains all the elements that have arisen in the course of study of its problem. There are still widely distributed special grants. The greater departmental activities still represent concentrated effort in specific fields. The increasing mutual support has not diminished initiative of the individual or of the group, but it has added an element which with the passing of time becomes more and more valuable, both in effort to concentrate upon special projects and in keeping that view of the larger field so desirable in long-continued researches.