seem generally desirable that such collections should to any great extent duplicate specimens or volumes already available nearby. They might well bring out the various features of the wealth of their respective states. They might be very valuable in connection with the development of the industries of their states.

In a broader way, helping ourselves by cooperation with others, our New Hampshire Academy is surely ready to take part in activities aiming toward the general cooperation of the academies that are affiliated with the American Association for the Advancement of Science. Because state academies of science are generally much alike in their organization and are confronted with similar problems, frequent and free interchange of ideas and experience would undoubtedly be beneficial to all. There is a strong movement toward the realization of such cooperative interchange and that movement has been fully recognized by the American Association, which has already furnished valuable aid to the academies affiliated with it. A special committee on academy relations has recently been formed, including representatives of all the affiliated academies and of the executive committee of the association. We expect the new committee to study the problems of academy work and to make suggestions and inaugurate facilities for much progress in inter-academy relationships, making use of the already well-established organization of the American Association. Each affiliated academy has a representative in the council of the association, being thus in direct touch with association affairs, and the academy secretaries are in close relation with the Washington office of the association. The permanent secretary, Dr. Burton E. Livingston, has informed me that he is enthusiastic about the new academy movement and that the facilities of the Washington office are at the disposal of the affiliated academies in all feasible ways. He has expressed the hope that the affiliated academies may soon become virtually local branches of the larger organizations. They stand for the advancement of science in their several states in somewhat the same way as the association has so long stood in the country as a whole.

Several tentative suggestions as to ways in which our New Hampshire Academy might cooperate with the other state academies and with the American Association are mentioned below, but it is clearly realized that considerable study by representatives of all the academies will be needed before such suggestions may be relatively evaluated.

1. We might invite representatives of other academies to our meetings.

2. We might encourage our secretary to carry on correspondence with the secretaries of other academies, reporting interesting points and suggestions to our council or to our academy as a whole from time to time, perhaps occasionally through the *News-letter* if such an arrangement can be made.

3. We might encourage a similar correspondence between our academy and the permanent secretary of the American Association. Dr. Livingston has said that he will be glad to do his part.

4. We might aid the American Association to secure the attendance of official representatives of the association at our meetings. The association has approved of such representation, but the plan has not yet been generally realized.

5. We might arrange for occasional joint meetings with near-by academies if that proves feasible.

6. We might aid the science workers of other states to establish state academies where there are none at present, hoping that newly-formed state academies might become affiliated in our group with the American Association.

7. We might do what we can toward securing the general realization of the common aims of all the academies through inter-academy cooperation and with help from the association.

With the "Backgrounds" shown us a year ago by Mr. Foster in his presidential address; with a realization of what the New Hampshire Academy of Science has accomplished in the eight years of its existence; and with the courage and faith of a Lindbergh to turn the opportunities of to-day into the realities of to-morrow, let us say, as did Professor B. S. Hopkins, the discoverer of Illinium, in his inaugural address before the Division of Chemical Education of the American Chemical Society at the Richmond meeting last April: "Hats off to the accomplishments of the past; coats off to the accomplishments of the future."

WILHELM SEGERBLOM

HESPEROPITHECUS APPARENTLY NOT AN APE NOR A MAN

IN February, 1922, Mr. Harold J. Cook, a consulting geologist and paleontologist of Agate, Nebraska, sent to Professor Osborn an isolated fossil molar tooth which he had found in the Snake Creek beds of western Nebraska. He regarded it as closely approaching the human type and requested Professor Osborn and his colleagues to examine and describe it. After careful study and comparisons Professor Osborn published an article in the *American Museum Novitates* (April 25, 1922) entitled *"Hesperopithecus,* the First Anthropoid Primate found in America." In this brief article the author described the molar as the type of a new genus and species, which he named

Hesperopithecus haroldcookii, "an anthropoid of the Western World, discovered by Mr. Harold J. Cook."

In the type specimen the crown of the tooth had been ground off by long wear to such a degree that the surface of the crown was entirely gone and only the very basal portion was left. This presented an evenly concave surface of wear that was strikingly similar to the worn-down surface of one of the upper molar teeth that had been found by Dr. Dubois at Trinil, Java, near the famous skull top of *Pithecanthropus erectus.* The Nebraska tooth also had a very wide root on the inner side, which was similar to the wide root on the inner side of the upper molars of *Pithecanthropus* and of many teeth of American Indians. Hence Drs. Gregory and Hellman, whose report was cited by Professor Osborn, were inclined to think that on the whole the nearest resemblances of the specimen were with men rather than with apes. Professor Osborn stated that "this second upper molar tooth is very distant from the gorilla type, from the gibbon type, from the orang type; among existing anthropoid apes it is nearest to m^2 (the second upper molar) of the chimpanzee, but the resemblance is still very remote." After comparing it with the upper molars of the known fossil apes of Asia, as well as with the tooth of an American Indian, the author concluded that it was a new and extinct type of higher primate and that we must seek more material before we could determine its precise relationships to hitherto known races of men and of anthropoid apes. Professor Osborn also alluded to the fact that since 1908 there had been in the American Museum another badly water-worn tooth from the same formation and that Dr. W. D. Matthew had long been inclined to regard that specimen as pertaining to an anthropoid ape.

The scientific world, however, was far from accepting without further evidence the validity of Professor Osborn's conclusion that the fossil tooth from Nebraska represented either a human or an anthropoid tooth. Many authorities made the objection "Not proven," which is raised to nearly every striking new discovery or theory, and in course of time nine suggestions were put forward by responsible critics as to what the type specimen of *"Hesperopithecus"* might represent other than any kind of ape or man. Accordingly, Professor Osborn requested Drs. Gregory and Hellman to consider these suggestions and to present a more detailed report on the already famous specimens.

The first report of these authors is given in the *American Museum Novitates,* January 6, 1923. In their analysis of characters of the type they endeavored to distinguish four categories: (1) characters due to long exposure to weathering, erosion and stream action; (2) characters due to extreme natural wear of the crown; (3) chief characters that the *Hesperopithecus* tooth shares with both man and the anthropoid apes; (4) characters peculiar to *Hesperopithecus.* They published a series of photographic views in which the type specimen of *Hesperopithecus* was compared with upper molars of the chimpanzee, of *Pithecanthropus* and of the modern American Indian. They gave a table of measurements in which the dimensions and proportions of the type were compared with similar data for the molars of chimpanzees, of *Pithecanthropus* and of American Indian, concluding that the *Hesperopithecus* type on the whole came nearest to the second upper molar of a chimpanzee. They also published a series of radiographs which showed marked resemblance in the pulp cavity and roots to both chimpanzee and Indian molars.

In the second report by Drs. Gregory and Hellman on the *Hesperopithecus* problem (published in the *American Museum Bulletin,* December 4, 1923) the chief results are that after extended comparisons the authors concluded that the specimen could not represent a lower molar of any carnivore, that none of the other suggestions as to its possible relationships had proved tenable, that the greater number of resemblances of the type appeared to be with the gorilla and the chimpanzee rather than with the orang. It was also noted that "one of us (M. H.) still regards the human resemblances as being of considerable significance, while the other (W. K. G.) leans toward the anthropoid affinities of the type. The range of variability in crown and root characters of the molars both in the Hominidæ (human family) and the Simiidæ (anthropoid ape family) is so great and so overlapping as to warrant either interpretation." In view of the foregoing, the authors concluded that the "exact generic diagnosis of *Hesperopithecus* must await further discoveries."

In the hope of discovering more remains of this highly interesting fossil, Professor Osborn sent Mr. Albert Thomson, of the Museum staff, to collect in the Snake Creek beds of Nebraska in the summers of 1925 and 1926. At different times Mr. Thomson was joined there by Mr. Barnum Brown, Professor Othenio Abel, of Vienna, Professor Osborn and the writer. Among other material the expedition secured a series of specimens which have led the writer to doubt his former identification of the type as the upper molar of an extinct primate, and to suspect that the type specimen of *Hesperopithecus haroldcookii* may be an upper premolar of a species of *Prosthennops,* an extinct genus related to the modern peccaries. Some of these teeth have the crown worn down and more or less similar to the type of *Hes-*

peropithecus, in others the crown is much less worn and directly comparable with the relatively unworn premolar crowns of *Prosthennops serus* (a well-preserved palate of which had been discovered in an earlier expedition), while still others reveal more or less intermediate conditions. Moreover, the lower teeth which are apparently associated with these upper premolars are unquestionably the same or nearly the same as the corresponding lower teeth of *Prosthennops.* The still weak link in the chain of evidence consists in the fact that in *Prosthennops* the premolars that approach the type tooth of *Hesperopithecus haroldcookii* have two inner roots, whereas the type tooth has a single broad root.

This apparent difficulty may perhaps be met by the hypothesis that the type specimen is a second upper premolar, a tooth which in *Prosthennops serus* has only a single root; on the other hand, the type is far larger than any known *Prosthennops.* This much may be said: Nearly every conspicuous character of the type can be matched in one or another of the *Prosthennops* teeth. Thus, the concave wearing surface of the type is closely approximated in a certain worn upper molar of *Prosthennops*; the sharp ending of the enamel on the neck is seen also in the same specimen; the form and direction of the roots are closely paralleled in a third. Another upper molar (found by Professor Abel) and identified by him as *Hesperopithecus,* in the light of later finds is demonstrably *Prosthennops.*

It is hoped that further exploration this summer (1927) will secure sufficient material to remove all doubt in this matter.

POSTSCRIPT

Last summer (1927) Mr. Thomson made further excavations in the exact locality where the type of *Hesperopithecus haroldcookii* was discovered. A number of scattered upper and lower premolar and molar teeth were found in different spots, but every one of them appears to me to pertain to *Prosthennops,* and some of these also resemble the type of *Hesperopithecus,* except that the crown is less worn.

Thus it seems to me far more probable that we were formerly deceived by the resemblances of the much worn type to equally worn chimpanzee molars than that the type is really a unique token of the presence of anthropoids in North America.

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A NEW THEORY OF POLYGENIC (OR NON-MONOGENIC) FUNCTIONS

IF we consider an independent complex variable

$$z = x + iy$$

and a dependent complex function

$$w = \varphi(x, y) + i\psi(x, y)$$

then in general the limit of the increment-ratio $\frac{\Delta w}{\Delta z}$ depends not only on the point (x, y) but also on the direction or slope m. The function is called *monogenic* in the classic case where the limit is independent of m, so that it has only *one* value at a point. I have proposed recently (in my lectures at Columbia University, and in communications to the National Academy and to the American Mathematical Society) the new term *polygenic* to describe the case where the limit has *many* values at a point, one for each slope. Thus for a polygenic function the derivative is not a function of (x, y) or z, but of x, y, m. We write therefore the derivative in the form

$$\frac{dw}{dz} = \gamma = \alpha + i\beta = F(x, y, m).$$

We plot $z = x + iy$ in a first plane, $w = u + iv$ in a second plane, and $\gamma = \alpha + i\beta$ in a third plane.

To each point in the first plane corresponds one point of the second plane, but ∞^1 of the third plane (which we also call the derivative plane).

The locus of these points is always a circle. This is true for any polygenic function. The equation of the circle is

$$(\alpha - H)^2 + (\beta - K)^2 = h^2 + k^2 = R^2$$

where

$$2H = \varphi_x + \psi_y, \qquad 2K = -\varphi_y + \psi_x,$$
$$2h = \varphi_x - \psi_y, \qquad 2k = \varphi_y + \psi_x.$$

(In the special case where the function w is monogenic the circles of course all shrink to points, since in virtue of the Cauchy-Riemann equations h and k vanish so that the radius R is zero.)

To the ∞^2 points of the first plane correspond ∞^2 circles (in general distinct), that is, a congruence of circles. We call this the *derivative circular congruence* of the given polygenic function.

Thus while the transformation from the first plane to the second plane is a point transformation, the passage from the first to the third plane gives rise to a contact transformation.

Many noteworthy classes of polygenic functions are obtained by specializing the congruence. Thus if the congruence degenerates into the ∞^1 circles with the center at the origin, the function is of the form

$$w = f(x - iy)$$

that is an analytic function (power series) of the conjugate complex variable. If the circles all go through the origin, the components φ and ψ are dependent, that is the Jacobian must vanish. If the centers all lie on the axis of α, then we obtain the special form

$$w = W_x + iW_y,$$

where W is an arbitrary function of x and y. And so on.