but by reference to Vol. I of the same work, under the heading "acreage and yields of British crops" in the articles on Agriculture, p. 398, is found the definition: "per imperial quarter, that is, 480 lbs. of wheat"; the imperial bushel is the same as that in use in the United States, 60 lb. of wheat.

Finally the International Institute of Agriculture of Rome, of which one function is that of coordinating and distributing agricultural information, has published a useful book: "Recueil de Coefficients et d'Equivalences" (4th Edition, 1922) in which are given the metric equivalents of the weights and measures of all countries. Here, on pp. 30 and 31 we find "quarter = 8 bushels = 2.90942 hl" and "1 Quarter froment = 480 liures = 2.17724 q."

Thus, despite the inconvenience caused by the failure of this country, as well as the United States, to adopt the metric system of weights and measures, there is no special difficulty in defining a quarter, and in converting it to metric units, if the source of reference is chosen with due regard to the context in which the term is used.

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THE LOGARITHMS OF NAPIER

READERS who have not consulted the original writings of John Napier will be misled by an article by Professor G. A. Miller in the Proceedings of the National Academy of Sciences, September, 1926, p. 537, denying the accuracy of the equation,

$$\text{Nap. log } x = 10^7 \log_e \frac{10^7}{x},$$

to explain the relation existing between the logarithms invented by Napier and the natural logarithms. We read:

> The inaccuracy of this equation, according to later historical studies relating to logarithms, results directly from the striking and well known theorem that the only pair of algebraic numbers which satisfies the equation $y = e^x$ is $x = 0$, $y = 1$. The tables which Napier computed imply that the rational integral numbers listed therein have rational integral logarithms, according to his use of this term. It is, therefore, obvious that no general exact rational relation can possibly exist between these so-called logarithms and those of base *e*, since Napier did not consider any operation leading to transcendental numbers. The fact that the given equation is approximately true is of secondary interest.

The tables of Napier represent in general only approximate values, as do all logarithmic tables. But the fundamental theory, as found both in his "Descriptio" and in the body of his "Constructio," leads to a "general exact rational relation" between his logarithms and those of base *e*, and involves transcendental numbers. Napier's conception of logarithms is kinematic and based on continuity. Napier uses expressions, such as, points "travel," "distances traversed in equal times," "ducenda sit linea fluxu . . . puncti," which can not be successfully interpreted without admitting continuity. We quote from the "Constructio" (§§ 25, 26) the specification for his "geometrically moving point" and his definition of "logarithm":

> A geometrically moving point approaching a fixed one has its velocities proportionate to its distances from the fixed one.
>
> The logarithm of a given sine is that number which has increased arithmetically with the same velocity throughout as that with which radius began to decrease geometrically, and in the same time as radius has decreased to the given sine.

Logarithms are here defined by the *velocities* of two points, and it is easy by aid of the calculus to derive, rigorously, the above equation, as is done in my "History of Mathematics" (2d ed., p. 150). Napier uses such phrases as "incommensurable number," "*bc*, the logarithm of the sine *dS*" (*bc* and *dS* being line segments), "the logarithm which *bc* represents," logarithms "may be included between near limits." These and Napier's mode of deriving his theorem on limits, and his fundamental theorem, "logarithms of proportional numbers or quantities are equally differing," make it very evident that, in the presentation of his theory, Napier looked upon logarithms as exact values deduced from the above kinematic definition of a logarithm.

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AN UNAUTHORIZED REPRINT

I should feel grateful, if you will permit me through your columns to draw attention to the issue in the United States without my permission of a reprint of my "Grammar of Science." That work without revision is hopelessly out of date, and it is not only an injury to the author but an insult, if it leads any purchaser to suppose that the treatment of the subject in 1911 is an adequate criticism of the state of physical science in 1927, and represents the present views of the writer.

Morality in such matters appears to differ very widely on the two sides of the Atlantic. I can only sympathize with a well-known British author who on seeing a multinational publisher enter a club-room remarked to me: "Now Barabas was a publisher."

KARL PEARSON