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CONTENTS

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THE GRAPHIC REPRESENTATION OF RELATIVE VARIABILITY[1]

IT has been the generally accepted biometric practice to use the coefficient of variation as the measure of the *relative* variability or scatter of frequency distributions. This constant is

$$V = \frac{100 \text{ (standard deviation)}}{\text{Mean}}$$

It gives the standard deviation of the distribution in terms of the mean value of the varying character. By expressing the scatter of the distribution in this way it becomes possible to compare the relative variabilities of characters measured in different absolute units.

But the coefficient of variation has never been an entirely satisfactory constant to biologists, at least. While formally correct enough, within the limits of its definition, it does not readily or instantly call up in the mind an adequate picture of the real degree of scatter of the distribution. This is, in part at least, because two things, the mean and the standard deviation, are involved in it. When one reads the value of the standard deviation of a particular distribution he recalls that roughly three times this quantity on either side of the mean includes the entire frequency and this gives at once some concept of the biological extent and meaning of the variation, in the particular case.

There would seem to be a place of usefulness for an adequate graphical method of depicting relative variability for comparative purposes, so that one may *see* the difference or likeness in the variation of a man and a mouse, for example, in respect of body-weight. It is the purpose of this paper to describe such a graphic method, and to illustrate its applications.

The method may best be approached through a concrete illustrative example. We have lately been studying in this institute the normal variation and correlation of the relative cell volume of human blood, in relation to age, body-weight and stature.[2] The present situation regarding the measurement and graphical depiction of variation in these four charac-

[1] From the Institute for Biological Research of the Johns Hopkins University.

[2] *Cf.* Pearl, R., and J. R. Miner. "A Biometric Study of the Relative Cell Volume of Human Blood, in Normal and Tuberculous Males." Johns Hopkins Hospital Bulletin. In press.

TABLE I

VARIATION CONSTANTS

Character	Mean	Standard deviation	Coefficient of variation (per cent.)
a. Age	30.59 ± .21 yrs.	5.22 ± .15 yrs.	17.06 ± .51
b. Body-weight	151.56 ± .82 lbs.	19.95 ± .58 lbs.	13.16 ± .39
c. Stature	68.13 ± .10 in.	2.45 ± .07 in.	3.60 ± .10
d. Relative cell volume	45.59 ± .10%	2.47 ± .07%	5.42 ± .16

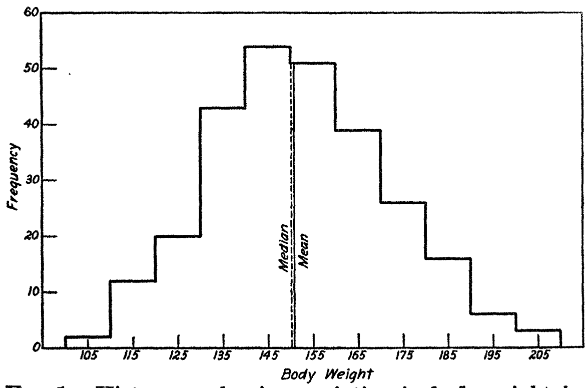

FIG. 1. Histogram showing variation in body-weight in a group of 272 normal males.

ters, in a series of 272 normal males, is fairly exhibited in Table I and Figs. 1 to 3.

Plainly the diagrams tell nothing whatever about the relative or comparative variability in this group of males in respect of the three characters, body-weight, stature and relative cell volume. They are correctly plotted histograms, but the unit of abscissal measure is different in each case and direct comparison is impossible.

From Table I we learn, through the coefficients of variation, that the group is from three to five times more variable relatively in respect of age and body-weight than it is in respect of stature or relative cell volume. But what does this mean translated into terms of distribution of frequency? A simple, direct and easily interpreted answer is not forthcoming.

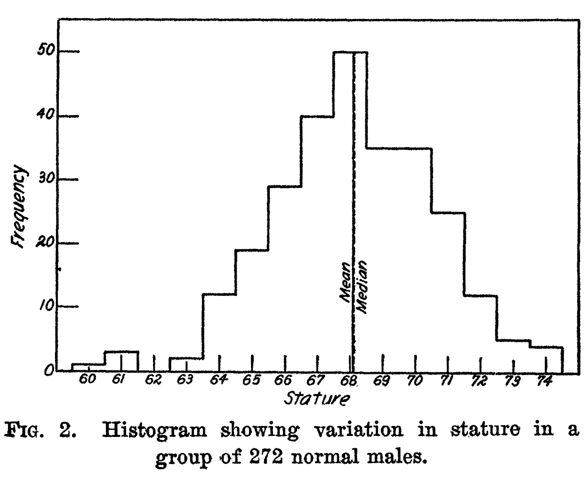

FIG. 2. Histogram showing variation in stature in a group of 272 normal males.

Suppose now we decide to express the age, the body-weight, the stature and the relative cell volume of each of these 272 individuals as a percentage of their respective mean values, the mean of each character being taken as 100 per cent. And further suppose we express the frequencies as respectively so much per one per cent. of the mean of each character. These are simple and entirely permissible transformations of the original data.

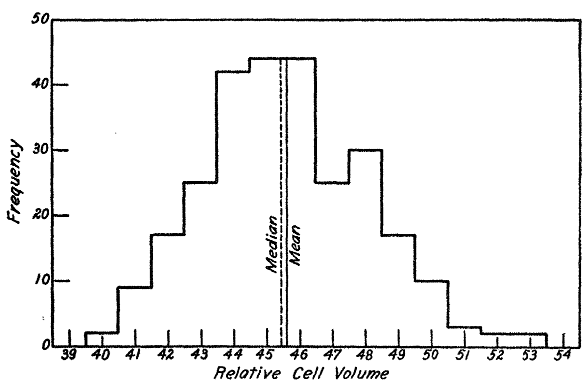

FIG. 3. Histogram showing variation in relative cell volume of the blood in a group of 272 normal males.

The data in their original form and after the transformation described are shown in Table II.

If now the figures in the columns headed A and B in Table II be plotted on arithmetically ruled coordinate paper we shall have a true picture of the relative variability of the four characters considered. This is done in Fig. 4. Each of the four frequency polygons has the same area, as a result of the transformations effected in the B columns.

This method of plotting superimposes the different polygons of variation on a common Cartesian coordinate grid, with the mean value for each of the compared variables at the same abscissal point. It constitutes no new method of *measuring* biological variation, but merely visualizes effectively what the coefficient of variation measures.

The method of plotting used in Fig. 4 shows at a glance that the 272 men of this group differ among

TABLE II

ABSOLUTE AND RELATIVE FREQUENCY DISTRIBUTIONS FOR VARIATION IN (a) AGE, (b) BODY-WEIGHT, (c) STATURE, AND (d) RELATIVE CELL VOLUME OF THE BLOOD IN 272 NORMAL MALES

Age				Body-weight				Stature				Relative Cell Volume			
		A	B			A	B			A	B			A	B
Class unit in years	Observed absolute frequency	Per cent. which mid-point of class is of mean	Absolute frequency per one per cent. of mean	Class unit in pounds	Observed absolute frequency	Per cent. which mid-point of class is of mean	Absolute frequency per one per cent. of mean	Class unit in inches	Observed absolute frequency	Per cent. which mid-point of class is of mean	Absolute frequency per one per cent. of mean	Class unit in per cent. of total volume	Observed absolute frequency	Per cent. which mid-point of class is of mean	Absolute frequency per one per cent. of mean
20–21.9	9	68.6	1.4	99.5–109.4	2	68.9	0.3	59.5–60.4	1	88.1	0.7	39.5–40.4	2	87.7	0.9
22–23.9	12	75.2	1.8	109.5–119.4	12	75.5	1.8	60.5–61.4	3	89.5	2.0	40.5–41.4	9	89.9	4.1
24–25.9	34	81.7	5.2	119.5–129.4	20	82.1	3.0	61.5–62.4	......	91.0	......	41.5–42.4	17	92.1	7.8
26–27.9	41	88.3	6.3	129.5–139.4	43	88.7	6.5	62.5–63.4	2	92.5	1.4	42.5–43.4	25	94.3	11.4
28–29.9	35	94.8	5.4	139.5–149.4	54	95.3	8.2	63.5–64.4	12	93.9	8.2	43.5–44.4	42	96.5	19.1
30–31.9	44	101.3	6.7	149.5–159.4	51	101.9	7.7	64.5–65.4	19	95.4	12.9	44.5–45.4	44	98.7	20.1
32–33.9	31	107.9	4.7	159.5–169.4	39	108.5	5.9	65.5–66.4	29	96.9	19.8	45.5–46.4	44	100.9	20.1
34–35.9	24	114.4	3.7	169.5–179.4	26	115.1	3.9	66.5–67.4	40	98.3	27.3	46.5–47.4	25	103.1	11.4
36–37.9	15	121.0	2.3	179.5–189.4	16	121.7	2.4	67.5–68.4	50	99.8	34.1	47.5–48.4	30	105.3	13.7
38–39.9	12	127.5	1.8	189.5–199.4	6	128.3	0.9	68.5–69.4	35	101.3	23.8	48.5–49.4	17	107.5	7.8
40–41.9	10	134.0	1.5	199.5–209.4	3	134.9	0.5	69.5–70.4	35	102.7	23.8	49.5–50.4	10	109.7	4.6
42–43.9	3	140.6	0.5	......	......	......	......	70.5–71.4	25	104.2	17.0	50.5–51.4	3	111.9	1.4
44–45.9	1	147.1	0.2	......	......	......	......	71.5–72.4	12	105.7	8.2	51.5–52.4	2	114.1	0.9
46–47.9	......	153.6	......	......	......	......	......	72.5–73.4	5	107.1	3.4	52.5–53.4	2	116.3	0.9
48–49.9	1	160.2	0.2	......	......	......	......	73.5–74.4	4	108.6	2.7	......	......	......	......
Totals......	272	......	......	......	272	......	......	......	272	......	......	......	272	......	......

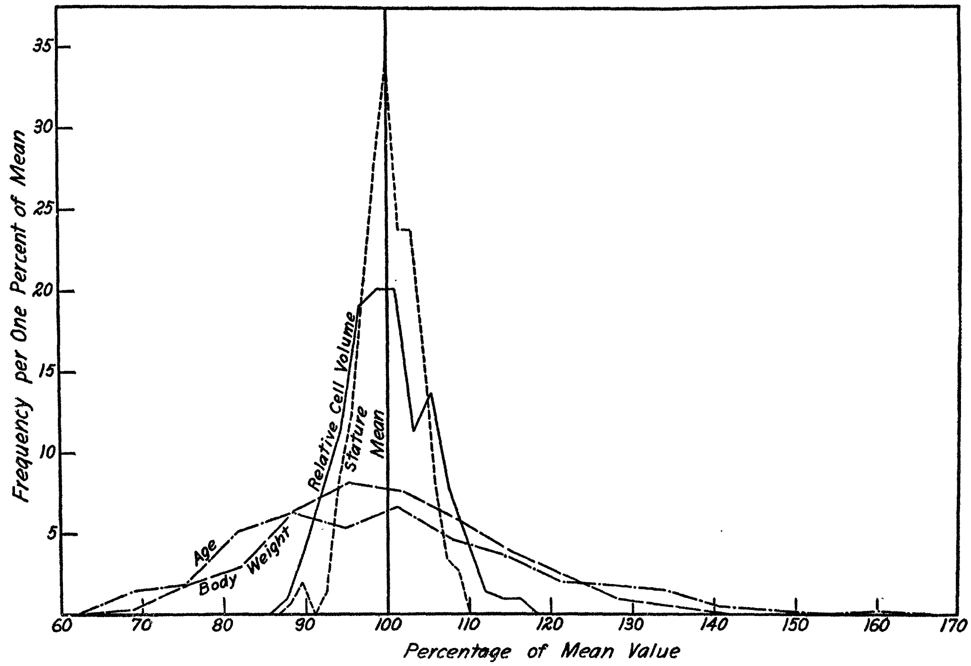

FIG. 4. Superimposed variation polygons for (1) relative cell volume, (2) stature, (3) body-weight, and (4) age, in 272 normal males. See text for further explanation.

themselves far more widely in respect of age and body-weight than they do in respect of stature or relative cell volume. The variation polygon for stature shows the least scatter. That for relative cell volume is somewhat, but not greatly, more spread. Those for age and body-weight are wide, flat distributions, indicating a relatively high variation in the group in respect of these characters.

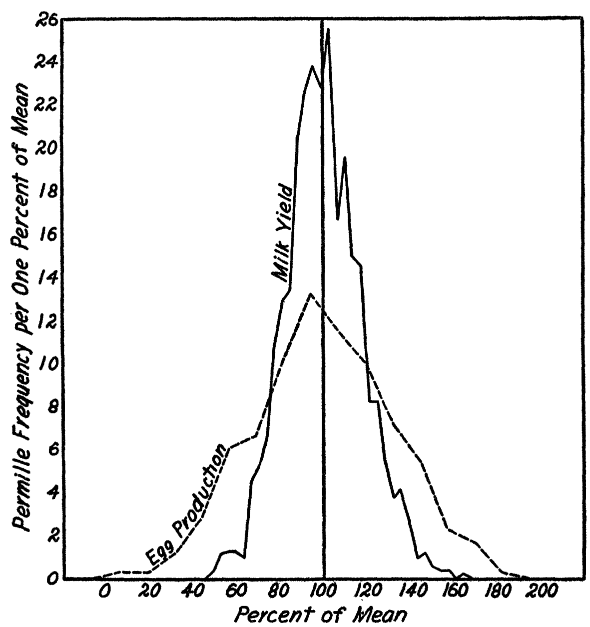

FIG. 5. Polygons showing the relative variability of cows in milk yield (solid line), and of hens in egg production (dash line). For further explanation see text.

One more example will be given. What is the comparative interindividual variability of cows in respect of milk production and of hens in respect of egg production? Table III gives the necessary data regarding (a) milk yield in gallons per week in three-year-old Ayrshire cows (combined years 1908–09),[3] and (b) annual egg production of Barred Plymouth Rock hens (1905–06, 150 bird pens).[4]

The coefficients of variation for the distributions of Table III are as follows:

Milk yield:	$V = 17.690 \pm .229$.
Egg production:	$V = 31.72 \pm 1.00$.

Using the data as given in columns A and C of Table III, Fig. 5 has been plotted. The transformation of the absolute frequencies per one per cent. of the means given in the B columns to the relative or per mille frequencies of the C columns is necessary in order to bring the two polygons to the same area, since the total observed frequency in one is 1,441 and in the other only 275.

[3] Pearl, R., and J. R. Miner. "Variation of Ayrshire Cows in the Quantity and Fat Content of Their Milk." *Jour. Agr. Research*, Vol. 17, pp. 285–322, 1919.

[4] Pearl, R., and F. M. Surface. "A Biometrical Study of Egg Production in the Domestic Fowl. I. Variation in Annual Egg Production." U. S. Dept. Agr. Bur. Anim. Ind. Bulletin 110, Part I, pp. 1–80, 1909.

TABLE III

Milk Yield					Egg Production				
		A	B	C			A	B	C
Class limits in gallons	Observed absolute frequency	Per cent. which mid-point is of mean	Absolute frequency per one per cent. of mean	Per mille frequency per one per cent. of mean	Class limits (number of eggs)	Observed absolute frequency	Per cent. which mid-point is of mean	Absolute frequency per one per cent. of mean	Per mille frequency per one per cent. of mean
6.50– 6.99	2	48.8	0.6	0.4	0– 14	1	6.3	0.08	0.3
7.00– 7.49	6	52.4	1.7	1.2	15– 29	1	18.8	0.08	0.3
7.50– 7.99	7	56.0	1.9	1.3	30– 44	4	31.4	0.32	1.2
8.00– 8.49	7	59.6	1.9	1.3	45– 59	10	44.0	0.80	2.9
8.50– 8.99	5	63.2	1.4	1.0	60– 74	21	56.5	1.67	6.1
9.00– 9.49	24	66.8	6.6	4.6	75– 89	23	69.1	1.83	6.7
9.50– 9.99	28	70.4	7.8	5.4	90–104	35	81.6	2.79	10.1
10.00–10.49	35	74.1	9.7	6.7	105–119	46	94.2	3.66	13.3
10.50–10.99	56	77.7	15.5	10.8	120–134	40	106.8	3.18	11.6
11.00–11.49	68	81.3	18.8	13.0	135–149	35	119.3	2.79	10.1
11.50–11.99	70	84.9	19.4	13.5	150–164	25	131.9	1.99	7.2
12.00–12.49	107	88.5	29.6	20.5	165–179	19	144.4	1.51	5.5
12.50–12.99	118	92.1	32.7	22.7	180–194	8	157.0	0.64	2.3
13.00–13.49	124	95.7	34.3	23.8	195–209	6	169.6	0.48	1.7
13.50–13.99	119	99.3	32.9	22.8	210–224	1	182.1	0.08	0.3
14.00–14.49	133	103.0	36.8	25.5					
14.50–14.99	87	106.6	24.1	16.7					
15.00–15.49	102	110.2	28.2	19.6					
15.50–15.99	78	113.8	21.6	15.0					
16.00–16.49	76	117.4	21.0	14.6					
16.50–16.99	43	121.0	11.9	8.3					
17.00–17.49	43	124.6	11.9	8.3					
17.50–17.99	28	128.2	7.8	5.4					
18.00–18.49	20	131.9	5.5	3.8					
18.50–18.99	22	135.5	6.1	4.2					
19.00–19.49	14	139.1	3.9	2.7					
19.50–19.99	5	142.7	1.4	1.0					
20.00–20.49	6	146.3	1.7	1.2					
20.50–20.99	3	149.9	0.8	0.6					
21.00–21.49	2	153.5	0.6	0.4					
21.50–21.99	2	157.1	0.6	0.4					
22.00–22.49		160.8							
22.50–22.99	1	164.4	0.3	0.2					
Totals	1441					275			

The greater relative variability in egg production is apparent.

The general principle here developed for the graphic representation of frequency distributions may, of course, also be applied to regression diagrams.

RAYMOND PEARL

THE BIOLOGICAL RELATIONSHIPS OF THE LAND, THE SEA AND MAN

UP to the present time life in the sea has always been treated and considered as quite separate and distinct from the more familiar life on land. But this idea can no longer be maintained. The life of all the world is one vast unit, dependent for existence on the same sources of supply. The ocean life, though to most of us so strange and unfamiliar, is but the aquatic fringe of the life on land, and for the most part is supported by the same materials which, washed into the sea, no longer are available for the support of the land creatures.

Heretofore we have been led astray in our contemplation of sea life by the interesting fact that about three times as many major types of animals live in the sea as are found upon the land; indeed, of the major types of animals no less than ten, nearly half again as many as all land living types together, are exclusively marine.

This great variety in the form and structure of sea animals obscures another interesting fact. About three fourths of all known kinds of animals live on the land, and only one fourth in the sea.