may not be a destruction of the zeolite molecule during extraction, as the leachings from a black alkali soil will often contain sufficient soluble aluminates and silicates to recombine under proper reaction conditions to form the insoluble zeolite with replaceable base properties, without evaporation. As stated above, our work shows that the zeolite molecule is most stable under alkaline conditions. This is doubtless due to the excess of OH ion present, which prevents hydrolysis. It is also very stable in the presence of an excess of the common metallic ion.

This work indicates that there is a great similarity between the soil zeolites as prepared either from acid or alkaline soils (acid extracts of the former and both water and alkaline extracts of the latter), and those artificially prepared on the one hand from aluminum chloride, silicic acid and sodium chloride and on the other from an alkaline aluminate and silicate. In fact, we are inclined to consider them as practically identical.

The small amounts of soluble aluminum present in acid soils probably result from the solution, with accompanying decomposition, of previously formed zeolites which are slightly soluble at hydrogen-ion concentrations below pH 5.0.

It is a widely known fact that black alkali (sodium carbonate) may be formed by the interaction between calcium carbonate and either sodium sulfate or sodium chloride. It may also be formed by the natural weathering of basaltic rocks. We are inclined to attribute the formation of zeolites in alkaline soils to the presence of black alkali formed as above (with attendant high pH), rather than consider black alkali as usually derived from sodium-zeolite hydrolysis. This latter explanation of black alkali formation appears to us to be putting the cart before the horse, for the major trend of chemical reactions in soils will be in the direction of synthesis of the least soluble product, which here, most assuredly, is the zeolite. That sodium hydroxide may be formed by the hydrolysis of sodium zeolite, after the latter has been formed, is of course well known, but the importance of this reaction in black alkali formation in soils is open to question.

To recapitulate briefly, our work thus far indicates that the steps in zeolite formation in alkaline soils are somewhat as follows: Sodium carbonate is formed from the interaction between white alkali and calcium carbonate; the sodium carbonate hydrolyzes to give sodium hydroxide,[3] which ionizes to produce a solution of high alkalinity (pH 10 to 11.5); sodium silicate and sodium aluminate are formed under these conditions from the more readily soluble aluminum silicate minerals; as the pH is lowered, due to washing out by rains or irrigation, or as the soil dries out, the sodium silicate and sodium aluminate combine to form sodium zeolite, which may remain as such or be converted into other zeolites, depending upon the salt concentrations within the soil solution.

A detailed account of this work soon will appear in the technical bulletin series of this experiment station.

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GROWTH AND GROWTH FORMULAS IN PLANTS

FOR some years the writer has been engaged in an investigation of the growth of plants under constant, controlled conditions, and especially of wheat, at the Food Research Institute of Stanford University. In the winter of 1925–26 he reached the conclusion that the first part of the growth cycle can not be represented by an autocatalytic reaction equation, while the later part of the cycle is only approximated by such an equation. These results were presented at the meeting of the Pacific Coast Branch of the American Association for the Advancement of Science in June, 1926. Since that time the writer has become acquainted with a paper by Gregory[1] in which the position is taken "that only the second part of the growth cycle can be represented by an autocatalytic reaction equation." As the full publication of the writer's work can not appear for some time and as his conclusions, while resembling those of Gregory in certain respects, differ in others, it seems of interest to students of growth to present at this time the writer's method of analyzing growth data.

Of the different theories and formulas which have been advanced in order to express growth of plants mathematically, the compound interest law of Blackman and Robertson's formula, which is identical with that of the autocatalytic reaction of chemistry, are the only ones which express growth as the effect of internal processes. As the latter formula

$$\frac{dx}{dt} = kx(a-x), \qquad (1)$$

in which a is the final length or final dry-weight, k, a constant and x, the length or dry-weight observed at a time t, can be written

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[3] In dilute solutions (N/20 or less), sodium carbonate is hydrolyzed to the extent of 65 per cent. or more.

[4] The authors are equally responsible for the presentation of this paper.

[1] Gregory, F. G., *Ann. Bot.*, 40: pp. 1–26 (1926).

$$\frac{1}{x}\cdot\frac{dx}{dt}=k(a-x), \qquad (2)$$

it is obvious that this formula is inconsistent with the formula of Blackman, which states that the relative growth rate is constant, or

$$\frac{1}{x}\cdot\frac{dx}{dt}=k. \qquad (3)$$

Hence, either both formulas are at best approximations, or the two formulas are valid for different stages of growth. Calculations based on data from the literature and on experiments of the writer upon wheat grown under constant conditions show not only that the second alternative is true, but also that Robertson's formula is but a first approximation.

In (1) it is assumed that k is a constant. Now, if we calculate k separately for different parts of the growth period, we may or may not find it constant throughout the period of growth. It may fluctuate in random fashion more or less about a mean; or it may display a secular change. In the latter case the differential equation (1) is applicable only if k is replaced by a function of x or t. But after such a revision of (1) the argument for an autocatalytic reaction would cease to be valid. In the data which Reed[2] and Reed and Holland[3] give for the length growth of different plants, which data these authors claim fit the Robertson formula very well, k decreases until about half the life cycle has been completed and then it rises to the end of the life cycle. For shoots of Bartlett pears[2] and Helianthus[3] the highest values of k are, respectively, 80 per cent. and 70 per cent. higher than the lowest values.

As, however, the values of k are small, the autocatalytic formula is very flexible and large variations of k in consecutive observations may occur without interfering with a rather good agreement between the observed and calculated data. A more severe test for the validity of this formula may be obtained, not by using the integrated formula, but by writing (2) in the form

$$\frac{1}{x}\cdot\frac{dx}{dt}=A-Bx. \qquad (4)$$

This formula states that, if the relative growth rate is plotted against the actual values of x, a straight line is obtained, whose intercept on the Y-axis is $A(=ak)$ and on the X-axis $\frac{A}{B}(=a)$. This criterion was tried out for the said data of Reed and Reed and Holland. Now there are difficulties in calculating $\frac{1}{x}\cdot\frac{dx}{dt}$. This derivative may indeed be estimated as equal to $\frac{\log_e x_2-\log_e x_1}{t_2-t_1}$, provided the time intervals between observations are sufficiently short. The data in question, however, show plainly that this method of approximation is much too crude. These difficulties were, however, overcome with the help of Dr. H. Hotelling, who has developed a method of calculating $\frac{dx}{x}\cdot\frac{1}{dt}$ in a way which corrects for the errors introduced by large values of t_2-t_1. Though these values calculated by the latter method are somewhat different from those obtained by calculating $\log. x_2-\log. x_1$, both methods gave the same results. If the relative growth rate is plotted against the values of x, for the data mentioned above, a curve for the observed data is obtained which is obviously not a straight line. If the values for x are recalculated according to the original formula (1) (by using an average k for all the data) this line is necessarily perfectly straight. For the observed data of the length growth of shoots of Bartlett pears and Helianthus a very good fit was obtained by the formula

$$\frac{1}{x}\cdot\frac{dx}{dt}=A-B.\ \log. x, \qquad (5)$$

i.e., if the relative growth rate is plotted against log. x a straight line results. For these data the formula (1) has to be changed into

$$\frac{dx}{dt}=kx(a-\log. x). \qquad (6)$$

For walnut trees[4] the relative growth rate is at first constant, then gradually slopes downward, but not according to (4).

The data of Kreusler[4] on dry-weight in corn show in some cases a difference of 25 per cent. between the calculated and the observed values. The relative growth rate increases during the first weeks instead of decreasing, which shows that neither the Robertson formula nor Blackman's law holds during this period.

This brief résumé shows that the autocatalytic formula is only a first approximation for a part of the growth curve. Therefore it is erroneous to regard growth as a simple autocatalytic process. That an analysis of these data and other data on the increase in dry-weight may be obtained in quite another way,[5] I hope soon to show.

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[2] Reed, H. S., *Journ. Gen. Physiol.*, 2: pp. 545–561 (1920).

[3] Reed, H. S., and Holland, R. H., *Proc. Nat. Acad. Sci.*, 5: pp. 135–144 (1919).

[4] Reed, H. S., *loc. cit.*

[5] See also van de Sande-Bakhuyzen, H. L., *Proc. Soc. Exp. Biol. Med.* (1926).