SCIENCE

Vol. LXIII June 11, 1926 No. 1641

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SCIENCE: A Weekly Journal devoted to the Advancement of Science, edited by J. McKeen Cattell and published every Friday by

THE SCIENCE PRESS

Lancaster, Pa. Garrison, N. Y.

New York City: Grand Central Terminal.

Annual Subscription, $6.00. Single Copies, 15 Cts.

SCIENCE is the official organ of the American Association for the Advancement of Science. Information regarding membership in the Association may be secured from the office of the permanent secretary, in the Smithsonian Institution Building, Washington, D. C.

Entered as second-class matter July 18, 1923, at the Post Office at Lancaster, Pa., under the Act of March 8, 1879.

EXPERIMENTAL SCIENCE AND WORLD GEOMETRY[1]

The major advances in science are many-sided. Though led by men of exceptional genius they have always been natural outgrowths of the concepts and problems met with in preceding systems of ideas. They have had natural setting not only in the main current of science in the narrower sense but even in the general thought of mankind. They need to be viewed in varied ways, interpreted by persons of varied outlook and equipment, in order that both their powers and their limitations may be understood.

For my part in the present group of papers I have chosen a point of view indicated by the title and best described in the body of the paper. Some reasons for this choice are found in the flavor of recent discussions concerning the bearing of particular experiments on the future of the relativity theory, in particular on the oft-heard question whether it is true or false. Now it is apparent that some of the diversities of judgment and expectation on these matters are to be ascribed to differences of opinion as to the nature and possibilities of scientific theories altogether. We may never reach unanimity on this point. It is perhaps even probable that there will always remain ways of thinking so contrasted that according to one's philosophy any particular theory can be justly called either true or false. Since one main motive of a genuine relativity scheme of thought is the explicit recognition of a variety of points of view or "frames of reference," it would be consonant with the spirit of such a theory to welcome a variety of modes of judgment even as to its own truth and virtue. But it is important to remember that part of the discrepancies of judgment are due to double meanings in the language used, not easy to avoid and therefore needful to be noticed.

We ask whether an ether exists, whether space is actually curved, whether time is truly a fourth dimension, whether a body is really flattened when it moves. The meaning of the questions needs some explanation.

Clearly we do not show that an ether exists in all the senses in which we show that a gas exists. We pump gas out of a vessel and stop the passage of sound. No one claims yet to have pumped the ether out of a portion of space and stopped the passage

[1] Third paper of symposium on relativity, joint meeting of American Physical Society and Section B, American Association for the Advancement of Science, Kansas City, Mo., December 30, 1925.

of light. But we seem to be justified in understanding that a vacuum has a dielectric coefficient as much as glass or oil does.

That the space of astronomy is curved, in the sense that the surface of the earth is curved, by test after the methods of geodetic surveying, is a proposition we never expect to verify. But that the concept of a curved universe is likely to be useful in the future interpretation of data on the stars and nebulae has already been abundantly shown in connection with the cosmological geometries of Einstein and DeSitter.

That time is a fourth dimension on a parity with length in all respects, for instance, as fully as right-and-left is on a parity with up-and-down, sounds fairly like nonsense. But that a unified geometry of space and time measurements together, in the abstract logical sense of mathematical thinking, is not only possible but in fact a more naturally scientific and fertile mode of thought than its predecessors should be clear from the simple facts that the new theory contains the whole vital content of the older ones and that it owes its increased power to the rejection of certain restrictions and prepossessions which are now seen to have been due to arbitrary assumptions or misunderstandings rather than to really experimental knowledge. Einstein's theory is not likely to be the last instance of the purification of science in such fashion.

Whether any particular body is flattened as it moves depends on how it is set in motion, and this question is therefore best regarded as a matter of experimental test and decision so far as the precision attainable in such measurements permits. But that the Heaviside ellipsoid, for example, is a convenient standard or ideal of reference in a moving system should be clear from the new simplicity that the Lorentz-Einstein system of variables, with its flattening factor and related concepts, has brought into the treatment of aberration, Döppler effect, and the like.

More important for our present purpose it is that the word "space" has itself a dual meaning, not always attended to. Sometimes it means something like the aggregate of natural objects in some vague and meager sense such as to suggest that we are noticing only such relations as order, position, accessibility. Sometimes it means an aggregate of ideal elements called points in a purely conceptual scheme described by mathematical postulate and definition, and thus open to purely logical examination. Some kind of correspondence between the two is implied in practically all scientific work and is understood to be established sufficiently by a suitable use, in combination, of relevant observational data and of a more or less arbitrary mode of computation or graphing.

Prerelativity physics used a sort of disjointed correspondence, mapping observed events into a flat three-dimensional length-space and a one-dimensional time-space, with only occasional hints that a closer affiliation between these two spaces might be of interest. The special relativity scheme of Lorentz and Einstein, interpreted geometrically by Minkowski, uses as conceptual space a single four-dimensional flat space, comprehending both length and time relations. The general relativity substitutes a suitable curved space and with the increased resources thus supplied is able to annex the entire realm of general mechanics to geometric theory.

This "geometrizing" of physics is sometimes objected to, but on grounds that are far from clear. The matter deserves extended notice, but a suitable discussion can hardly be attempted here. A few other objections need at least to be referred to.

It is sometimes said that the relativity theory is mathematical rather than physical, as though these terms were mutually exclusive. One finds it hard to see how a physical theory of such scope could have the needed precision without mathematical formulation, and there seem to be reasons for conjecturing that "physical" occasionally means "metaphysical." Sometimes the theory is called too abstract, but one could hardly expect a combination of comprehensiveness and manageability without abstractness. Then it is said to work entirely with principles, not at all with physical pictures. This is in the main perhaps more a difficulty than an objection but it touches a point vital for many scientists and invites some comment. One aspect of the distinction thus made helps to set in a clear light, not only why the relativity movement was opportune, but why further developments of similar type are now urgently needed, especially in the field of what is commonly called molecular physics.

The distinction between abstract principles and physical pictures runs somewhat parallel to that other distinction, roughly possible in most scientific thinking, between primary and secondary theories, foundation and superstructure, main outlines and specific details. The secondary and usually more picturesque concepts ride, as it were, on the underlying primary principles, the latter being often taken for granted almost unconsciously because of long familiarity. It is almost characteristic of these secondary theories that they serve to improve the representation of nature in particular aspects, act as correction terms, so to speak, in their respective places of usefulness. They prove serviceable so long as they are nearly enough in harmony with the primary principles and are fertile in suggestions for experiment and in means of amplified control of observed relations. But they are usually too specific, gratuitously rich in picturesque detail, and there come times when, because of inadequacy of the primary structure, these secondary concepts are stretched so far as to interfere with each other or come into conflict with the basic principles, thus leading to confusion of meanings and bewildering uncertainty as to what is fact and what is fancy. The relativity ideas emerged when there was a plain need for some radical improvement in primary theory, in view of the dilemma of the ether and absolute motion. We are now in the presence of another such dilemma in the physics of light and electron-quantum relations and it has already been demonstrated that the relativity program has resources for this new task as well.

To be sure, the listing of any particular set of ideas as primary or secondary in the sense used is neither precise nor permanent. During the growth of a science there can usually be traced a steady progress of its concepts from the picture stage toward greater abstractness and generality. Einstein's scheme, however, came as fruition of a long period of development in a mature science, and considered by itself had from its beginning the form of a primary theory of specially comprehensive type. But the full history of such ideas really goes back to the dawn of science. We are now reading the latest chapter of the long story of geometry, mechanics and electrical theory, and are entering an epoch when these will be joined with chemical theory in a single time-space geometry.

We are now familiar with the important distinction between logical geometry, which is pure mathematics, and concrete geometry, which is really a field of experimental physics. It seems wise to keep steadily in mind the similar dual aspect of all physical theories. Every such has an abstract logical framework, which by itself is in the main mathematical, though frequently only informally arranged. It becomes a physical theory by the addition of a scheme of instrumental interpretations whereby a correspondence is indicated between the abstract terms and the language of experiment, so that observed relations can be compared with those implied in the theory. Changes in theory may relate to either item, and normally there is steady mutual influence. It is readily seen then why, according to the placing of emphasis, theories may be and have been viewed in such manifold ways, for example, as logical systems merely suggested by experiment, as sources of suggestion for experiment, as origins of predictions concerning the outcome of new experiments, as statements of "objective reality" adhered to with a tenacity not easy to understand.

For to-day let me emphasize a sort of intermediate view, according to which we may understand that, by a combination of an abstract chain of ideas with a scheme of instrumental meanings, a theory defines a system of expectations, equipped with which we essay the task of arranging and interpreting our observations. We like to say that a theory is to this extent true, that findings agree with expectations. But whether this occurs or not, it is standard practice to report the results of experiment by comparison with expectations, by giving residuals between measured and computed values, for instance. Moreover, the residuals in successful cases are merely small, not exactly zero, and there is even much arbitrariness in the meaning of smallness because of freedom of choice in the mode of scaling or plotting.

For this purpose, of supplying a standard of comparison and a terminology for communication, it seems not so important that a theory should be "true" as that it should be definite. No matter how large the residuals may be, the measured data may be justly describable in terms of them, provided the theoretical values used for comparison are really unambiguously determinate. Uncertainty may arise, either because suitable logical inferences have not been properly made, or because the intended experimental meanings of the abstract terms are not clear enough. When Einstein found a way out of the ether dilemma by giving an enriched meaning to the relativity of motion he uncovered an actual experimental ambiguity in the meaning of the time-scale, even in connection with so elementary a matter as the simultaneity of two spatially separated events. We now see that previous confidence in an absolute time-reckoning was connected with a risky extrapolation from the facts of experience, since science was not yet in possession of a method of transmitting signals with infinite speed, or of any other method of testing the synchronism of separated time-pieces so as to determine uniquely the required reduction of their readings to the alleged absolute scale. The time-scales of the new theory, relative to particular frames of reference, have a really improved directness of experimental setting. It is simpler to think of adjusting clocks to a domestic symmetry of readings in the case of to-and-fro signals in a moving system than to seek the particular departure from symmetry required in order to match some foreign scale.

Viewed in the way suggested a theory may seem to have fundamental values quite distinct from the mere matter of success in defining expectations which turn out to agree completely with findings. In fact, no theory has ever had such perfect success, though the Newtonian celestial mechanics reached a striking approximation to it; and no theory has ever been discarded merely because the residuals were not zero, although, to be sure, if they are uncomfortably large we look for a new theory to improve our expectations.

The relativity plan is then to take the FitzGerald contraction as the normal behavior of a body set in motion, with the understanding that a different outcome is to be explained by some added special condition. Similarly, a fixity of the fringes in the revolving interferometer is a standard of reference; and any observed shift is to be taken, not as discrediting the primary theory, but as something which this theory leaves to be interpreted by some superposed theory. Ideal solar wave-lengths, displaced according to Einstein's formula, are suggested as origins from which to estimate the departures due to such special conditions in the source as are not defined in terms of the primary space-time geometry alone. These are only a few of the many instances of altered standards suggested by the relativity theory, all closely knit and interlocked in the most comprehensive and strongly unified synthesis of physical theory yet reached. It is also to be remembered that there is a large range of phenomena that now come under sufficiently exact control by the primary theory itself, while they were formerly treated by more special devices. It would take a long time to give an adequate account of these cases. In the main it has surely been the habit of science to test theories of large scope as working methods or as systems of scientific thinking rather than by the results of individual supposedly crucial experiments.

In spite of some apparent strangeness in the newer ideas, their roots lie deep in the past, and if approached with sufficient historical setting are seen to be abundantly natural developments from some of the most vital things in previous science. To outline such a survey of this recent synthesis let us look at a few items of prerelativity physics, partly respecting their actual place in history and partly for brevity by way of an imaginary personal revision of some basal thoughts and experiments.

If we should make some straight-edges by sighting or triple matching, use them as guides in planing a drawing-board flat, mark off scales by firmly constructed dividers, then survey a great variety of drawings, we should likely, after much trial and disappointment possibly, be able to codify something much like practical Euclidean geometry and to idealize it into an abstract system as well. Then furnished with this system, if we should by sighting mark what we would call straight scratches on a frozen pond, lay off isosceles triangles and so on, and measure separations by stretched tapes, we should find that the surface of the ice so observed very closely met a natural and intelligible expectation that it would be found flat in the sense of the adopted geometry. If then we were able to go to much greater distances we would naturally harbor the *a priori* supposition that in a smoothed-off sense the whole earth would prove to be flat. Perhaps we might insist that it must be, because the pond was found to be.

But to proceed scientifically with a special problem, suppose we lay off equal large radial distances from home, measure the distance around on each locus so fixed, and tabulate the numbers so as to confront them with the expected linear relation $c = 2\pi r$. We find that observed drops below computed relatively more and more the larger the radius. If by this time we have acquired also the ideal three-dimensional Euclidean geometry in sufficient development, as we may have been led to do by experience with building construction say, and especially if we are convinced of the wisdom of geometrizing our experimental results when we can find a way, we change our expectations, and perhaps try a sine-curve, as belonging to a special curved surface whose ideal properties are familiar. But if the "flat" notion of the earth is not of our own making but rather traditional for many generations and habitually used in speculations, we may insist on plotting geographic places on a flat map and seeking a "physical" explanation of the regular "expansion" of the measuring tape when laid transverse to a beeline leading home. One wonders what the early course of world geometry would have been if we were giants living on a planetoid so small that we could walk around it before breakfast. Perhaps it is not really strange that we pygmies chained to a single planet are slow to reconsider our long-standing insistence on a flat space for the universe of stars.

A further example is at hand in the same connection. When we make explorations in three dimensions, to compare with the ideals of our space-geometry, we find on closer inspection that measuring-rods which agree when held horizontally do not agree when held vertically, when pressed on, or taken near a fire. We do not, however, abrogate our ideal geometry as a physical theory, but superpose on it a systematic experimental and mathematical study of what we call elasticity, and thermal expansion, using perhaps mystical language but really building a rational scheme that appears as a suitable extension of what we had called geometry.

To continue our studies so as to include motions of various objects, we construct as preparation certain periodic mechanisms to exhibit as consistently as we can a concrete parallel to our instinctive notion of a time-sequence, perhaps even noticing that in a mathematical sense we are adding a new and initially apparently independent geometry of one dimension. First we send various objects sliding over the skating-rink, and find that when nothing else is observed to interfere they move nearly along our straight scratches and reel off distances nearly proportional to the number of clock-beats. We therefore adopt a first law of motion defining a norm for horizontal motions, then find that we can correlate departures with interferences by pushes, pulls, bumps, under the general heading of force. Then, knowing already what we mean by straight lines and distances in three dimensions, we formulate the three-space analogue of our first law and expect that projectiles while free from interference will behave approximately in the way defined. The results are widely different from the expectations.

Galileo tells us we can think of the projectile as moving with constant speed along its initial straight line, provided we add the idea that the line is thought of as dropping in a certain regular way, the same for all cases, thus clearly foreshadowing the Einstein principle of equivalence. Newton shows us how to arrange a mathematical account of the types of departure from our expectations, even in the much more extensive case of the solar system; tells us we can save our faces by speaking of a mystical force of gravitation or attraction, but warns us not to prefer hypothesis to rational system. Ignoring this warning our successors actually make hundreds of mechanistic hypotheses during the next two centuries, adding nothing to any real control of the matter. Then Einstein shows that we could have accounted for the whole of the large residuals from our first law ideal by simply reinterpreting that ideal in connection with a more generous geometry, by improving our mechanical geometry in much the same way though more complicated as that in which we had already improved our geographic geometry.

And now, dropping our play-acting and turning to our own century, let us glance at some of the things that have happened. The Newtonian mechanics has been extended to cover wide ranges of special phenomena by fuller and more precise development of its original principles. An extension to a supposed structure of matter, too fine to be surveyed directly, has led through an elementary kinetic theory of matter to the realm of general statistical mechanics, rich in bright lights and deep shadows. Other residual phenomena, some very striking, have been codified more or less separately under electrical theory, and this has absorbed the bulk of working ideas in the interpretation of light, so far as something more specific than a general wave-theory has been felt to be needed in optics.

Electrical theory was first modelled after a Newtonian pattern, but for some time its main outlines have been those drawn on an essentially independent plan by Maxwell. Since Minkowski's paper of eighteen years ago we can see that the real first step in the passage from Newtonian to Einsteinian relativity was taken by Maxwell when he introduced into his equations the term representing dielectric displacement current, since this rounded out what we may now call the isotropic form of these equations in time-space four dimensions.[2] But this was noticed only later, found out partly step-by-step by Lorentz and others, when in their labors on the pressing problem of the electro-mechanics of moving bodies they worked their way up to the now famous transformation from "ether" coordinates to others better suited for use in a moving system.

The new variables were first introduced merely as a mathematical device. The present era of relativity then got its full start when Einstein gave his simple experimental interpretation of the transformation, showed that all such systems of variables really belonged on a par with each other to a kind of democracy of systems of reference, and founded an improved kinematics free from the perplexities that had been specially noticed in the earlier blend of mechanics and ether physics.

The essential character of the new scheme was made clearer by the work of the next few years, led by Minkowski's mathematical revision. The germ latent in Maxwell's equations grew into a full-fledged time-space geometry and vector analysis, which exhibited the original transformation of variables as accompanying a change of axes or vectors of reference, with the components of electric and magnetic field vectors acting like area projections. By virtue of analogies with ordinary geometry and in other ways the results threw new light on the physical relations gathered by preceding science. Practically all main features of earlier physical theories proved to be adaptable, mostly with trivial changes if any, to the new scheme of arrangement, which was not only more perspicuous and better unified, but supplied a finer and more complete tabulation of the concepts and relations that had emerged during the long history of physical measurements. Thus to survey the new scheme fully would be to make a treatise on theoretical physics altogether. I want to dwell on one feature that has a general bearing on the relativity motive in scientific thinking and a particular bearing on the curvature theory of gravitation.

Like Euclid's geometry, like ordinary vector analysis and the far-reaching Ausdehnungslehre of Grassmann, now renamed tensor algebra, like such mathematical systems as the theory of invariants, the space-time vector analysis owes its simplicity to its direct attention to certain central concepts rather than to the partial and manifold representations obtainable by arbitrary modes of dissection. Time and space intervals are projections of a single vector, electric and magnetic intensities are components of a single

[2] It is noteworthy that this very item in Maxwell's scheme which led toward the relativity geometry is also the one which led to the notion of electrical waves and the later developments in wireless transmission.

property of the field, and like these examples the multiplicity of physical quantities appear as parts or components of a smaller set of basal properties. We recognize that a part of the complexity of our science can be understood as due to the manifoldness of the possible modes of partial representation or projection. This idea goes further than merely to cover the many cases so representable by coordinate analysis in the narrower sense. Thus time-lag, aberration, Döppler effect, are individual phases of the relation of a monochromatic source to the observer, and are separated only when an arbitrary though useful disjointing is carried out. We thus have a much more elaborate example of the same kind of simplicity as that which appeared in the study of the nebulae when it was pointed out that the variety of observed forms might correspond to various orientations of fewer intrinsic types.

Still, in the scheme of 1905–1908 a limitation was manifest which the spirit of the new theory itself made to appear rather artificial. The interpretation was limited to the comparison of frames of reference moving uniformly with respect to each other. Moreover, the physical mystery of gravitation remained essentially as Newton left it, together with the apparent absoluteness of rotation. Einstein guessed that these items belonged together. For it is clear that a more generous scheme would involve not only a relativity of direction and speed, but of straightness and of acceleration, and therefore some kind of relativity in the estimation of a gravitational field. A sweeping extension was thus conceivable, and a guiding principle was at hand.

Since Newton's time it had been customary to think of an observed gravitational acceleration as partly true attraction, partly centrifugal, but this might be another of the arbitrary dissections spoken of, and so it proved. This idea yielded the famous principle of equivalence which led to the goal sought, although it is not now recognized either as a permanent postulate or as a strictly correct proposition in the wider forms of the definitive theory. To the mathematical framework of this generalized theory of relativity and gravitation reference will be made shortly. The special results concerned with the planet Mercury, eclipse stars and spectrum shift are familiar. It is naturally out of the question to put such a large scheme of ideas into purely illustrative form, but various special examples are readily at hand, using rotating discs, falling elevators, and the like. The main idea may be described as the substitution of a wisely chosen curved space-time map for the previous flat one.

The curvatures introduced are such as to allow the exhibition of the planetary motions as instances of the first law of motion alone, to allow the definition in terms of curvature of all the general concepts of mechanics, and the deduction, as purely geometric theorems, of the general mechanical laws. With the actual data for planets and terrestrial objects the curvatures required are so minute as perhaps to seem strangely unpromising for such a sweeping extension of the powers of geometry as a physical theory. But herein lies the explanation of two prominent items; first, why such a simple scheme as Newton's could give such accurate results, second, why these curvatures are open to theoretical use for the purpose without being subject to challenge by direct test, since only the most precise geodesy could hope to detect the disparity between old and new estimates of intervals by length and time measurements alone.

There is to be seen here a fresh instance of a procedure widespread in scientific method, which may be called magnification by indirection. We may like to define a straight segment as the shortest path between two points, but we know that the test of straightness by sighting is vastly more delicate in practice. The vernier, the slow beats between rapid vibrations, interference measurements, stroboscopic devices, colored indicators, are a few from the myriad of examples. Einstein's identification of mass and energy gives another, according to which the processes of calorimetry may be thought of as yielding highly magnified symptoms of minute changes of inertia. These cases suggest a certain general motive of thought that seems likely to be of enduring importance and give hints of further power that may be latent in world geometry.

The instrumental method whereby some feature of nature is first revealed as admitting quantitative examination is usually crude in comparison with the means later found for its detailed analysis and measurement. The discovery of a phenomenon of some novelty is commonly followed by a systematic search for improvements in technique whereby its details can be magnified in some sense for accurate estimation and inspection. The two examples given from the relativity theory of mechanics illustrate a kind of theoretical analogue of this experimental procedure and suggest something of its further possibilities when combined with the resources that mathematics has to place at the disposal of physical theory for the purpose of representing measurements that are necessarily of limited precision or resolving power.

According to the new interpretation curvatures so minute as to escape detection by direct length measurements serve for representing all phenomena open to what have long been called mechanical experiments, while the latter are still not precise enough to show the delicate mass changes with which calorimetry deals rather accurately under another name. Since optical phenomena are in the main still more delicate it is not surprising that correspondingly delicate details in the geometry should suffice to give at least a partial representation of light in a way that seems to promise complete removal of the traditional dualism between matter and radiation in energy relations. Not only is the black body problem thus given a new setting, but it seems practically certain that when the needed revision is carried out the entire realm of thermodynamics will prove to be geometrically interpretable in the time-space manifold. Clearly, in our present physico-chemical thermodynamics it seems vital that increments of heat, of work, of matter of various kinds should be distinguishable in some unambiguous way. If they are all to be interpreted as mass changes, the distinction may turn out to be another of the artificial dissections spoken of above.

This suggested identification of mass and energy is a central feature in the new mechanics and supplies a rather ironical comment on the earlier history of the theory of heat, when the conservation of energy was understood as a substitute for the caloric theory. It is the beginning of a needed reconsideration of the relation of thermal and mechanical concepts, which will include an interpretation, suitable for the relativity scheme, of the various kinds of matter recognized as distinguishable by the phase rule. Thermodynamics has thus far received unduly little attention under the relativity program, and some of the few adjustments already proposed seem unlikely to remain satisfactory. Thus, since entropy has been commonly taken as an extensive quantity associated with a three-dimensional volume, it is not natural that it should appear as a four-space invariant. The relativizing of physical probability remains to be undertaken, but on account of the close connection with molecular physics we may notice two ways in which the perplexing subject of statistical mechanics is likely to be clarified.

First, the characteristic postulates of this subject, particularly those clustering about the troublesome ergodic hypothesis, are plainly conditioned by the aloofness of the time-scale and serve mainly to introduce into the statistical treatment of motions, especially of irreversible processes, something of the kind of dynamical determinism that was so prominent in the Newtonian mechanics. In the unified kinematic geometry it should be possible to replace these postulates by mathematical definitions and theorems and thus remove some of the faults that have hitherto been felt to be personal to this subject. Second, it seems likely that, in the light of relative time, the property of irreversibility may prove to be relative to particular modes of projection rather than intrinsic to the space-time configuration, hence possibly not justifying the sweeping consequences often insisted on in connection with the second law. Moreover, a really satisfactory solution of these questions would cover the thermodynamics of radiation, and the fact that the quantum hypothesis had a statistical origin is one reason for expecting that relativity should place the quantum problem in a new light.

Although its fine-structure theory is a significant exception, the quantum theory has in the main proceeded independently of the relativity revision. It is therefore not surprising that a reconsideration of the quantum phenomena from the point of view of a general world geometry should indicate the possibility of reinterpretation without the quantum hypothesis. It has in fact been found that under a proposed interpretation of light, using an enriched and adapted form of wave-theory, a difference-structure in spectrum relations would be present, and that the Einstein quantum relation of voltage and frequency is a corollary of the difference-structure and the relativity of the time-scales. The apparent necessity for a really novel type of discontinuity disappears and a close affiliation with "classical" theories is restored.

But even ignoring special problems we may see that the astronomical side of relativity should have a natural counterpart in molecular physics, since the arbitrary outward extension of the Euclidean geometry assumed during two centuries of celestial mechanics has long been matched by its inward extension or interpolation for the representation of the fine structure of matter. The important meaning now known for curvatures affecting the mutual relations of distances like 10^{13} centimeters may well be matched by a far-reaching significance in curvatures affecting distances like 10^{-13} centimeter. There are abundant reasons for expecting that in the not distant future the whole of electrical and optical theory and a rational physico-chemical theory of the varieties of matter will be aspects of a fine-scaled time-space geometry. To exhibit one feature of this outlook of special interest for chemical theories let us notice one striking fact about the mathematical side of the Einstein movement.

The earlier Greek geometry grew in close contact with concrete experience, but it was later systematized in more abstract form, and thus furnished the first great example of a highly developed mathematical theory, cultivated partly for its own intrinsic interest. When Kepler reviewed the planetary kinematics he found the deductive geometry of the conic sections awaiting the wide range of physical applications later gathered. Then, roughly speaking from Newton to Fourier, there followed an epoch when mathematics and physical theory grew for the most part together, quite largely under the same leaders; with germs of mathematical principles arising in physical problems and their developments reacting on physical theories. This process has continued, but for about a century past there has been discernible a considerable degree of separateness, with only a minority of the leaders effective in both respects. During this period of intense cultivation of pure mathematics for its own sake a number of mathematical systems grew to high stature in advance of any but elementary contact with experimental science. Among these is the Riemann geometry and its associated general calculus, unwisely named "absolute," which was elaborated some time before Einstein found his remarkable physical interpretation for the four-dimensional case, although Riemann and Clifford foresaw something of the possible physical meaning of curvature. Thus, in contrast with the Newtonian system, the Einstein mechanics found its natural mathematical machinery already highly perfected. In a more general philosophical sense it may be remarked that the postulational movement in abstract mathematics represents a kind of relativity movement, with its reaction on the concrete sciences just beginning. There will be other examples like the Riemann-Einstein of anticipation and fruition, and in particular if time permitted it could be pointed out how some of the existing fields of mathematics seem peculiarly adapted to the special needs of chemistry, though their possible value in this way is only meagerly appreciated.

This same epoch of independent mathematics has witnessed a great increase in the variety of physicochemical phenomena experimentally studied and the variety of measurements and technical devices used in their exploration. But chemistry is still only approaching the stage of mathematical formulation, and in both sciences there has been a marked growth in the relative prominence not only of what have been called secondary theories but of shadowy notions, mystical hypotheses and vague analogies. The art of the experimenter has outrun the vision of the logical physical theorist, while pure mathematics, not keeping such fullness of contact with its concrete origins, has been discovering more consciously its own natural character and powers. But the new wealth of experimental knowledge may become a bewildering burden if its theoretical counterpart remains too much a maze of special and even partly contradictory hypotheses. It is needful that a wider range of experimental results be brought under control by simple and comprehensive primary theory, and there is now a wealth of mathematical material at hand for the purpose. The world geometry has made one great stride in this direction and opened other paths just beginning to be followed. It is clearly desirable that the promise and power of its way of thinking should not be masked by undue emphasis on its relation to special experiments.

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MYSTERIOUS ACOUSTIC PHENOMENA IN YELLOWSTONE NATIONAL PARK

It is highly gratifying that the American people are making yearly increasing use of the Yellowstone National Park and that each season many thousands become for the first time acquainted with its beauties and wonders. Among future visitors there will be some who will experience the strange and bewildering musical sounds that for many years have been noted in certain parts of the park. As a partial contribution to the solution of the mystery, a personal observation on one manifestation of the weird phenomena may be of interest.

This subject has received scant mention in various published reports and articles, and by some people has been relegated to the category of yarns and myths which helped to make Jim Bridger famous. The best account seems to be that contained in the best work[1] on the park, that of General Chittenden, from which the following quotation is taken (pages 288–289):

> A most singular and interesting acoustic phenomenon of this region, although rarely noticed by tourists, is the occurrence of strange and indefinable overhead sounds. They have long been noted by explorers, but only in the vicinity of Shoshone and Yellowstone Lakes. They seem to occur in the morning and to last only for a moment. They have an apparent motion through the air, the general direction noted by writers being from north to south. They resemble the ringing of telegraph wires or the humming of a swarm of bees, beginning softly in the distance, growing rapidly plainer until directly overhead, and then fading as rapidly in the opposite direction. Although this phenomenon has been made the subject of scientific study, no rational explanation of it has ever been advanced. Its weird character is in keeping with its strange surroundings. In other lands and times it would have been an object of superstitious reverence or dread and would have found a permanent place in the traditions of the people.

In the summer of 1919, during a visit to the park in connection with governmental fish-cultural and fish-planting work therein, the present writer, in company with Mr. A. H. Dinsmore, of St. Johnsbury, Vermont (a former superintendent of fish hatcheries in the park), made a camping trip to Lewis and Shoshone Lakes, employing pack horses, with helper, and having as a part of the equipment an Oldtown canoe transported by motor truck from Yellowstone Lake to Lewis Lake.

About eight o'clock on the morning of July 30, after having camped for two days and two nights on the shore of Shoshone Lake at its outlet at the

[1] "The Yellowstone National Park," by Hiram Martin Chittenden. New and enlarged edition. Cincinnati, 1915.