reducible p-adic factors of $f(x)$ and to speak of the corresponding cycles as the first, second, etc., cycles for the prime p. Thereafter the order numbers of any number $R(\varepsilon)$ of the corpus correspond in a perfectly definite manner to these cycles, the form of $R(\varepsilon)$ determining the correspondence of the p-adic factors of $F(X)$ in (8) to the p-adic factors of $f(x)$ in (7). It might be that $R(\varepsilon)$ is itself a root of the equation $f(x) = 0$ other than ε, in which case the order numbers $\tau_1, \ldots, \tau_r$ of ε for the prime p would also be order numbers of $R(\varepsilon)$ but would in general correspond in some other order $\tau_\alpha, \ldots, \tau_\beta$ to the first, second, etc., cycles.

Considering a number of primes simultaneously we shall distinguish them by suffixes. If the number of cycles corresponding to a prime p_λ is designated by r_λ and the order numbers of a given number ϵ corresponding to the different primes $p_1 \ldots, p_\lambda \cdots$ be designated by

$$\tau_1^{(\lambda)}, \tau_2^{(\lambda)}, \cdots, \tau_{r_\lambda}^{(\lambda)}, \lambda = 1, 2, \cdots$$

and those of another number e of the corpus by

$$t_1^{(\lambda)}, t_2^{(\lambda)}, \cdots, t_{r_\lambda}^{(\lambda)}, \lambda = 1, 2, \cdots$$

the necessary and sufficient conditions for ε to be divisible by e, that is for ε/e to be an integral algebraic number, is given by the inequalities

$$\tau_\mu^{(\lambda)} - t_\mu^{(\lambda)} \gtreqless 0, \lambda = 1, 2, \cdots, \mu = 1, 2, \cdots, r_\lambda.$$

The order numbers τ and t here in question are integral multiples of the reciprocals of certain integers

$$\nu_1^{(\lambda)}, \nu_2^{(\lambda)}, \cdots, \nu_{r_\lambda}^{(\lambda)}.$$

In the field of the rational numbers we can always construct a number possessing as its order numbers an arbitrarily assigned set. This, however, is not the case with the numbers in any algebraic corpus which is more general than the field of the rational numbers. For example, we would not in general find in the corpus a number all of whose order numbers are 0 with the exception of a single coincidence $1/\nu$ corresponding to a definite cycle of a given prime p. Also a coincidence $1/\nu$ might be a constituent element in each of the order number sets of several integral numbers of the corpus no one of which could be factored within the corpus. These circumstances must have rendered futile any attempt to find in an algebraic corpus other than that of the rational numbers an analogue to the prime numbers in the sense of ultimate and unique factorization of the integral numbers of the corpus. The generalization from the factorization properties of rational numbers relative to the primes p lies rather in the extension of these properties as expressed in terms of prime order numbers to the properties of algebraic numbers in general as determined by their sets of p-adic order numbers. In the consideration of problems of factorization then the ultimate elements involved are the individual coincidences $1/\nu$ associated with the different cycles in the corpus.

In the light of what has been said in the preceding the conception of an ideal may be elucidated as follows: Assign an order number for each cycle corresponding to every prime p such order number being an integral multiple of the corresponding number $1/\nu$ and only a finite number of the order numbers being different from 0. The system of order numbers so assigned we shall designate by the notation (τ) and call a basis. A number of the corpus, we shall say, is built on the basis if no one of its order numbers is less than the corresponding order number furnished by the basis. The aggregate of the numbers of the corpus built on the basis (τ) we call an ideal. If none of the order numbers furnished by the basis is negative the ideal determined by the basis we call an integral ideal. If, apart from 0-order numbers, the basis (τ') consists of the single order number $1/\nu$ corresponding to a given cycle we say that the ideal determined by the basis is a prime ideal. A prime ideal then consists of all the integral numbers of a corpus whose order numbers for a certain cycle are $\geqq 1/\nu$ where ν is the order of the cycle in question. Where the basis (τ) consists of the actual order numbers of a number of the corpus the ideal determined by the basis is a *principal* ideal (Hauptideal).

What has been said in the foregoing with regard to the cycles of a quadratic corpus suffices to determine all the ideals in the case of such a corpus.

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CONVERSATIONS WITH EUROPEAN MYCOLOGISTS[1]

DURING July and August of the present year, 1925, a botanical mission abroad was undertaken by the writers for the purpose of exchanging opinions with fundamental problems pertaining to the rusts (Uredinales). The project was heartily sponsored by the National Research Council, although efforts to partly or wholly finance the journey were fruitless, chiefly due to the brief interval after the plan was developed. The available time to be employed was necessarily limited, permitting but six weeks on European soil, and consequently it was only possible to visit Germany, Sweden, Norway, Denmark and En-

[1] Presented at the Kansas City meeting of the Mycological Section of the Botanical Society of America, December 30, 1925. Joint contribution from the Botanical Department of the Purdue University Agricultural Experiment Station and the Botanical Department of the Pennsylvania State College.

gland. Fortunately, we were able to meet most of the mycologists of these countries who have given marked attention to the rusts, especially in regard to their intimate structure, their modes of development and their relationships.

Our method of procedure was simple. Letters were sent a few days ahead announcing our coming and the purpose of our visit. The interviews were individual in each case, and occupied from two to four hours on an average. We had five or six typewritten articles with us, being the preliminary draft of two chapters of a forthcoming handbook of the rusts, one giving a brief synopsis and one the morphology and cytology; a restatement of a portion of the first chapter, together with the same in German; a much condensed statement of our views regarding rust development and the terminology required, couched in the shape of dicta; and a somewhat elaborated and changed form of the latter; all of which were considered by us to be of a tentative nature. These were used as a convenient basis for discussion and to expedite the presentation of our views, one or more being brought out each time as seemed best, the elaborated dicta most often of all.

There was no attempt at propaganda. What we desired was a frank expression of opinion regarding the deductions that may reasonably be drawn at the present time regarding the fundamental forms of the rusts, their general course of development and the terms best suited for indicating such conceptions. There has been, and still is, wide divergence of opinion regarding these matters. So far as the writers know, no one has ever before undertaken to compress these features into a statement that would include all forms and sorts of rusts, under all conditions of growth, and with absolutely no exceptions. We were bringing forward, therefore, a novel way of viewing the rusts, and naturally felt great uncertainty about the kind of reception such views were likely to meet, even presupposing the nationality of their advocates had no adverse influence.

In order that the reader may better understand the nature of the subject to which we were calling attention the dicta, as we used them, are here given, although it must be borne in mind that many other matters entered into the conversations, sometimes to the exclusion of the formal dicta. The other papers that we used are too long to be reproduced.

DICTA

Relating to life-cycles and sori of the rusts.

(1) The conception of rusts should first regard the vegetative body (mycelium) and secondarily the fruiting structures (sori) arising from it.

(2) The mycelium (usually uninucleate) possessing indications of ancestral sexuality, is regarded as gametophytic; the mycelium (usually binucleate) following in the same life-cycle, but discontinuous, is regarded as sporophytic.

(3) The long-cycle rusts have both kinds of mycelium; the short-cycle rusts have only gametophytic mycelium.

(4) The sori either produce spores exhibiting no or slight further development (pycnia), or they produce spores capable of definite growth and multiplication of cells (aecia, uredinia, telia, aeciotelia).

(5) The criteria for determining the morphologic and phylogenetic standing of the sori with efficient spores are:
  (a) The relation to the vegetative generation.
  (b) The relation to basidial production.

(6) The terminology of the sori with efficient spores and their distinguishing characters, based on this method, are as follows:
  I. *Aecia:*
    (a) The sori borne on the gametophytic mycelium.
    (b) The spores not directly producing basidia.
  II. *Uredinia:*
    (a) The sori borne on the sporophytic mycelium.
    (b) The spores not directly producing basidia.
  III. *Telia:*
    (a) The sori borne on the sporophytic mycelium.
    (b) The spores directly producing basidia.
  IV. *Aeciotelia:*
    (a) The sori borne on the gametophytic mycelium.
    (b) The spores directly producing basidia.

(7) There are structural characters useful in indicating the nature of sori but no such characters, single or combined, are sufficient to delimit any sorus, consequently, sori of wholly different origin may exhibit structural parallelism.

The subject was presented in more or less detail to Drs. Klebahn, Sydow, Kniep and Dietel, in Germany, Drs. Eriksson, Lagerheim and Juel, in Sweden, Dr. Jørstad, in Norway, Dr. Butler, Mr. Ramsbottom and Miss Wakefield, in England, and by a fortunate coincidence to Dr. Gäumann, of Switzerland. Many other botanists were met, with but slight opportunity, however, to discuss with them the purpose of our mission.

In every instance we were greeted with the utmost cordiality, and the object of our mission given respectful and unbiassed consideration. Every one, botanists and others alike, showed us courtesy, and there were no annoyances, wherever we went. We could discern no indication that nationality had any bearing upon the attitude with which our discussion was approached; in fact it was a surprise to find no trace of the jealousy and suspicion that in former

years was prevalent among botanists on the continent, and doubtless among students in other lines, causing much repression of their individual opinions and knowledge.

As it is clearly impractical for want of space to give an account of our interviews in detail, we must be content with a brief summary. It is to be understood that in this presentation we are not quoting actual statements, but rather expressing our conception of the opinions and beliefs of the various workers, based on our discussions with them.

In the first place there was practically uniform agreement that our statement respecting the development of the rusts, as shown in the dicta and their elaboration, was logical. Furthermore, the attempt to get a conception of rusts from the point of view of plant bodies was generally considered well worth while (Gäumann, Lagerheim). It is possible, we were told, to consider the basidiospores to be the most important spores in the life of the fungus, the aecia being the most important sori, and the other sori relatively unimportant (Jørstad), and that in such genera as *Coleosporium* there are no true teliospores, only groups of basidia (Gäumann, Jørstad). It was thought, however, that these matters do not materially affect the scheme we present, neither does the opinion that the rusts will eventually be found to be heterothallic (Kniep). Only once did we learn that somewhat similar views to ours regarding development had been entertained, and new terms devised to apply in part to the new concepts (Lagerheim).

It was almost unanimously agreed that the new terms we suggested followed naturally from our logic, but whether the terms were suitable or practical for general use was another matter that called out diverse opinions. There was no hesitancy to consider as a true aecium what in Germany and America is usually called the "primary uredo." It was pointed out in this connection that the term "primary uredo" is confusing, as it is often and more properly used to designate the first sorus following aeciosporic infection, which sometimes differs from the secondary uredinia that follow (Lagerheim, Butler).

There was practically uniform objection to the use of aeciotelium, partly because the term too strongly suggests Endophyllum, although admitting that the term is logically required (Klebahn, Ramsbottom), partly because all short-cycle sori can not be designated by a single term (Juel), and partly because the short-cycle sorus is apparently not materially different from long-cycle sori and so requires no separate term (Jørstad).

The terms employed should be adequate to express the ideas, we were told, even if new ones must be found (Gäumann), and we were materially assisted in one instance in devising new terms (Lagerheim), and yet usage can not be dictated (Butler).

As to adopting the terms pycnia, aecia, uredinia and telia in their own writings some offered no expression (Dietel, Gäumann), some thought the old terms could be modified or stretched to cover the new concepts (Klebahn, Juel, Wakefield), some were willing to employ them after they had been used in standard works (Sydow, Butler), others were inclined to put them into use soon (Jørstad, Lagerheim, Ramsbottom).

We did not go to Europe, however, to ask that our concepts or our terminology should be subscribed to or adopted, but to ascertain what our confreres would think about them, and if they had anything better to offer along these lines. What they have to offer will doubtless appear in due course of time. Our reaction to what they said has found expression in the following modified form of the dicta, which we believe is essentially in harmony with the views of our transatlantic coworkers among the rusts, whom we had the good fortune to meet.

REVISED DICTA

(1) The conception of rusts should give consideration both to the vegetative body (mycelium) and to the fruiting structures (sori) which arise from it.

(2) The vegetative body is either macrocyclic (long cycle), consisting of two unlike and discontinuous generations, or microcyclic (short cycle), consisting of one continuous generation.

(3) The fruiting structures, the sori, contain spores that are either inefficient, having no power to bring about infection, or efficient, having the power to bring about infection.

(4) The sori with inefficient spores (pycnia) are present or absent in both macrocyclic and microcyclic rusts, in the former only with the first generation.

(5) The sori with efficient spores in a macrocyclic rust are the initial forms (aecia) and final forms (telia), both essential, with or without intermediate repeating forms (uredinia); the sori with efficient spores in a microcyclic rust resemble either the macrocyclic aecia or telia, and may be designated by the same terms.

In conclusion we wish to express our appreciation for encouragement in undertaking the mission to members of the National Research Council, and other American friends. The cordial manner and frank cooperation of our European confreres made the mission a complete success. Any amount of correspondence, entailing much labor and time, could not have accomplished as satisfactory results. Not only has a better understanding between the workers in this field of science been established, but the general cause of science has been measurably advanced, and the feeling of international good will promoted.

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