

## DISCUSSION AND CORRESPONDENCE

### PRESSURE ENERGY IN AN INCOMPRESSIBLE FLUID AND BERNOULLI'S PRINCIPLE

PROFESSOR E. H. KENNARD in SCIENCE for September 11 gives a correct statement of the "energy transfer" formulation of Bernoulli's principle, but it seems to me that he is mistaken in his contention that the "pressure energy" point of view is absurd.

In an open body of incompressible liquid under the action of gravity the potential energy per unit volume of a given portion of the liquid is, of course, partly dependent on gravity, and it is permissible to think of it as partly due to pressure. The first part of the potential energy per unit volume is equal to  $hdg$  where  $h$  is the height of the given portion above a chosen reference level,  $d$  is the density of the liquid, and  $g$  is the acceleration of gravity; and the second part of the potential energy per unit volume is equal to the pressure  $p$  of the portion of the liquid.

Let us take the pressure at the surface of the liquid as zero so that the pressure  $p$  at any point in the liquid may be thought of as gauge pressure. Then to carry unit volume of the liquid from the surface to a place where the pressure is  $p$  an amount of work equal to  $p$  must be done in overcoming the forces exerted on the unit volume by the surrounding liquid. This work has been handed on to other portions of the body of fluid (it does not reside in the portion of fluid which has been carried from  $A$  to  $B$ ), but the location of what we choose to call potential energy is never a matter for consideration. The notion of potential energy is legitimate when the work done to effect a change of configuration is a function only of the change of configuration.

It is, of course, entirely proper to consider where the work done on a portion of fluid to carry it from  $A$  to  $B$  has gone to, that is to say the energy aspects of a fluid in motion can be formulated on the basis of transfer of energy in the fluid, and there is some advantage in this method because it involves definite things which are ignored in the method in which we assign potential energy due to pressure to each portion of the fluid. Potential energy is always an idea which makes up for things ignored.

WM. S. FRANKLIN

MASSACHUSETTS INSTITUTE OF  
TECHNOLOGY

IN an article on Bernoulli's theorem (SCIENCE, Sept. 11, 1925) Dr. Kennard objects to the name *pressure energy* for the so-called pressure head, and especially to the idea that a pound or a cubic foot of

liquid carries pressure energy with it along a tube of flow.

The validity of the objection can not be tested conclusively by deriving the theorem from the principle of energy, because this involves the point in question. Let Bernoulli's theorem be obtained as an integral of Euler's equations in the case of irrotational, frictionless, stream-line flow; it is then merely a mathematical affair awaiting any useful and usable interpretation. Two of the terms in it are kinetic and position energy, one depending on the velocity and the other on the position of the element of liquid. The third term is energy and since it depends on pressure the name pressure energy is surely not inappropriate. Every one of the three terms varies with the mass and the position of the element; therefore each quantity of energy may at least be regarded as belonging to and traveling with the element.

Pressure energy in this sense is distinctly different from compression or elastic energy. For example, if a rod is to be used for transmitting a push  $P$  with a possible displacement  $p$ , the pressure energy possessed by the rod is  $Pp$ ; this is like potential energy  $Wh$ . If  $P$  shortens the rod an amount  $e$  the compression energy is  $\frac{1}{2}Pe$ . Similarly, when energy is transmitted by water flowing through a horizontal pipe, the input, omitting the kinetic energy, is  $p/w$  ft-lbs per lb, the friction loss is  $h_f$  ft ft-lbs per lb, and the output is thus  $(p/w - h_f) vAw$  ft-lbs per sec. where  $v$  is the velocity of flow,  $A$  the section area of the pipe,  $w$  the specific weight of the water, and  $p$  the pressure (force per unit area) at the input end of the pipe. Problems solved in this way become simple exercises in book-keeping on the energy transactions of a pound of water.

Dr. Kennard's value of 46 ergs should be 23 ergs, because the average pressure is half of  $1.031 \times 10^6$  dynes per cm.<sup>2</sup>

R. F. DEIMEL

STEVENS INSTITUTE OF TECHNOLOGY

### A FOSSIL FISH OF THE FAMILY CALLICHTHYIDAE

IN the fresh waters of South America north to Panama may be found small catfishes (*Hoplosternum punctatum* Meek and Hildebrand) of a peculiar type, the sides of the body covered with a double series of vertically elongated plates. They were revised by Mrs. Marion Durbin Ellis in 1913, and since then not much has been added to our knowledge. When recently collecting fossil insects in the green Tertiary rock at Sunchal, Province of Jujuy, Argentina, I was fortunate in finding the first fossil representative of the family. Although it is at least sev-