ARITHMETIZATION IN THE HISTORY OF MATHEMATICS

Arithmetization represents in the development of mathematics a general tendency of the widest application and hence it is comparable with the principle of evolution in the natural sciences. Notwithstanding the fact that traces of our modern arithmetical algebra appear in an ancient Egyptian work written by Ahmes about 1700 B. C., yet the first really developed algebra was the geometrical algebra of the ancient Greeks, and this has been slowly arithmetized —first by the Greeks themselves, as may be seen in the *Arithmetica* of Diophantus, and later by authors of various other nations.

One of the most influential writers on the arithmetization of the Greek geometrical algebra was R. Descartes (1596–1650), who, in particular, emphasized the fact that he used the symbols a, a^2, a^3 algebraically, and that he did not understand thereby a line, a surface and a solid, respectively. Even F. Vieta (1540–1603), who is commonly called the father of modern algebra, still adhered to the ancient custom (which was, however, not universal) of representing a line by the symbol a, while the symbols a^2 and a^3 were used to represent a surface and a solid, respectively. This usage is reflected in the modern terms linear, square and cube, respectively.

The arithmetization of trigonometry may be illustrated by the change in the definitions of the trigonometric functions. With possibly a few exceptions these functions were regarded as line segments until the time of the great Swiss mathematician L. Euler (1707–1783), and even Euler did not explicitly define them as abstract numbers, as is now commonly done, but he obviously regarded them as such numbers. In the earlier works the values of these functions depended not only on the angle but also on the radius of the circle, or the length of the hypotenuse of the right triangle, with respect to which the functions were determined. For instance, the sine of 90°, that is, the *sinus totus,* had a large number of different values, including the following: 60, 120, 3438, 600,000, 10,000,000,000, etc.

Although analytic geometry and the calculus were founded comparatively recently, yet we find in their development also evidences of the growing tendency towards arithmetization in the development of mathematics. In particular, in analytic geometry the coordinates of a point were for a long time regarded as line segments instead of abstract numbers. A change in this respect was effected by the work of Euler, Lagrange and Monge. In fact, Euler began the use of complex coordinates but did not proceed as far in this direction as C. F. Gauss and A. L. Cauchy.

In the preface of the "Calculus of Observations," by Whittaker and Robinson, 1924, it is noted that "when the Edinburgh Laboratory was established in 1913, a trial was made, as far as possible, of every method which had been proposed for the solution of the problems under consideration, and many of these methods were graphical. During the ten years which have elapsed since then, the graphical methods have almost all been abandoned, as their inferiority has become evident, and at the present time the work of the laboratory is almost exclusively arithmetical. A rough sketch on squared paper is often useful, but (except in descriptive geometry) graphical work performed carefully with instruments on a drawing-board is generally less rapid and less accurate than the arithmetical solution of the same problem."

Thus far we have spoken only of the principle of arithmetization in the history of elementary mathematics. In advanced mathematics this principle was especially emphasized by Weierstrass towards the close of the preceding century, and James Pierpont, of Yale University, treated the subject in a valuable article published in volume 5 of the *Bulletin* of the American Mathematical Society. The object of the present note is mainly to point out that in the development of mathematics there is clearly seen a broad fundamental principle of coordination, and that the study of the history of mathematics is facilitated by an explicit recognition of this principle. The great emphasis which the ancient Greeks placed on geometry gave to their mathematical developments an undue geometric bias, and this bias was naturally transmitted to the students of Greek mathematics for a long period of time. The later tendency towards arithmetization may be partly explained by this well-known situation. L. Kronecker expressed the arithmetization principle of evolution in the history of mathematics as follows: "God made integers, all else is the work of man."

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AVOIDABLE DIFFICULTIES WITH TERMINOLOGY IN COMPARATIVE ANATOMY

Contradictions in anatomical terminology are one of the many disturbing factors with which a student in comparative vertebrate anatomy has to struggle. Courses in comparative anatomy have at least two purposes: to acquaint the student with the vertebrate plan and to serve as a preparation for embryology, human anatomy and medicine. Since the majority of the students often are premedical students, the course should be so arranged that continuity with human anatomy and embryology is attained.

One of the greatest difficulties is the inconsistency in terminology in current text-books and manuals on the various vertebrate types. Consideration of a single organ system will illustrate this. In an anatomical work of recent date, admirable in execution and illustration, the mesonephros of the shark is called a "kidney," without any qualification or explanation of the term. The same criticism applies also to many manuals in use in courses in elementary zoology, in which the mesonephros of the frog is fully discussed as a "kidney." In other manuals the accessory nephritic duct of the shark is usually called a ureter, although it has no relation to the ureter of the amniotes.

In dealing with the genital system, the old term vas deferens is used almost exclusively in disregard of the preferable B.N.A. term "ductus deferens." The latter the medical student must know. At the base of the ductus deferens or Wolffian duct of the shark and the amphibian is a small dilatation which is called the "seminal vesicle." This term the student learns, and perhaps remembers, only to discover later that this terminal dilatation is not comparable to the seminal vesicle of human embryology, but corresponds to the ampulla ductus deferens, the mammalian seminal vesicles being evaginations of the Wolffian duct, which occur only in certain mammals, including man. In a like manner the enlargement of the oviduct of many amphibians is called a uterus, although it is more nearly homologous to the shell gland of certain sharks, and certainly should never be confused with the eutherian uterus.

This same criticism of terminology may be extended in a similar manner to the other systems and is the source of much confusion to the student. Assuming that the premedical student will carry some small bits of information and some few anatomical terms into the first year of medicine, it is evident that he must not only learn more terms, but must unlearn and relearn many of those current in general zoology and comparative anatomy. Otherwise he will not gain a clear idea of the homologies between the organs of the lower classes of vertebrates and man, and will lose one of the most valuable lessons of comparative anatomy—the position of man in a phylogenetic system. It is just as easy to present accurate terms in the first instance when all terms are new and equally unfamiliar to the student, as to use those which are anatomically incorrect and which must be abandoned later.

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LUMINOUS SPIDERS

THE issue of August 21 contains a very interesting letter from Barnum Brown on his discovery in Central Burma of a luminous spider whose abdomen glowed with light while "fireflies sparkled here and there." May not this be analogous to the effect obtained by that prank of childhood when we caught and fed fireflies to the ordinary hop toad and then turned him loose on the lawn in front of the veranda to the consternation of the older folks, who could see but not comprehend the bouncing light. Maybe the spider had feasted plentifully on the abundant fireflies.

EDWARD PIERCE HULSE

In my opinion, there are three explanations of this luminosity: First, the eating of the luminous portions of fireflies by the spiders; second, injection by bacteria or fungi; third, a true luminous organ. Mr. Hulse touches upon the first possibility in his letter and the answer to this, I think, is to be found in the habits of spiders. Spiders are provided with sucking mouth parts, and do not devour the material as a whole. If this individual had selected only the luminous portions of fireflies, light would have shown not only on the abdomen, but through the thorax and head as well. I think the answer to the second possibility, injection by luminous bacteria and fungi, would hold equally true, that both thorax and head would have shown luminosity had the spider been injected. As a matter of fact, I was sufficiently close to determine accurately that only the abdomen glowed. I think that it was definitely provided with a true luminous organ.

BARNUM BROWN

SCIENTIFIC BOOKS

Telephone Communication. By C. A. WRIGHT and A. F. PUCHSTEIN.

"TELEPHONE Communication," written by Professor C. A. Wright in collaboration with Professor A. F. Puchstein, is a text-book intended for use in engineering schools. It deals primarily with the functioning of a telephone system in transmitting and reproducing speech sounds. In this connection it discusses sound, the operation of telephone transmitters and receivers in performing the conversions between sound and electrical energy; the transmission of electrical currents in lines and impedance networks, and the means of measuring and specifying the transmission efficiency of telephone circuits and apparatus.