ARITHMETIZATION IN THE HISTORY OF MATHEMATICS

ARITHMETIZATION represents in the development of mathematics a general tendency of the widest application and hence it is comparable with the principle of evolution in the natural sciences. Notwithstanding the fact that traces of our modern arithmetical algebra appear in an ancient Egyptian work written by Ahmes about 1700 B. C., yet the first really developed algebra was the geometrical algebra of the ancient Greeks, and this has been slowly arithmetized —first by the Greeks themselves, as may be seen in the *Arithmetica* of Diophantus, and later by authors of various other nations.

One of the most influential writers on the arithmetization of the Greek geometrical algebra was R. Descartes (1596–1650), who, in particular, emphasized the fact that he used the symbols a, a^2, a^3 algebraically, and that he did not understand thereby a line, a surface and a solid, respectively. Even F. Vieta (1540–1603), who is commonly called the father of modern algebra, still adhered to the ancient custom (which was, however, not universal) of representing a line by the symbol a, while the symbols a^2 and a^3 were used to represent a surface and a solid, respectively. This usage is reflected in the modern terms linear, square and cube, respectively.

The arithmetization of trigonometry may be illustrated by the change in the definitions of the trigonometric functions. With possibly a few exceptions these functions were regarded as line segments until the time of the great Swiss mathematician L. Euler (1707–1783), and even Euler did not explicitly define them as abstract numbers, as is now commonly done, but he obviously regarded them as such numbers. In the earlier works the values of these functions depended not only on the angle but also on the radius of the circle, or the length of the hypotenuse of the right triangle, with respect to which the functions were determined. For instance, the sine of 90°, that is, the *sinus totus*, had a large number of different values, including the following: 60, 120, 3438, 600,000, 10,000,000,000, etc.

Although analytic geometry and the calculus were founded comparatively recently, yet we find in their development also evidences of the growing tendency towards arithmetization in the development of mathematics. In particular, in analytic geometry the coordinates of a point were for a long time regarded as line segments instead of abstract numbers. A change in this respect was effected by the work of Euler, Lagrange and Monge. In fact, Euler began the use of complex coordinates but did not proceed as far in this direction as C. F. Gauss and A. L. Cauchy.

In the preface of the "Calculus of Observations," by Whittaker and Robinson, 1924, it is noted that "when the Edinburgh Laboratory was established in 1913, a trial was made, as far as possible, of every method which had been proposed for the solution of the problems under consideration, and many of these methods were graphical. During the ten years which have elapsed since then, the graphical methods have almost all been abandoned, as their inferiority has become evident, and at the present time the work of the laboratory is almost exclusively arithmetical. A rough sketch on squared paper is often useful, but (except in descriptive geometry) graphical work performed carefully with instruments on a drawing-board is generally less rapid and less accurate than the arithmetical solution of the same problem."

Thus far we have spoken only of the principle of arithmetization in the history of elementary mathematics. In advanced mathematics this principle was especially emphasized by Weierstrass towards the close of the preceding century, and James Pierpont, of Yale University, treated the subject in a valuable article published in volume 5 of the *Bulletin* of the American Mathematical Society. The object of the present note is mainly to point out that in the development of mathematics there is clearly seen a broad fundamental principle of coordination, and that the study of the history of mathematics is facilitated by an explicit recognition of this principle. The great emphasis which the ancient Greeks placed on geometry gave to their mathematical developments an undue geometric bias, and this bias was naturally transmitted to the students of Greek mathematics for a long period of time. The later tendency towards arithmetization may be partly explained by this well-known situation. L. Kronecker expressed the arithmetization principle of evolution in the history of mathematics as follows: "God made integers, all else is the work of man."

G. A. MILLER

UNIVERSITY OF ILLINOIS

AVOIDABLE DIFFICULTIES WITH TERMINOLOGY IN COMPARATIVE ANATOMY

CONTRADICTIONS in anatomical terminology are one of the many disturbing factors with which a student in comparative vertebrate anatomy has to struggle. Courses in comparative anatomy have at least two purposes: to acquaint the student with the vertebrate plan and to serve as a preparation for embryology, human anatomy and medicine. Since the majority