LENGTH OF LIFE OF OPALINAS IN VARIOUS LABORATORY MEDIA

	In clear fluid	With cloacal content	With piece of cloaca
Locke's solution	28 hrs.	35 hrs.	73 hrs.
Sea-water, 50 per cent.	22 "	25 "	45 "
Pond water	25 "	34 "	36 "
Ringer's solution	32 "	26 "	35 "
Physiological salt solution	21 "	25 "	25 "
Tap water	13 "	13 "	13 "
Distilled water	9 "	9 "	9 "
Kroniker's solution	4 "	4 "	4 "

frog and opening it in a watch glass half filled with the desired medium. The most satisfactory results are obtained by dividing the material among two or three dishes so that each dish has a piece of the cloaca and a part of the cloacal content. The watch glasses should be kept covered to prevent too great evaporation and consequently too great concentration of the salts. The Opalinas in such a solution may be expected to remain alive and in good condition for two days or more and in the case of *Nyctotherus* for as long as six days.

The above solutions are easily made up and sea-water may be readily obtained at any biological supply station or be made synthetically according to Mayer.[1]

A more complete report on the longevity of *Opalina obtrigonidea* in various media together with other observations is in preparation for publication later.

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[1] "The relation between degree of concentration of the electrolytes of sea-water and rate of nerve-conduction in *Cassiopea*," by A. G. Mayer, from Papers from the Tortugas Laboratory of the Carnegie Inst. of Wash., Vol. VI, 1914. Publication No. 183.

SPECIAL ARTICLES

SURFACE TENSION DETERMINED BY THE RING METHOD

IN recent years a considerable amount of attention has been given to the problem of finding a method by means of which the surface tension of a liquid can be measured rapidly and with some degree of accuracy. An apparatus in which the pull on a ring is measured by means of the torsion of a wire evidently meets the requirements of convenience and rapidity. Such an apparatus has been devised by du Noüy.[1]

According to the simple form of the theory underlying the use of the ring method, the surface tension of the liquid is equal to the pull on the ring at the instant of rupture of the films of liquid divided by twice the circumference of the ring. Unfortunately the values obtained in this way are certainly too high. Paul E. Klopsteg[2] has attempted to explain the high values obtained by the method of the torsion-balance on the ground that as the ring is lifted out of the liquid the zero of torsion no longer corresponds to the zero of the scale. He suggests that as the torsion on the wire is being increased the vessel containing the liquid must be lowered so that the arm will at all times be in its position of zero-balance. This procedure is undoubtedly correct. But I do not think that the simple theory of the experiment even with the procedure advocated by Klopsteg can lead to accurate values of the surface tension.

It is well known that, after the ring has been detached from the liquid, droplets frequently adhere to it. Klopsteg suggests that a correction must be applied by adjusting the zero-balance of the instrument with the droplets adhering to the ring. I hope to show that the magnitude of the pull on the ring is independent of whether droplets are formed on the ring or not.

In this discussion I shall use the following symbols:

R = average radius of ring.
r = radius of circular cross-section of wire used in making ring.
p = total pull on ring in dynes divided by $4\pi R$.
α = surface tension in dynes per centimeter.
s, g = density of liquid and acceleration of gravity respectively.
a^2 = specific cohesion $= \frac{2\alpha}{sg}$
$\alpha = p$ (on the basis of the simple theory).

During the last two or three years, Dr. R. G. Green, of the department of bacteriology, has been carrying out measurements with platinum rings having values of r from .015 to .05 cm and of R from 0.3 to 1.3 cm. We soon found that the value of p is a function of r and R, increasing rapidly with increase in r and diminishing slightly with increase in R. We also observed that a maximum pull is reached before the film breaks. If there is in fact a maximum pull, it is evident that its magnitude will be independent of such phenomena as the actual breaking of the film and the adherence of droplets of liquid to the ring.

At this stage in our studies, I came across an ar-

[1] *Journal of Gen. Physiology*, I, 521–524 (1918–19).
[2] SCIENCE, October 3, 1924.

ticle by M. Cantor[3] in which the present problem is treated in a very complete manner. This article seems to have been overlooked by recent writers on the subject. I shall therefore present some of the results obtained by Cantor and discuss the range of their validity, assuming for the sake of simplicity that the liquid and the ring have zero "contact-angle." At any stage in the process of detaching the ring from the liquid, imagine a vertical plane passing through a diameter of the ring and giving a circle as cross-section of the wire. The inner and outer films may touch this circle at the points A_1 and A_2. If O is the center of this circle, let OA_1 and OA_2 make angles γ_1 and γ_2 with a radius drawn vertically downwards from O. Let k be the height of the lowest part of the wire, and let y_1 and y_2 be the heights of A_1 and A_2 above the general level of the liquid. Let γ and y be the corresponding values as R approaches infinity. Cantor assumes (1) that r is small in comparison with R; (2) that R is sufficiently large to make it possible to write $y = \frac{y_1 + y_2}{2}$ and $\gamma = \frac{\gamma_1 + \gamma_2}{2}$. How large R must be to satisfy these conditions, we shall discuss later. With these assumptions, Cantor obtains the equations:

$$p = \alpha \sin\gamma + krsg \sin\gamma + \frac{r^2 sg}{2}[\sin\gamma\,(2 - \cos\gamma) - \gamma]$$

$$k = \sqrt{\frac{2\alpha}{sg}(1 + \cos\gamma)} - r(1 - \cos\gamma).$$

It is easy to show that p has a maximum value for a certain value of k or γ. Cantor applies his results to a ring made of material of rectangular cross-section. It is, however, not difficult to obtain the corresponding results for a ring made of wire of circular cross-section. Thus we obtain for the state in which p is a maximum,

$$\gamma = \frac{\pi}{2} - \frac{r}{a}\left(1 - \frac{r}{2a}\right)$$

$$k = a - \frac{r}{2} + \frac{3r^2}{8a^2}$$

$$p = \alpha + r\sqrt{2\alpha sg} - \frac{4}{\pi - 1} r^2 sg \qquad (A)$$

Equation (A) gives the relation between p (which on the simple theory is equal to the surface tension) and α, r, s and g. For practical purposes it is convenient to have α expressed in terms of the other quantities. Algebraic manipulation of equation (A) results in the following:

$$\alpha = p - r\sqrt{2psg} + \left(\frac{\pi + 3}{4}\right) r^2 sg \qquad (B)$$

[3] Capillaritätsconstanten, Wied. Ann. 47, 399—423 (1892).

For purposes of illustration assume that water at 20° C. is being examined using a ring with r = .025. Suppose that p = 81. Then the term $r\sqrt{2psg}$ is approximately 10 and the term $\frac{\pi + 3}{4} r^2 sg$ is approximately 1. The important rôle played in these experiments by the magnitude of the radius of the wire is manifest.

It will be remembered that Cantor assumed that R is so large that the average height of the points A_1 and A_2 may be taken as equal to the height of either when R = ∞. When $\frac{r}{a}$ is a small fraction, the angle γ does not differ much from $\frac{\pi}{2}$. Let us therefore consider the values of y_1 and y_2 when the ring is replaced by a vertical cylinder. In these circumstances it is well known that to a first approximation, $y_1^2 = a^2\left(1 + 0.6095\frac{a}{R}\right)$ or $y_1 = a\left(1 + 0.3047\frac{a}{R}\right)$ and $y_2^2 = a^2\left(1 - 0.6095\frac{a}{R}\right)$ or $y^2 = a\left(1 - 0.3047\frac{a}{R}\right)$. For $R = \infty$, $y = a$. Accordingly we find $y = \frac{y_1 + y_2}{2}$. But this equality holds only when in the expression for y_1 or y_2 we neglect terms in $\frac{a^2}{R^2}$ and higher powers of $\frac{a}{R}$. Now this term in $\frac{a^2}{R^2}$ will have the same sign in both expressions. Accordingly we may infer that Cantor's results are valid when the term in $\frac{a^2}{R^2}$ is small in comparison with unity. By a method of approximation, I have extended the expressions for y_1 or y_2 to include the term in $\frac{a^2}{R^2}$. The results follow:

$$y_1^2 = a^2\left(1 + 0.6095\frac{a}{R} + 0.173\frac{a^2}{R^2}\right)$$

$$y_1 = a\left(1 + 0.3047\frac{a}{R} + 0.040\frac{a^2}{R^2}\right)$$

$$y_2 = a\left(1 - 0.3047\frac{a}{R} + 0.040\frac{a^2}{R^2}\right)$$

These equations, which I submit with some reservation, would indicate that the error committed in using equations (A) or (B) will not amount to more than one per cent. if the term $0.040\frac{a^2}{R^2}$ is not greater than 0.01; in other words if R is greater than 2a. In experiments with water, therefore, R should be greater than 0.75 cm.

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