SCIENTIFIC BOOKS

Calculus of Variations. By GILBERT AMES BLISS. Published for the Mathematical Association of America by the Open Court Publishing Company, Chicago (1925), pp. xiii, 189. Price, $2.00.

WHEN it was announced some time ago that the first number of the Carus Mathematical Monographs was to be written by Professor Bliss on the subject to which he has made so many important contributions, it was felt by some that Professor Bliss had set himself a very difficult task. The main purpose of this series of monographs is the "diffusion of mathematics and formal thought as contributing to exact knowledge and clear thinking, not only for mathematicians and teachers of mathematics, but also for other scientists and the public at large," and it seemed that of all branches of mathematics the Calculus of Variations might easily prove one of the most difficult to present in a popular manner. This must have been, especially, the thought of those who were, like ourselves, introduced to the subject in a physical treatise expounding Hamilton's conception of the proper mode of statement of the laws governing the development of a dynamical system. We can still see in our mind's eye the formidable pages of δ's which seemed to differ in some mysterious way from the mannerly differentials of the preceding and following paragraphs; which had a ghostly superiority since they could never physically be realized; but yet which could be interchanged as to order of procedure with the more homely and familiar time differentiations of the subject. Could it be possible to present this subject in an elementary and straightforward manner, intelligible to the ordinary student of physics, with the usual familiarity with simple differentiation and integration?

There can be no doubt as to the answer in the mind of any one who reads Professor Bliss's book. The exposition is masterly and is as lucid as the best French mathematical treatises. Beginning with an interesting historical introduction, typical problems of the Calculus of Variations are described, such as, for examples, the shortest distance problem, the brachistochrone problem and the problem of the surfaces of revolution of minimum area (the soap film problem). Before taking up the general theory the author adopts the pedagogically sound procedure of treating these special problems in detail and he is able, without ever abandoning the "elementary" viewpoint, to illustrate in these problems the general modes of solution of problems in the Calculus of Variations and to point out the kind of difficulties that may be expected to arise. The presentation is very "modern" and the consistent use of Weierstrass's ingenious concept of a "field of extremals" and of Hilbert's invariant integral makes the theory straightforward and easy.

Jacobi's interesting theorem as to conjugate points here presents no difficulty. The final chapter, headed "A more general theory," is easy reading after the previous chapters dealing with special problems have been mastered and the book ends with some interesting historical remarks and a well-considered and useful list of bibliographical and historical references.

On the whole we think this book an illustration of the truth of Benjamin Franklin's dictum that no theory is difficult if properly and methodically presented. No student of physics of chemistry needs to be told of the importance of the subject-matter, for most physical theories seem incomplete until they can set up a Hamiltonian integral dominating the subject by the fact that its variation must be zero. Or, to give an illustration which will be more appealing to those interested in quantum theory, it may be said that any conservative dynamical system in which the coordinates are q and the momenta p lives in such a way that its life line in its representative (q, p) space is an extremal of the integral $\int \Sigma\, p\, dq$ where all comparison curves satisfy the equation $H = \text{constant}$ (H being the total energy of the system). We heartily recommend Professor Bliss's book either as a text for a short course on the Calculus of Variations or to any serious scientist for private reading. May the succeeding monographs of the series maintain the standard set by this first one!

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THE NORWEGIAN IDENTITY BOOK

AT the International Emigration Congress, held in Rome from May 15 to June 6, 1924, a proposition was introduced by the Norwegian delegate Fjelstad which, if feasible and viable, may go far toward solving the vexed question of the identification of the individual in intranational and international relations. It was devised by Drs. Jon Alfred Mjoen and Jon Bö at the Winderen Laboratory, Christiania, and proposes, in brief, the biological registration of a given population by means of an identity book,[1] so arranged as to include and, for public purposes, replace the ordinary certificates of birth, baptism, vaccination, school attendance, university matriculation and graduation, marriage, tax-payments, life insurance policies, military, naval, police court or prison relations and other personal papers establishing the true status of the individual in society. Its object is to facilitate and obviate bookkeeping in passport offices, courts of justice, at the ballot box or in any other situation

[1] J. A. Mjoen and J. Bö: *Eugenics Rev.*, Lond., 1924, XVI, 183–188.

in which the identity of the individual as a reliable or dubious social unit becomes of paramount importance. But the very object of this Scandinavian invention is to protect Abel against Cain, to see to it that the decent and honorable (the mass of mankind) do not suffer unduly for the "crimes, whims, ineptitudes and insanities of the few," to prevent the miserable, unwarranted espionage and hold-up policies which have harassed and tormented the peoples of Europe and America ever since the beginning of the World War. As to the other side of the shield, our Scandinavian authors speak in no uncertain language:

In all the belligerent lands, there were actually regulations which prevented criminals from being sent to death. It is equally characteristic of the organization of our modern society that the law-abiding are restricted in the exercise of their right of self-determination, because there exists a growing minority of law-breakers. In the human ant-hill there are none to exterminate the parasites. On the contrary, these parasites flourish and strut around as they have never done before and make their way with well-filled valises through all entrance gates, from the Riviera to Gjedser-Warnemunde, from Ukraine to Castle Garden. They manage to cross frontiers and pass through one land to another without standing in a queue on Alexander-platz, whilst the unpracticed, respectable citizen of another country must run from consulate to consulate, from one police station to another, to be cross-examined, suspected, nay, even thrown into prison, until, wearied out and discouraged, he gives up all further attempts to get across the border. The drunkards make it necessary to fashion regulations which fall heavily upon the temperate, the international criminal necessitates a system of passports, of control, and finally of punishment, which becomes more and more intolerable for the industrious members of society. The fault lies mainly in the fact that in the passport office, or in the sick insurance, at the ballot box or in the court of justice, no sincere effort has hitherto been made to identify a human individual or to connect the individual with his own past. The basic principle for the modern treatment of the individual at all the cross-roads of life is this: "Because there are to be found individuals with a vicious past, we must also deal with you as if you were equally vicious." The claim that every human being has a right to make, namely, the claim to be held responsible for his own acts, for his own motives and for his own way of life, is rendered difficult by the fact that we have hitherto had no means of identifying the individual. We have lacked a system which could promptly and in a practical manner establish the fact that "I am I, and nobody else." It is not very reassuring to see the powerlessness which is displayed by the so-called authorities of the state, even in the leading civilized lands, when it is a question of ascertaining the material composing the population and its movements, or when law and order need to be maintained and social evils combated. Milliards of money are granted annually for education, public health and the prison system, with the result that hospitals, lunatic asylums and prisons must be built larger and larger. And the prisons are filled, although most of the criminals still remain at liberty.

Our national and international conditions afford in reality to the criminal the fullest opportunities of constantly extending his fields of operation. Even in those cases where anti-social activities do not display criminal methodicalness and foresight, we see that these anti-social race-elements inflict great damage on the community. For they infect society biologically and morally. The biologically inferior—the mentally deficient—fill the ranks of the chronic drunkards, castaways and loafers, and may spread venereal disease. With the increasing movements of the populations which are now taking place these race-elements are being dispersed more than ever over new fields of operation.

In our human age it is becoming—and rightly so—constantly more and more difficult for society to rid itself of the inferior individuals by brutal methods. Formerly recourse was had to capital punishment. The method was radical and killed all evil growths at the root. But in our modern society, with its increasing feelings of mercy, the difficulties augment themselves with every new generation. The upright and industrious individual is often placed under control as if he or she belonged to the class of drunkards, prostitutes or criminals, especially if they remove themselves some distance from their native place. The demand for a new solution of these problems is making itself more and more clearly heard.

That such a plan would defeat its own object if controlled by unscrupulous political henchmen of jack-in-office type is self-evident. It is also obvious that the loss of his identity book would put the individual in a tight place, in the absence of his personal papers, and that the book itself would have to be safeguarded against falsification. But it is urged that the immense amount of time consumed and of energy dissipated in identification by ordinary methods has become so costly to government (for in modern life time is money) that the identity book will not only protect and defend citizens who have been slandered, rendered suspect, falsely accused or wrongfully arrested, but will also constitute a check upon the habitual criminal and recidivist. At the same time, a given book, if lost, will be at least valueless to the finder. In virtue of the identity book, the complement of an emigrant ship can be easily filtered, more people will vote at the polls when the troublesome formalities are dispensed with, the character of servants and other people seeking employment will be easily ascertained and registration of taxpayers facilitated, as also military mobilization, the prevention of smuggling, the apprehension of spies, the relief of deserving poor and of distressed sailors, while the passport system at frontiers will be enormously simplified. On the death of the holder, it is proposed that the identity book be returned to the Biological Registration Office as a public record. In view of its important bearings upon emigration and immigration, the proposition was

printed *in extenso,* in English, French and Italian, and referred to a special conference. The conclusions of the conference will be awaited with interest.

SPECIAL ARTICLES

A HIGHLY SENSITIVE PHYSICAL METHOD FOR DETECTING PROTEINS IN A SOLUTION

IT is generally believed that immunological reactions are the most sensitive, as they make it possible to detect such minute amounts as 0.00000005 grams of proteins (1/20000000). H. G. Wells, in his excellent book on the "Chemical Aspects of Immunity," remarks that "these figures give a striking illustration of the delicacy of the immunological methods and their value in studying certain problems in protein chemistry. In no other way would such minute amounts of protein be detected in a solution."

This statement, which expresses the conditions actually prevailing, led me to make some preliminary experiments which are reported in this paper and which show that it is possible, by a purely physical method, to detect the presence of still smaller amounts of protein, namely, 0.00000002 gr. or 1/50000000, in two hours or even less.

This method is based on the use of the tensiometer previously described[1] and on a phenomenon mentioned in 1922[2] and later in this journal,[3] under the title "Antagonistic action of colloids."

When a trace of powdered sodium oleate is added to pure water, or to a salt solution contained in a watch-glass, the surface tension decreases instantaneously and becomes very small. If the amount of sodium oleate added is smaller than 1/1000, the drop will continue for over one hour until a certain minimum value is attained. This value will then remain practically constant.[4] But when the same amount of sodium oleate is added to the same watch-glass containing some other colloid in solution, instead of pure water or saline, the surface tension, after reaching almost instantaneously its bottom value, starts up immediately and in a few minutes, according to the concentrations, tends towards its original value or even reaches it.

The presence of 1/1000000 of a gram of protein has no effect on the value of the surface tension of water, or very little, as the monomolecular layers are formed for egg albumin, for instance, at dilutions comprised between 1/80000 and 1/210000. Consequently it was thought that an indirect method, based on the above described "antagonistic phenomenon" was more likely to bring results. For this purpose a solution of 1/300000 of pure sodium oleate was prepared, and 1 cc was poured in a watch-glass; on the other hand, 1 cc of solution of 1/25000000 of crystalline egg albumin was added to it. The final dilution of the solutes was consequently 1/600000 for sodium oleate and 1/50000000 for egg albumin. Then, the drop of the surface tension of this mixed solution was compared with that of a solution of pure sodium oleate at 1/600000. The results of three series of experiments were as follows:

SURFACE TENSION IN DYNES

	Initial values			Static values (2 hours)		
No. of experiment	1	2	3	1	2	3
1. Water	75.4	75.3	75.3	75.3	75.3	75.3
2. Egg albumin crystal. 1/50 000 000	75.1	75.1	75.2	75.3	75.3	75.3
3. Sod. oleate. 1/600 000	75.0	74.9	74.7	70.8	72.0	71.0
4. Sod ol. + egg albumin same concentrations as above	75.0	75.0	75.0	75.1	75.2	75.2
Difference between the static values of Sol. No. 3 and 4, due to the presence of egg albumin				4.3	3.2	4.1

Consequently, the presence of 1/50000000 of egg albumin in 2 cc of liquid will prevent the lowering of the surface tension of water by sodium oleate in the proper dilution. This phenomenon is observed only when the sodium oleate is highly diluted. It would certainly be at a maximum for a concentration of 1/750000 in the watch-glasses which we use, filled with 2 cc of solution, as under such conditions, I have shown that an organized monolayer is formed.[5] However, it is difficult to prepare sodium oleate in such state of purity that it will entirely dissolve in true solution, and it happened that the sample used in the above reported experiments was not absolutely perfect. Nevertheless, the phenomenon is quite striking and clear at 1/600000. It is very probable that the sensitivity of this reaction can still be increased, and we have already secured evidence that it might show the presence of 0.00000001 gram of protein. Experiments are now being carried on to study the phenomenon more thoroughly.

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[1] du Noüy, P. L., *J. Gen. Phys.,* 1919, i, 521; *La Nature,* 1920, No. 2391, p. 63; Holmes, H. N., "Manual of Colloidal Chemistry," New York, Wiley, 1922.

[2] du Noüy, P. L., *J. E. M.,* 1922, xxxvi, I, 115.

[3] du Noüy, P. L., SCIENCE, 1924, lx, No. 1554, 337.

[4] du Noüy, P. L., "Surface equilibrium of colloidal solutions. I.," SCIENCE, 1924, lix, No. 1539, 580.

[5] du Noüy, P. L., *Phil. Mag.,* 1924, xlviii, 264 and 664.