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THE TREND OF THOUGHT IN PHYSICS[1]

THE retiring vice-president for Section B is in the position of an electron who two years ago was hurled from his normal state of repose into a new orbit in which, according to true classical laws, he does nothing, and is not expected to do anything, but from which he is not allowed to return without emitting his quantum. And so, I appear before you to-day to emit my quantum in the form of an address. I am not unmindful of the fact that the emission of quanta takes place according to laws which are restricted by no precedent, so, on this occasion, I feel free to wander as I please in the realms of speculation.

From the quietude of a quarter of a century ago, when it appeared to many that physics had exhausted itself and all that was worth discovering had been discovered, and when would-be Ph.D.'s went about like roaring lions seeking something to measure and finding nothing but the density of a gas or the viscosity of a solid, from a state where the line of demarkation between what we could hope to probe further and what was apparently beyond our reach forever was so sharp, from a state where the mechanism of what was known was so very clear and the mechanism of what was unknown was so very obscure, we have, in a few short years, evolved a situation in which nothing is very clear and nothing is very obscure, but in which we have found courage to peer into the innermost shrines of nature with a hope of comprehension of what we may there find. And what have we found? We have found as part of the same mechanism phenomena which take place according to our most primitive and naïve conceptions and phenomena which appear to violate some of our most cherished ideas as to how we think nature ought to work. We might have been serenely happy in a state of mind where electrodynamics remained untouched, and it was necessary to appeal to the occult to explain the atom; but, when the atom draws upon electrodynamics just sufficient to spoil it and yet leave us bound to it, we may be excused for a feeling of confusion and for a tendency to join in the cynicism of the wag who said that atomic theory is classical

[1] Address of the retiring vice-president and chairman of Section B—Physics—American Association for the Advancement of Science, Washington, December 30, 1924.

electrodynamics on Mondays and Wednesdays and quantum theory on Tuesdays and Fridays.

THE REASONABLENESS OF PHYSICAL HYPOTHESES

I suppose that if one should ask the theoretical physicist what he is trying to do, he would be fairly safe in replying that he is trying to understand the universe. And what does this understanding of the universe mean? It means different things to different people. To most physicists, however, it means the realization of a process of logical reasoning by which the phenomena of nature may be deduced as a consequence of certain hypotheses which are accepted without further inquiry. We are all familiar with the satisfaction which we feel when things are accounted for on the basis of certain hypotheses which we like. But an attempt to establish even the most plausible of these hypotheses on any basis other than pure hypothesis usually results in sorry failure. The average physicist would doubtless be much pleased if he could see some way by which gravitation could be transmitted as the result of strain in an elastic medium. If some philosopher should ask him what he means by an elastic medium, he may be wary and give a formal definition, but the thing which really gives him satisfaction about the medium will be the thought of a piece of elastic and the belief that the medium acts in a way somewhat analogous to that piece of elastic. If the philosopher should ask why the elastic pulls, he will reply, "Well, that is a matter of cohesion, but we believe that when the molecules of the elastic are separated from each other, they tend to come together again." "But," says the philosopher, "why do they do this?" "Well," replies the physicist, "although we speak crudely of their being separated, we really believe that they are embedded in a medium which has elastic properties, so that it resists their separation." "But what do you mean by the medium having elastic properties?" says the philosopher. "That it is in some way analogous to a piece of elastic," says the physicist. "But why does the elastic pull?" says the philosopher. And so we are back where we started.

Well, has nothing then been accomplished by our physicist's theory? Is he really as big a fool as he looks? I think not, for if he had realized his ambition in creating an elastic theory, he would at least have shown that this phenomenon, gravity, which he does not understand, acts in the same sort of way as that other phenomenon concerned with the elastic which he also does not understand although he thinks he does. There is at least a unification of ignorance in the matter. Psychologically the theory satisfied our physicist because he had become familiar with the elastic before he was old enough to think about it and be astonished at its power to pull; and he had arbitrarily accepted this pulling property as something which needed no explanation or at any rate needed much less explanation than most other things. And so we all have what I may call criteria for reasonableness, in terms of which we think. They are functions of the age in which we live, and of the phenomena which have been studied and made familiar to us by our immediate predecessors.

Three centuries ago, the astronomer Francesco Sizzi, arguing against Galileo's discovery of Jupiter's moons, spoke thus:[2]

> There are seven windows in the head, two nostrils, two eyes, two ears, and a mouth; so in the heavens there are two favorable stars, two unpropitious, two luminaries, and Mercury alone undecided and indifferent. From which and many other similar phenomena of nature, such as the seven metals, etc., which it were tedious to enumerate, we gather that the number of planets is necessarily seven.
>
> Moreover, the satellites are invisible to the naked eye, and therefore can have no influence upon the earth, and therefore would be useless, and therefore do not exist.
>
> Besides, the Jews and other ancient nations as well as modern Europeans have adopted the division of the week into seven days, and have named them from the seven planets; now if we increase the number of the planets this whole system falls to the ground.

Now, argue as we may, this man was probably no fool. He was doubtless a man of eminence in his day. His criteria for reasonableness were different from ours. And there is a certain element of truth in his statements. There *are* seven days in the week, seven openings in the head, and so forth, and if he had pushed his ideas far enough, he might even have predicted something about the periodic table. His hypothesis that all things which exist have some use is not at all a bad hypothesis, and would correlate many facts in biology. It would suggest to us that the laws of electrodynamics could not be expected to hold in the atom, for to build an atom in this way would be very bad since it could not be permanent. Of course, his criteria for reasonableness seem very naïve to us. He did not know as much as we do, and did not have so many things to explain. If he had, he might have joined us in saying:

> If we set a tuning fork into vibration by means of another vibrating fork placed at a distance, we can, if we wish, trace out the air motions by which the vibrations are transmitted. We can demonstrate that the energy is handed on through the medium. A similar thing can be demonstrated when a cork in one part of a pond is made to oscillate through the agency of a stone thrown into the pond at another place, or when an object

[2] Quoted from Sir Oliver Lodge's "Pioneers of Science."

at one end of a table is shaken by vibrations communicated at the other end. Hence, if things do not take place in this way when an electron falling upon the target of an X-ray tube causes the ejection of an electron somewhere else, all of this is meaningless.

THE FACTORS WHICH HAVE MOULDED THOUGHT ON THE LINES OF CLASSICAL DYNAMICS

I believe some bygone astronomer said that the planets moved in circles because the circle is the perfection of symmetry. Now that would not have been a bad law if it had only been right, and if it had been a little more explicit as to the relation between the sizes of the orbits and the speeds of the planets; for, after all, a circle is a very nice curve for a self-respecting planet to move in. However, the language of celestial mechanics speaks otherwise. It says that the planet moves in a circle, or rather in an ellipse, because it would really like to move in a straight line, but the sun prevents it. Now passing over for the moment the question of what is meant by a straight line, I think that we should make a philosophical error in supposing that a planet should prefer a straight line to any other curve if left to itself, whatever that might mean. The fact is that if we analyze the motion of the planet and pick out the acceleration, we find that it is always directed towards the sun, and that its magnitude depends only upon the distance from the sun, following, in fact, the well-known inverse square law, so that at great distances from the sun, the motions of the planets tend more and more towards the rectilinear constant velocity type. There are many other cases where the motion of a body is completely described by the statement that its acceleration is a function only of its position in relation to other bodies, as, for example, the case of a stone swinging around at the end of a piece of elastic. In the case of the stone and the elastic, it is necessary to attach, as a coefficient of the acceleration, a mass factor depending on the stone, in order that the quantity so obtained shall be determined entirely by the position. It is because things are such that in a large number of cases the mass times the acceleration is determined entirely by the position of the body that it has been convenient to give a name to that quantity, and another name to the function of the position to which it is vectorially equal. Nobody could have prevented our defining force as proportional to the acceleration had some other law been true, but nobody would have done it. The orientation of our thoughts, the adjustments of our criteria for reasonableness so that the motions of the planets and other bodies shall be understood by us in terms of those criteria, is one where we define force to be absent if the body is moving without acceleration. There is no question of rightness or wrongness. It is a matter of definition. Absence of acceleration represents the state of nothingness which the mind takes as its origin from which to elaborate its thoughts. That the situation is not even one inherent in our primitive instincts is obvious when we recall the difficulty experienced by beginners in physics in understanding why a heavy body can be kept moving with constant velocity without the application of a force. I have a good deal of sympathy with them. For, why the second time derivative of the motion rather than the first or the third or the fifth should be elevated to fundamental importance is a question whose answer can not be elaborated out of nothing. The state of mind to which our experiences have adapted us is one in which, if the body is at rest or moves with constant rectilinear velocity, we are content, no questions are asked. If it moves otherwise, we have decided to ask why, and we have been led to inquire particularly as to the acceleration, because that is the quantity which in most of the cases which have interested us in the past may be expressed readily in terms of the position of the bodies in relation to other bodies. The law is simple. Of course, it is a very interesting thing that it should be possible to express the matter in any manner which is simple; but, in pondering over the reason for the simplicity of nature's laws, it is well to remember that that simplicity is imparted largely because mankind, being unconsciously wise, has chosen to give names to those things which are simply related. Herein is the origin of the conservation of energy. For the fact that the mass accelerations of the bodies are functions only of their positions leads by direct analytical procedure to the result that the sum of the one half mv^2 plus a certain function of the positions of the bodies is always constant. Hence, the desirability of giving names (kinetic and potential energies) to the two quantities which possess such a simple and striking property. So much does the mind love the idea of something remaining constant that directly the laws of any new branch of physics are formulated, they are immediately searched for something which these laws keep constant in such a way as to permit of its being christened energy.

And so we have grown up in a frame of thought which speaks of forces, masses and accelerations. The law of action and reaction and its extension in the principle of d'Alembert merely place restrictions on the types of the forces which occur, and the equations of Lagrange, the principle of least action and the Hamiltonian principle merely form analytical elaborations of the scheme suitable for application to special types of problems.

THE DESCRIPTION OF NEW LAWS IN THE LANGUAGE OF THE OLD

When the accelerations are no longer functions of the position alone, but involve, for example, the velocities, we retain the form of the language with the stipulation that the forces depend upon things other than the relative positions. In this way arise such concepts as viscous forces. So wedded are we to the language which we have created, and in terms of which we think, that we will go to almost any length to preserve it. Thus, in the case of electrodynamics, the principles of the subject have led to a form of statement of the law of motion of an electron in which we say that the electron moves in such a way that

$$\iiint \varrho \left(\mathbf{E} + \frac{\mathbf{u} \times \mathbf{H}}{c}\right) d\tau = 0$$

where $\mathbf{E}$ and $\mathbf{H}$ are the electric and magnetic fields, ϱ is the electric density, $\mathbf{u}$ the velocity of an element of the electron, and the integral is extended all over the electron.

We do not like this statement. It does not look like an equation of motion, so we first divide the nothing into two parts (a large portion of theoretical physics is derived by dividing nothing up into two parts) and write it as

$$\iiint \varrho \left(\mathbf{E}_o + \frac{\mathbf{u} \times \mathbf{H}_o}{c}\right) d\tau + \iiint \varrho \left(\mathbf{E}_i + \frac{\mathbf{u} \times \mathbf{H}_i}{c}\right) d\tau = 0$$

where subscript i denotes the field of the electron's charge, and subscript o the field of the other charges of the system. The integral involving the subscript i is of course calculable when we know the shape and change of shape of the electron with motion; and when we work it out and expand it as a function of the velocity, acceleration and higher time derivatives, we obtain, even for the case where motion is rectilinear, an equation of the form

$$\iiint \varrho \left(\mathbf{E}_o + \frac{\mathbf{u} \times \mathbf{H}_o}{c}\right) d\tau = a_1 \ddot{\mathbf{x}} - a_2 \dddot{\mathbf{x}} + \cdots \text{ etc.}$$

where the a's are functions of the velocity of the electron. We thus have an equation in which the acceleration of the electron appears. All the rest of the terms are transferred to the left-hand side and christened "Force," the term

$$\iiint \varrho \left(\mathbf{E}_o + \frac{\mathbf{u} \times \mathbf{H}_o}{c}\right) d\tau$$

being the external force, and the other terms radiation reactions. Thus is our language "Force equals mass times acceleration" maintained. It is true that since the coefficient of the acceleration is a function of the velocity it is necessary to speak of a mass varying with the velocity; and, moreover, since that coefficient of the acceleration is different when the velocity is perpendicular to the acceleration from what it is when it is parallel thereto, it is necessary to speak of a longitudinal and a transverse mass.

The limit of straining of ideas would of course be reached if the equation of motion had not involved the acceleration at all. We could still have inserted it on the right-hand side provided that we had put in a corresponding term on the left-hand side to cancel it, but nobody would believe that the formal retention of the words "Force equals mass acceleration" for this case would provide any simplification.

I of course grant that the smallness of the variation of mass with velocity, and the relative unimportance of the terms other than the acceleration term in the usual cases where the equation is employed afford a justification for the process in the ordinary applications to electron theory, and I simply discuss the case because the complications involved in retaining formal adherence to the ideas of force and mass acceleration give, even in this case, a fairly strong threat of the trouble we invite, if we expect to be able to throw every equation of motion which we encounter into this form.

To many minds the appeal of some sort of minimum principle is almost as strong as the more primitive appeal of forces, kinetic reactions and the like. The earliest of these principles is the principle of least time, so well known in optics, a principle which is, of course, the analytical equivalent of the ordinary laws of reflection and refraction. Probably the primary cause which has given rise to the desire for minimum principles has been a feeling that a properly designed universe would naturally be constructed so that something should be as small as possible. It should not be wasted; and the problem remains to discover what it is that is of such value in the universe that it should not be wasted in this sense. The dignity of the process suffers to some extent when we recall that, since the condition that the time integral of a certain function of certain variables shall be a minimum is a set of differential equations, it is not surprising that given any set of differential equations describing the motions of bodies we should be able to construct an integral for which these equations are the conditions that it shall be a minimum. After the discovery of such an integral it only remains to christen it, and its importance in the science becomes established. The minimum principles expressed in the Hamiltonian principle and the principle of least action are of course merely the analytical equivalents of the equations of motion of Lagrange; and, in spite of the fact that almost every expounder of a new theory tries to put his equations in the form of a minimum principle, it is doubtful whether any greater significance is to be attached to the process than a crystallization of the equations in a compact form.

A characteristic feature of theoretical physics has been the moulding of new theories in the forms of those which have gone before, by a judicious christening of the quantities concerned. One of the classical examples of the process is to be found in Maxwell's moulding of the laws of electrodynamics into the form of the equations of ordinary dynamics. To some, this achievement appears as a proof of the electrodynamic laws from dynamical principles; but a careful perusal of what has been done will show rather that it constitutes a discovery of what quantities it is necessary to christen kinetic and potential energies, and in terms of what coordinates it is necessary to talk in order that the application of the dynamical principles in the form of the equations of Lagrange, or, in later developments, in the form of the Hamiltonian principle, shall lead to the results stated in the laws of electrodynamics.

DEFINITION AND SCAFFOLDING IN PHYSICAL THEORY

The content of a physical theory is frequently obscured owing to the fact that certain of the quantities occurring in it are visualized in one sense, defined in another and used in yet another. To illustrate my point, let me for a moment remind you of the well-known equations of electrodynamics

$$\frac{1}{c}\left(\varrho\,\mathbf{u}+\frac{\partial \mathbf{E}}{\partial t}\right)=\operatorname{curl}\mathbf{H}\quad(1);\quad -\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t}=\operatorname{curl}\mathbf{E}\quad(3)$$

$$\varrho=\operatorname{div.}\mathbf{E}\quad(2);\qquad 0=\operatorname{div.}\mathbf{H}\quad(4)$$

$$\iiint \varrho\left(\mathbf{E}+\frac{\mathbf{u}\times\mathbf{H}}{c}\right)d\tau=0\qquad(5)$$

where the triple integral is extended over the whole of one electron. Now to one who has pictured the moulding of these equations from dynamical principles, it will be recalled that **E** is visualized as an ethereal displacement, while equations (1) and (4) really constitute the definition of **H**. On the basis of another form of dynamical development applicable to the case where no charges are present, **H** appears as a velocity of the ether, while **E** appears as the curl of an ethereal displacement. On the other hand **H** is frequently defined as the force on a unit pole and **E** the force on a unit charge. On the basis of a separate definition of all the quantities concerned, each of these equations may be regarded as the expression of a law which might or might not be true. But how is **H**, for example, *used,* in most of the important applications of these equations at any rate? Certainly not as the force on a magnetic pole. For the realization of a magnetic pole involves the conception of a magnet infinitely thin in the macroscopic sense, but thick enough in the sub-macroscopic sense to contain a large number of molecules per unit of length, for example. Now when we are talking about the magnetic field at a point inside an electron, and we do have to talk of this in many problems in electrodynamics, how are we to conceive of that enormously large magnetic pole, which indeed only has existence in the macroscopic sense, as being placed in an element of volume of the electron which is infinitesimal in the sub-macroscopic sense. The point involved is not one of experimental difficulty, but of logical absurdity. Yet, if we do not trouble ourselves about the matter, the calculations go along merrily as though all were well. They do this because we really do not use the quantities **E** and **H** in the sense in which the unit charge and pole defined them. If we solve the equations (1) to (4) the solutions can be put in the well-known form

$$\mathbf{E}=-\frac{1}{c}\frac{\partial \mathbf{U}}{\partial t}-\operatorname{grad}\varphi\quad(6);\qquad \mathbf{H}=\operatorname{curl}\mathbf{U}\quad(7)$$

where

$$4\pi c\mathbf{U}=\iiint\frac{[\varrho\mathbf{u}]}{r}d\tau\quad(8);\qquad 4\pi\varphi=\iiint\frac{[\varrho]}{r}d\tau\quad(9)$$

and where the square brackets indicate retarded potentials.[3] The content of the scheme is then contained in the statement that the motion of the electron is such that

$$\iiint \varrho\left(\mathbf{E}+\frac{\mathbf{u}\times\mathbf{H}}{c}\right)d\tau=0$$

where the integral is extended over the electron in question, with the understanding that the values of **E** and **H** are to be calculated by (6) to (9), by taking account of the contribution from all the electricity, including that in the electron whose motion is under investigation. The law really expresses how the electrons move in relation to one another. The vectors **E** and **H** constitute the scaffolding for relating the motions of the charges. In practice, as already stated, the plan is to separate the integral into two parts—one corresponding to the field of all the electrons other than the electron under investigation and another part contributed by that electron itself, this latter part being then expressed as a func-

[3] The restricting relation $\operatorname{div}\mathbf{U}=-\frac{1}{c}\frac{\partial\varphi}{\partial t}$, which is not here written down, but which holds between φ and **U**, merely provides for the equation of continuity, in preventing us from assigning in our problem change motions and density changes which might violate that equation, an equation which is contained in (1) and (2) themselves. If we automatically provide for the equation of continuity in stating our problem, the restricting relation between φ and **U** need not be stated explicitly.

tion of the motion of the electron. In this way, the motion of the electron is related to the fields of the other electrons.

To any one who would object to the use of quantities, or symbols such as $\mathbf{E}$ and $\mathbf{H}$ in respect to which no definition has been given, let me cite the following very simple example. Suppose I desire to express the equation of a parabola in parametric form. I may write

$$x = u\,s \qquad (10)$$

$$y = \frac{1}{2} g\, s^2 \qquad (11)$$

and state that the desired curve, which is of course $x^2 = (2u^2/g)y^2$, is to be obtained by eliminating s. Nobody asks me what s is. If by chance they do, I reply that it is merely a parameter which becomes eliminated in the final result, and the equations which have been written down constitute a definition of a process of calculation rather than a relation between magnitudes. If I am pressed for a definition, I shall tell my inquisitor to take the first of the equations as the definition of s. He will then be happy, but will make a calculation no different from mine. His morale would probably be improved enormously if I permitted him to believe that the curve under discussion was that of a body with horizontal velocity u, falling with acceleration g, for he would then be able to regard s as the time, but that would not alter his curve.

In the same way as equations (10) and (11) define a process of calculating the relation between x and y, so equations (1) to (5) define a process of calculating the relation between the motions of the various charges of the system. If I am pressed for a definition of $\mathbf{E}$ and $\mathbf{H}$, I define them by equations (1) to (4) or, what is the same thing, by (6) and (7). If anybody asks me whether equations (1) to (4) are right, I do not know what he means. Of course they are right. They constitute the method of calculating $\mathbf{E}$ and $\mathbf{H}$. Truly, it may turn out that the quantities calculated may be of no use; and the essence of the whole scheme lies in the fact that it states that the motion of an electron may be evaluated in terms of the $\mathbf{E}$ and $\mathbf{H}$ so calculated from the motions of the other electrons. Any disagreement of the classical electrodynamics with the quantum theory or anything else is to be sought not in equations (1) to (4), which are usually regarded as most characteristic of it, but in the force equation (5).

Conservation of Energy in Electrodynamics and in Experience

Now it is a pure analytical consequence of the circuital relations that for any field obeying them,

$$\iiint \rho(\overline{\mathbf{u}.\,\mathbf{E}+\frac{\mathbf{u}\times\mathbf{H}}{c}})\,d\tau = -\frac{\partial}{\partial t}\iiint \frac{1}{2}(\mathbf{E}^2+\mathbf{H}^2)\,d\tau - c\iint (\mathbf{E}\times\mathbf{H})_n\, dS \qquad (12)$$

where the volume integral is extended throughout any volume, and the surface integral over the corresponding surface. This equation suggests what quantities it is appropriate to christen energy density, energy flux and so forth in order that the form of the principle of energy may be preserved. If all parts of an electron move with the same velocity so that the vanishing of

$$\iiint \rho(\mathbf{E}+\frac{\mathbf{u}\times\mathbf{H}}{c})\,d\tau$$

insures also the vanishing of

$$\iiint \rho(\overline{\mathbf{u}\cdot\mathbf{E}+\frac{\mathbf{u}\times\mathbf{H}}{c}})\,d\tau$$

for each electron, then the addition of the force equation (5) insures that the left-hand side of (12) shall be zero, and a christening of $c(\mathbf{E}\times\mathbf{H})$ as flux of energy per square cm and $\frac{1}{2}(\mathbf{E}^2+\mathbf{H}^2)$ as energy per cc leaves us with the statement that the inward flux of energy through a surface is equal to the rate of increase of energy within it, which energy in this case works out approximately as $\Sigma\frac{1}{2}mv^2$ for all the electrons within the surface. If all parts of the electron do not move with the same velocity, the left-hand side does not vanish, and what remains of it must be associated with the other volume integral as a rate of increase of the internal energy of the electron in order that the language of the conservation of energy shall be maintained. Now directly the force equation ceases to be true, and it certainly is not true in problems dealing with photoelectric effect and the like, all this evaporates. It even evaporates partially with the force equation holding, in the practical case where the velocities of different parts of the electron are not the same. This does not mean that any astonishing violation of the principles of nature is involved, but simply that, for the cases where the force equation does not hold, the quantity $\frac{1}{2}(\mathbf{E}^2+\mathbf{H}^2)$ and the Poynting flux $c\,(\mathbf{E}+\mathbf{H})$ are unsuitable quantities to have been christened as we have christened them. When we are speaking of the conservation of energy, I think we have to realize that the vital point at issue is this. Suppose we have a self-contained system in which are to be found a number of electrons moving with constant, though possibly different velocities, and we measure $\Sigma\frac{1}{2}mv^2$ or, if we like, the more elaborate expression

$$T=\Sigma\, m_0c^2\left[\left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}}-1\right]$$

corresponding to the theory of relativity. Then suppose that after these electrons have bombarded the various things around, and a steady state in which the velocity of each electron is once more constant has been attained, we once more measure T for all the electrons in the system. In the spirit of its practical application, all that conservation of energy requires is that the sum shall be the same as before. Now such a condition is provided for by the most radical forms of the quantum theory. When we speak of the quantum theory violating the spirit of the conservation of energy, our thoughts are apt to be concentrated upon the idea of how it may be possible for the Poynting vector of electromagnetic theory to pump as much $\Sigma\frac{1}{2}\,mv^2$ into some photoelectron as it appears to get. The trouble here is not with the conservation of energy, but with the picture of how we would like the process to occur. If it turns out that the things which long ago were christened energy flux and energy density do not have the properties which energy flux and energy density should have, the trouble is to be found in the christening.

THE PRESENT STATUS OF THE CIRCUITAL RELATIONS

If, however, the force equation is wrong, and that is the only part of the electromagnetic theory which is not scaffolding, what ought to be done with the scaffolding? May it not serve for another structure? It possesses some very desirable features. It is true that these features lie in the forms of the equations rather than in any connection which they may have with electricity. For the valuable features of the equations are preserved if we should replace ϱu_x, ϱu_y, ϱu_z and ϱ by anything else, as, for example, some set of quantities S_x, S_y, S_z, S_t. All that the equations will require of this set of quantities is that it shall obey the relation

$$\frac{\partial S_x}{\partial x}+\frac{\partial S_y}{\partial y}+\frac{\partial S_z}{\partial z}+\frac{\partial S_t}{\partial t}=0$$

which, in the electrical case, is the equation of continuity. The equations will talk as glibly in terms of the quantity S assigned as a function of x, y, x, and t as, in electromagnetic theory, it speaks in terms of $\varrho\mathbf{u}$ and ϱ.

The valuable feature of the equations is of course contained in those properties which, if we were speaking in a language which talked of transference of energy through an ether, we should term the properties necessary to produce transverse waves.

Now suppose that the equations be applied to any problem where interference patterns are obtained, such as the problem of a point source and a mirror. What certainly is true is that where the equations predict a zero value for $\mathbf{E}$ nothing ever happens, photographic, photoelectric or visual. Where they predict a value other than zero, something may happen; for example, a photoelectron may be ejected from an atom there. The unsatisfactory element in the situation lies in the fact that we should have preferred to say with certainty that the event will or will not occur. The indecision of the statement might be converted to certainty by supposing that something about the electron or the atom in which it existed determined whether it would or would not be ejected. This solution seems psychologically repugnant to the mind. For one thing it would violate the spirit of the conservation of energy, not necessarily, it is true, in a practical form in which we discuss it only statistically, but in a form which many would like to retain. For, according to it, that particular phenomenon which we speak of as the emission of a quantum from a radiating atom and caused possibly by the absorption from an electron of $\frac{1}{2}\,mv^2$ equal to $h\nu$ might, occasionally, be accompanied by the emission of more than one electron from other atoms, each with gain of $\frac{1}{2}mv^2$ equal to $h\nu$, although of course in the statistical case which concerned a large number of atoms, practical conservation might be obtained.

Not only does the scheme of circuital relations predict where photoelectric and other phenomena will certainly not occur and where they may, but the probability of occurrence of one of these phenomena in unit time in any element of volume as a result of the emission of a quantum of radiation from some point is, for a given frequency and type of phenomenon, proportional to the vector which on classical electromagnetic theory represents the Poynting flux of the primary wave there, although the actual $\frac{1}{2}mv^2$ gained by a single electron may be equal to the time integral over the period of emission of the surface integral of the Poynting flux over the whole surface of a sphere drawn with the source as origin.

The crude notion of a quantum of energy shot out from an atom in the form of a ball or of a wave filament provides a satisfying picture as regards an interpretation of the inner significance of the probability, which otherwise stands as a sort of confession of ignorance, and it provides a picture by which the mind can think of the energy as being handed on from point to point from source to goal. It does this, however, at the expense of enormous complications as regards interference. Moreover, it has other difficulties. In its primitive form, the small ball notion is inconsistent with interference experiments altogether; and as regards the filamentary wave, we have the difficulty pointed out by Lorentz that Michelson's experiments on interference over long paths show that the quantum must have a considerable length, and his measurements of the diameter of stars, in which interference is obtained between two portions of a wave front a considerable distance apart show that the quantum must have by no means a small width, so that altogether it is a pretty big thing. Then, the experiments of Wien and Dempster on the decay of the light emitted by positive rays during their journey along a vacuum tube suggest that appreciable emission occurs over a period of the order of 10^{-8} second. Yet nobody doubts that if he should shut off half of the path of emission, and use the light of the tail end of the pulse (if it is a pulse) to produce a photoelectric effect, he would get the full velocity of electronic emission predicted by the Einstein formula.

How then do we stand in this matter? One thing is certain. If we believe that the equations of classical electrodynamics mark out the space in such a way that where they predict darkness, nothing ever happens, any quanta emitted from the source must follow paths such that they never cross a region where darkness is predicted by the classical theory. In other words, the quanta must follow the paths of that vector which in classical theory was called the Poynting flux, and which even with the abandonment of its meaning as a flux of energy still has a definable direction and magnitude. A full appreciation of the artificiality of the belief that all things when left to themselves should travel in straight lines with constant velocities will leave one who is sufficiently sophisticated with no difficulty as to the logical possibility of the quanta traveling along paths which are not straight. To make a long story short it would appear that the following picture would provide a satisfactory correlation of most of the phenomena in terms which should be agreeable to one who wishes to retain the idea of a special handing on of energy from source to goal.

A Possible Correlation of Classical Theory with Quantum Emission

We recall that the quantum theory speaks in terms of the existence of certain stationary states of electronic motion in the atom, in which no energy is emitted, the emission of energy occurring only during the transition from one state to another. Now even in the stationary states there is no question of the existence of a field at external points in space. This is there as a matter of definition even though the field should not do anything. We are under no compulsion to imagine that the field is associated with the radiation of energy in the sense that the electron should have to alter its orbit. I am not particularly concerned with the question of whether this field is allowed to do anything or not. Perhaps there is something to be said on the side of consistency for allowing it to do something, so that the atomic orbits may influence each other, and provide, for example, for such a phenomenon as the magnetization of one substance by the presence of another, an effect ultimately attributable to interactions between the orbits in the stationary states. Any actions of this kind which we imagine to take place will have to be founded on equations of motion which give rise to no trouble. Thus, for example, the equation of motion

$$e\left(\mathbf{E}+\frac{\mathbf{u}\times\mathbf{H}}{c}\right)=\frac{d}{dt}\frac{m_0\mathbf{u}}{\left[1-\frac{u_x^2+u_y^2+u_z^2}{c^2}\right]^{1/2}}$$

for an electron, where $\mathbf{E}$ and $\mathbf{H}$ refer to the external field, is an equation consistent with restricted relativity and one which would permit the existence of the permanent orbital motion of an electron about a nucleus without diminution of amplitude, since the terms involving higher time derivatives which occur in the complete equations of classical dynamics but which have never been observed experimentally are absent. Interactions between different orbits may take place and will give rise to no trouble of a fundamental nature, except perhaps in cases of complete resonance. The acquirement of energy by free electrons as a result of the fields due to the orbital motions presents some difficulties in the matter of realization of a steady state. However, I do not intend to concern myself greatly with these difficulties. If they become too acute, the avenue of escape lies through denial of any influence between the electrons during their steady state motion, and the main thing which I desire to imply is that the course of events in the picture we have sketched is one of which we would have but little direct experimental knowledge. Hardly any of the phenomena with which our experiments deal would be realized. No electrons would be ejected from an atom so as to give rise to photoelectric effect, no photographic action would take place, and the eye, which operates, presumably, through photoelectric agency, would be unconscious of what was happening.

Now suppose that, owing to some agency, for example, the advent of a high speed electron from somewhere an electron transition from one orbit to another occurs in an atom, and suppose that somehow or other unifrequentic radiation is emitted during this process of transition. I should perhaps not use the words radiation of energy, for all I mean is that the field changes produced outside the atom are those corresponding to an electron moving with simple harmonic motion. We shall suppose that this field is calculable from strict classical theory; but, in calculating its effect on other atoms, we shall suppose the electrons there replaced by virtual linear oscillators in the sense prescribed by Bohr, Kramers and Slater.[4] The use of these virtual oscillators must, of course, be regarded as a temporary expedient designed to enable us to proceed in our speculations without awaiting a complete solution of the story of how the atoms influence each other. Through the agency of these virtual oscillators, as Bohr, Kramers and Slater show, all the phenomena of classical theory, such as reflection, refraction and polarization, take place. The virtual oscillators of the reflecting mirror produce waves which conspire with the waves originating in the primary transition in such a way as to produce reflection. If there be polarizing media present, the electronic motions set up in these are such as to cooperate with the primary field, again in true classical manner. The peculiar thing about the whole situation is that we should be all unconscious of it. For none of these phenomena are associated with things we observe. It is not necessary for us to regard these fields as carrying energy and therefore invite trouble in the matter of accounting for it. But now suppose that during the transition something else happens. Let us suppose that a quantum, in the form of a ball if we like, is emitted from the electron, and let us suppose that its probability of emission in a fixed element of time, at any instant, and from any element of surface, is proportional to the magnitude, at that time and place, of the vector, which in classical theory would be the Poynting vector associated with the *irreversible radiation field* of the virtual oscillator,[5] the magnitude of the quantum being, of course, $h\nu$. The factor of proportionality in the expression for the probability is of course such that the probability that quantum is emitted somewhere at some time during the transition is unity. Let us suppose that the quantum then follows the path of the Poynting vector[6] emanating from its point of emission, this Poynting vector being constructed from the resultant field composed of the radiation field associated with the initial transition and all the secondary fields to which it gives rise through its action on the virtual oscillators in the other atoms which are influenced by it. If, following the line of a particular Poynting vector, the quantum comes into the vicinity of one of the electrons of a virtual oscillator, we expect that it may be caught by the electron in such a way as to enable it to induce a transition there if its energy is greater than that necessary for the transition. The effects of this transition, at least in the limiting case of ionization, are the things physically observable as photographic, photoelectric or visual effects.

As regards their practical aspects, all interference phenomana follow as in the classical theory, except that there is no interference of quanta. At the places where the classical theory indicates darkness there is darkness on the present view because no quanta succeed in getting there. Polarization phenomena do not invoke any polarization in the quanta themselves, but are provided for by the polarization of the vectors **E** and **H** as calculated by the classical process. All that is concerned in the question of whether or not quanta shall enter any region of space is the question of whether or not there is any path leading there from the place of the original transition by way of a finite Poynting vector.

An experiment such as that of Wien on the decay

[4] The quantum theory of radiation, *Phil. Mag.*, S-6, Vol. 47, pp. 785–802, 1924.

[5] It may be a comfort to some to regard the field originating in the transition and operating on the virtual oscillators as consisting of vectors of a kind different from those to which the electrons give rise during their motions in their stationary states, although we may suppose them to obey the same laws in relation to their determination in terms of the singularities which produce them. One may even go so far as to postulate, as operating to produce the field associated with the transition, a singularity other than the electron, which singularity moreover performs the function of the moving element in the virtual oscillator. Its binding to the ordinary electron which makes the transition exists simply in its acting as a sort of trustee for the quantum which it receives from the electron and disposes of according to its own judgment as specified in the law of probability here stated. It may be that we shall have to look to the nucleus for the seat of the virtual oscillator.

[6] Since this was written, there has come to my attention a paper by Louis de Broglie, *Phil. Mag.*, S. 6, Vol. 47, pp. 446–458, 1924. In this paper the author presents the idea of light quanta following the paths of the "rays" of optics, an idea related in some degree to the above. The details of the theory are, however, quite different from that here sketched. I must also mention that during the meeting at which the above address was presented, I learned in conversation with Dr. Slater that, in work as yet unpublished, he had given some attention to the matter of light quanta following the Poynting vector.

of intensity along the paths of the positive rays receives a natural explanation from the fact that during the emission of the unifrequentic radiation the amplitude of the vibrations and the magnitude of the Poynting vector diminishes, so that in regions of the beam far from the point of excitation the observed intensity is small because the Poynting vector is small there, and the probability of emission of the quantum at those places is small compared with its probability of emission at an earlier stage. The quantum, for a given frequency, is the same thing wherever it is emitted, however, so that its property of ejecting electrons with the true Einstein frequency does not depend upon the point of emission along the beam, and of course only one quantum is emitted for each transition. It is of interest to observe that the Wien experiment points to a real damping of the unifrequentic oscillation of the virtual oscillator.

The picture here presented permits us to indulge in a rather closer speculation as to the mechanism of the processes involved. For, if, in free space, we travel with the energy along a tube whose walls are composed of the Poynting vector, and measure the area of cross-section of the tube as we go along, the product of the area and the Poynting vector will be constant, so that the probability that a quantum emitted with and following the Poynting flux shall pass within any given area perpendicular to the flux, in any given time, is proportional to the quantity which on classical electromagnetic theory would represent the energy which has passed through that area in that time. Now, what happens when absorbing atoms are present in the field? Suppose we imagine such an atom replaced by a virtual oscillator and consider the vibrating electron acting as one constituent of the oscillator. The motion of this electron under the influence of the field will of course bring about such a resultant orientation of the Poynting vector about itself as to cause "energy"[7] (in the classical sense) to feed into its vicinity.[8] The greater the absorption of energy in the classical sense, the greater the probability that the quantum emanating from the original center of disturbance shall be directed towards the absorbing atom in such a way as to produce a transition. In the case of resonance of the virtual oscillator, the probability that the particular atom would receive the quantum would be enhanced, since resonance enhances the flux of the Poynting vector into the region occupied by the oscillator.

A complete theory following the lines above indicated would involve specification of the conditions which determine the capture of a quantum by the atom. One is immediately tempted to set up a correlation between the absorption of energy by the oscillator in the classical sense (the fizzling out of the Poynting vector in the regions around the oscillator), and the capture of the quantum. Detailed discussion of this matter is impossible without going into great length. However, it would appear that reasonable assumptions, designed to maintain the principle that the probability of the passage of a quantum through a given element of area shall be represented by the ratio of the time integral of the total Poynting flux across that area to the time integral of the total Poynting flux from the original center of disturbance, would also provide for the conclusion that the probability that a quantum of suitable energy would produce a transition in any given atom is equal to the ratio of the "classical energy" which that atom would absorb from the wave train emitted from the center of disturbance to the total "classical energy" of that wave train. Such a law would appear to be in harmony with experience.

One may demur against our refusal to regard the Poynting vector itself as a true flux of energy, on the basis that the parts of the virtual oscillator are actually set in motion in such a way as to relate their energy (in the $\frac{1}{2}mv^2$ sense) to what on classical theory would be the energy flux represented by the Poynting vector. The objection is to be met by the statement that this motion disappears as the oscillation of the virtual oscillator dies down. It never becomes apparent to us, and does not figure in a scheme which regards the only experimentally recognizable phenomena as those resulting from ejection of electrons from their orbits, or more generally the acquirement of energy by electrons in quanta.

An interesting situation arises when the quantum, following the path of the Poynting vector, gets caught in the vicinity of an atom and finds itself without enough energy to produce a transition, or when it gets caught in the vicinity of a free electron. We must suppose that, in the former case, it becomes scattered, unaltered, while in the latter case it gives rise to the Compton effect. A characteristic feature of the Compton effect is the emission of different

[7] We use the term "classical energy" for want of better words to denote the integral of $1/2(E^2+H^2)$ over the region concerned, although, as already implied, we are not regarding this as in any way the equivalent of the energy transferred by the quantum. In fact we are not regarding it as energy at all.

[8] Since this address was delivered my attention has been called to a paper by Richard Becker, *ZS. f. Phys.*, 27, 3, p. 173, 1924, in which the probability of a transition is made to depend upon the surface integral of the Poynting flux taken all over a sphere surrounding the atom. The author does not appear to use the idea in connection with quanta following the Poynting vector, however.

frequencies in different directions. The situation in this connection has been discussed by Bohr, Kramers and Slater,[9] who, in representing the action of the free electron, invoke a moving linear oscillator whose Doppler effect gives rise to the different frequencies, in different directions, and who, in representing scattering by an atom, invoke (presumably) a linear oscillator emitting a frequency corresponding to the primary quantum and modified now to a negligible extent by the Doppler effect. Leaving the justification of these assumptions to the arguments presented for them in the paper referred to, we may incorporate the result in a theory of quanta of the type here sketched, for the purpose of correlating the phenomena of the Compton effect and atomic scattering, by supposing that the energy of the original quantum, having been received by the system, becomes available for total or partial reemission in varied amounts, the probability of emission of a quantum from the virtual oscillator at any point of the surface of its oscillating electron in unit time being proportional to the magnitude of the Poynting vector corresponding to the irreversible radiation emitted from that point, and the magnitude of the quantum emitted being determined by the frequency of the wave emission from that point, the subsequent history of the quantum after emission being determined in its relation to gratings, prisms, etc., upon which it may fall by the same sort of considerations as those which determine the history of the primary quantum.

It is then necessary to endow the quanta with the characteristics of momentum in such a way that the energy and direction of emission of the electron associated with the scattering follows as in Compton's calculations. The feature which a view of this kind adds to the more primitive theory is first, in harmony with the view of Bohr, Kramers and Slater, its consistency with the undulatory phenomena required by the properties of the scattered radiation in relation to its analysis by a grating, and second, its formal attempt to assign definite probabilities to the scattering of quanta of different magnitudes, and in different directions, these probabilities being calculable in terms of the magnitudes of the Poynting vector at the appropriate point of emission in the sense outlined above.

Apart from its power to provide a visual picture of the passage of energy from one place to another, its power to give through the law determined by the Poynting vector a physical interpretation of the probability of a transition induced by the quantum, and a numerical magnitude to that probability, the concept of a quantum operating according to the laws above described provides the feature of an instantaneous emission of the energy, in spite of the association of that emission with a wave emission extending over a finite time. Such a feature as the last named seems essential to a satisfactory explanation of the fact that, in such a phenomenon as that of the Wien experiment above cited, the "light" emanating from any part of the track of the positive ray, no matter how small, possesses the power to produce the full Einstein photoelectric velocity in any electron which it ejects from an atom in its path.

Finally, it may again be emphasized that if we decide to talk in terms of quanta at all, and accept as an experimental fact that no quantum ever crosses a region where the classical theory predicts darkness, in other words, if we accept the classical theory as an empirical description of interference phenomena and the like, we practically constrain the quanta to follow the paths of the Poynting vector in the sense outlined above.

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(To be concluded)

[9] *Loc. cit.*

THE BIOLOGICAL SURVEY OF THE MOUNT DESERT ISLAND (MAINE) REGION[1]

THIS survey was undertaken in the season of 1923 at the suggestion of Mr. William Procter, a member of the board of trustees of the laboratory. The object of the survey is to gain a knowledge of the flora and fauna of the region, principally the marine forms, which will be of use to the scientific research workers who contemplate coming to work at the laboratory, as well as to present a picture of the ecology of the forms, the numbers as to kinds and individuals, their distribution with regard to season of year and over periods of years, kinds of water and bottoms that they live in, temperature conditions that influence them, their feeding habits, mating habits and seasons, habits of offense and defense and other ecological relationships.

The work may be divided into two more or less distinct parts: First, a series of intensive surveys by individual workers of restricted areas, and second, a more general and comprehensive treatment of the whole area which will take a much longer period of time but from which interesting results have already become apparent.

In the first part the following papers have already been published or are in course of preparation:

[1] By the members of the staff of the Mount Desert Island Biological Laboratory and Associated Naturalists.