duct can be easily seen. The primary and secondary tubules are so isolated that little overlapping occurs and a clear conception of the form and extent of an individual tubule is obtained. In this respect the preparation made by intraperitoneal injection is much more instructive than one made by injections through the Wolffian duct in which every tubule contains more or less color mass.

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SPECIAL ARTICLES

WAVE-LENGTH SHIFTS IN THE SCATTERING OF LIGHT[1]

BOHR[2] has called attention to a remarkable prediction made by Smekal[3] and by Kramers and Heisenberg (unpublished), to the effect that when monochromatic light falls on a multiply periodic electromagnetic system, the scattered radiation may contain not only the incident frequency, but also combinations of this frequency with those characteristic of the scatterer. The purpose of this note is to direct attention to a conclusion pointed out by one of the writers some two years ago that the wave-length shift may be very large in certain favorably chosen cases of the scattering of ordinary light. It appears that a part of this frequency shift is the quantum analogue of the phenomenon predicted by Kramers and Heisenberg. In general, the wave-length change is due to three influences—the Doppler shift caused by the initial motion of the atom, the Compton shift due to the recoil of the atom or the ejected electron, and the shift due to alteration in the internal energy of the atom. To a first approximation these are additive.

A. H. Compton's original theory[4] of the wave-length shift of scattered radiation was based on the assumption that each unidirectional quantum of incident radiation is deflected by a single electron, which recoils from the momentum of the quantum. For scattering at angle φ with the primary beam the increase in wave length (in centimeters) should be

$$\Delta\lambda = \frac{2h}{mc}\sin^2\frac{\varphi}{2} \tag{1}$$

where h, m and c are Planck's constant, the mass of the electron, and the velocity of light, respectively. In Ångstrom units the shift is

$$\Delta\lambda = .0484 \sin^2\frac{\varphi}{2}$$

Ross[5] attempted, without success, to observe this shift in the visible spectrum by scattering the light of the mercury line 5461 A from paraffin, and by multiple reflections from silvered glass.

We shall not attempt to discuss the latter experiment, but there are at least two possible reasons why no shift of the order of .02 A was observed in the case of paraffin. First, it seems improbable that there are "free" electrons in paraffin, and presumably light of wave length 5461 A is unable to eject electrons from atoms of this substance. Under such conditions the scattering must be attributed to the atom as a whole (or even to larger aggregates) and the shift is inappreciably small. Second, even if electrons were ejected, the shift should not be given by the equation (1); for in its derivation both the work required to separate the scattering electron from its parent atom, and the final momentum of the atom, were neglected. Compton[6] has taken these influences into account in a revised theory. There are not enough equations to determine all the unknowns in this problem. For example, the recoiling atom and electron as well as the scattered quantum will in general possess angular momentum with respect to the center of gravity of the system. The distribution of this angular momentum can not be specified unless we know the dynamics of the collision. The result is that the momentum of the atom and the direction cosines of its trajectory, which are unknown, appear in the formula for $\Delta\lambda$. In all probability these quantities are dependent on the relative phases of the incident quantum and the internal motions of the atom, so that $\Delta\lambda$ is not a constant for a given frequency, angle of scattering and material. This circumstance is probably contributory to the fact that the shifted peak is wider than the unshifted one. At any rate, the formula shows that $\Delta\lambda$ lies between infinity and $\lambda^2/(\lambda-\lambda_s)$, where hc/λ_s is the ionization potential for the atomic energy level from which the scattering electron is ejected. That is, $\Delta\nu$ lies between $-\nu$ and $-\nu_s$. In unpublished calculations of Ruark and Ellett these results are still further extended, though the numerical result is not changed appreciably. A formula is derived for the frequency ν_2 obtained when a needle quantum of frequency ν_0 falls on an atom and is scattered without ejecting an electron. Let E_0 be the rest-energy of the atom and let it have a velocity $v_0 = \beta_0 c$, making an angle ϑ_0 with the direction of the quantum. E_2, β_2 and ϑ_2 denote the corresponding quantities for the final state of the

[1] Published by permission of the Director of the Bureau of Standards, U. S. Department of Commerce.

[2] *Naturwissenschaften* 12, 1115, 1924.

[3] *Naturwissenschaften* 11, 873, 1923.

[4] *Bull.* Nat. Research Council, Vol. 4, No. 20, 1922.

[5] *Proc. Nat. Acad. Sci.*, 9, 246, 1923.

[6] Compton, *Phys. Rev.*, 24, 168, 1924.

atom, while E_1 is the rest-energy which the atom would possess if it absorbed and retained the incident quantum. Then

$$\nu_0 \frac{1-\beta_0 \cos \vartheta_0}{\sqrt{1-\beta_0^2}} \frac{2hE_0}{E_1^2-E_0^2} = \nu_2 \frac{1-\beta_2 \cos \vartheta_2}{\sqrt{1-\beta_2^2}} \frac{2hE_2}{E_1^2-E_2^2}$$

When the values of β_2, E_1, etc., obtained from the solution of the dynamical problem are put in this equation, it specifies the entire frequency change caused by the initial motion of the atom, its recoil from the quantum and the change in its quantized energy. Just as in Compton's revised theory, the shift will be indeterminate if we suppose that an electron is ejected.

It is quite possible that this wave-length change is the cause of some of the mysterious continuous spectra lying in the region of the ordinary spectrograph. Suppose, for example, that a vapor for which $\nu_s = 40{,}000$ cm.$^{-1}$ is illuminated with light of wave length 2,000 A. That part of the scattered radiation which is not of wave length 2,000 A will lie at frequencies lower than 10,000 cm.$^{-1}$, corresponding to 1 μ. In other words, it will appear as radiant heat. Shorter incident wave lengths might give a spectrum extending into the visible. Therefore, it seems important to photograph the spectra of vapors which are scattering intense radiation of short wave lengths.

It may be pointed out that spark lines having frequencies greater than the highest series limit of the neutral atom should give rise to a scattered line spectrum provided that $\Delta\lambda$ does not depend on the relative phases of the atom and the incident quantum. Spectrum lines of this character would be strictly analogous to the displaced lines of Kramers and Heisenberg. Apparently such lines do not exist in the spectra of the alkaline earths.

It remains for us to consider whether the scattered combination frequencies of Kramers and Heisenberg can be detected experimentally. When sodium vapor is illuminated with light of the second member of the sodium principal series, the resonance radiation contains both the first and second members of the series (and presumably 4 pairs of lines in the infrared). Bohr states that phenomena of this kind constitute a special case of those predicted by Kramers and Heisenberg. Here the D lines are to be considered as a difference frequency, in accordance with the equation

$$1s - 2p = (1s - 3p) - (2p - 3p).$$

The emission of a summation frequency may be obtained by illuminating mercury atoms in the $2p_1$ state with light of the line $2p_1 - 3d_2$, for example. The emitted light will contain $2p_2 - 3d_2$, and

$$2p_2 - 3d_2 = (2p_1 - 3d_2) + (2p_2 - 2p_1).$$

In these instances the atomic resonator is so sharply tuned that a very slight deviation from the correct frequency causes the amount of scattering to decrease tremendously. Similar remarks apply to molecular resonance spectra excited by monochromatic light. It seems very doubtful that the combination frequencies can be detected in the case of ordinary Rayleigh scattering.

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AMERICAN ASSOCIATION FOR THE ADVANCEMENT OF SCIENCE

BOTANICAL SCIENCES AT THE WASHINGTON MEETING

(*A report for Section G appeared in Science for February 6.*)

The Botanical Society of America

President, William Crocker.

Secretary, Ivey F. Lewis, University of Virginia, University, Va.

(*Report by Ivey F. Lewis, unless otherwise noted*)

The Washington meeting of the Botanical Society of America was the most largely attended and one of the most successful in the history of the society. The names of 88 new members were added to the roll of the society, and the following corresponding members were elected: Professors V. H. Blackman and A. C. Seward, of England, and Karl Goebel, of Germany. The officers elected for 1925 are: J. R. Schramm, *president*; W. J. V. Osterhout, *vice-president*; G. E. Nichols, *treasurer*. C. E. Allen was elected to represent the society in the National Research Council, Division of Biology and Agriculture, and H. C. Cowles was elected to the editorial board of the *American Journal of Botany*. The society endorsed the proposed deep sea exploration of the United States Navy. Forward steps were also taken in conjunction with the Ecological Society of America and the American Phytopathological Society, looking toward the convening of an International Botanical Congress in Ithaca in August, 1926. The exhibits by members proved exceptionally interesting and attractive. These included, among others, cytological preparations by W. R. Taylor, R. E. Cleland and A. M. Showalter, and a very fine exhibit of plant pigments by F. M. Schertz. The Botanical Society will meet with the American Association in Kansas City in 1925. The program, as usual, was carried on by sectional groups.