sistant professor of petroleum engineering at the Colorado School of Mines.

JAMES MONTAGU FRANK DRUMMOND, director of research for the Scottish Society for Research in Plant-breeding, has been appointed Regius professor of botany in the University of Glasgow, in the room of Professor Bower, who has resigned.

DR. WILHELM EITEL, professor of physico-chemical mineralogy and petrography at Königsberg, has been invited to the chair of mineralogy at Freiburg-im-Breisgau.

## DISCUSSION AND CORRESPONDENCE

### THE POPULATION OF CANADA

MALTHUSIANISM has returned. The discussion of the problem of population is widespread. East's "Mankind at the Crossroads" and the scientific papers by Pearl and Reed now available in Pearl's "Studies in Human Biology" forecast populations or tell us of the dire consequences that may soon ensue. This summer in the Institute of Politics at Williamstown and in the meeting of the British Association at Toronto the problem was thoughtfully discussed. I heard at Toronto G. Udne Yule's clear and sympathetic exposition of the Pearl-Reed curve $1/P = A + Be^{-nt}$ followed by an expression of scepticism on the part of many economists. In conversation it was recalled that according to press reports Lloyd George when in Canada not so many months ago predicted a future population of 300 million for that great dominion. This seemed somewhat extravagant in view of the Pearl-Reed forecast of only 197 million for the United States and I resolved to examine the figures by the method of these scientists to check up on the politician and statesman.

We have had eight censuses in Canada decennially, commencing in 1851. The figures in millions are:

| 1851 | 1861 | 1871 | 1881 | 1891 | 1901 | 1911 | 1921 |
|---|---|---|---|---|---|---|---|
| 2.38 | 3.16 | 3.69 | 4.32 | 4.82 | 5.37 | 7.21 | 8.79 |

By various methods I reached the solution

$$1/P = -.0286 + .3967\ e^{-.0157t}$$

which was checked by Dr. W. J. Luyten of the Harvard College Observatory by treating it as a first approximation to be corrected with a least-squares solution. His result was

$$1/P = -.031 + .430\ e^{-.0155t}$$

t being measured in years from 1851. The solutions look decidedly different, but we must remember that in a minimum problem a considerable variation of the variables (here, A. B, n) may not effectively change the minimum value. For the two equations the fit is in fact equally good. The departures between the observed and the fitted values were in the 8 cases, respectively, about +5, −6, −5, −4, +3, +10, 0, 0 per cent., giving a mean algebraic error of less than one half per cent. and a mean arithmetic error of 4 per cent. Forecasting from the equations, it appears that about the year 2013 the population of Canada reaches the value 300 million set by Lloyd George.

Further, it appears that on some day apparently in the year 2020 the Canadian population will become infinite.

*Dies irae, dies illa!*

Somewhat disconcerted by this result I tried to fit the curves

$$P = d + \frac{1}{A + Be^{-nt}}$$

$$P = \frac{k}{1 + me^{at + bt^2 + ct^3}}$$

which are also used by Pearl and Reed when they desire a better fit than that given by the simple 3—constant equation. The last equation has five constants to fit to eight points. The results, however, remained disconcerting.

*Solvet sæclum et favilla—*

But I am not a good curve-fitter.

Now what, if anything, may these results mean? First, they can not in any way impugn Pearl's biologic postulate that populations must tend to saturation. Canada will never have an infinite population, nor yet 300 millions, nor perhaps more than one fifth of that figure. Second, they do not cast doubt upon his findings that experimental (laboratory) populations do follow his growth curves, nor upon his further demonstrations that human populations (for so long as good censuses have been available) in many cases have followed his curves with a high degree of fidelity and already disclose a tendency to saturation. The results do show, I think, that the Canadian population (and other instances could be given) so far fails to indicate approaching saturation as actually to show a runaway tendency.

And why should this not be so! The Malthusian dictum that population tends to increase in geometric progression whereas food tends to an arithmetic increase really implies in its latter half the economic law of diminishing returns (as applied to agriculture). It would be as silly to doubt this law, which is analogous to that of Chatelier in physical chemistry, as to question the saturation of population or the fact that increasing the pressure diminishes the volume. But all such laws are based upon the underlying conception of an (at least approximate) equilibrium, and in fact may be taken as definitions of states of equilibrium. The principle "To him that hath shall be given" is not a definition of equilibrium nor an illustration of the law of diminishing returns!

We have in physical chemistry conditions leading to explosions and in economics we have had for a century similar conditions wherein the law of diminishing returns is in abeyance. Given on the one hand concentrations of population with more or less fixed habits and on the other hand extensive new agricultural territory to bring under cultivation, it is not necessary that more persons to feed should make it harder for all to get food—they may make it easier, for a million new farmers in Canada may supply not only themselves but from their surplus an additional million or more in Europe. Hence the population curve for Canada may well for a time show an explosive rise whether by an unusually large birth-rate or by an eager immigration or by both. Much the same conditions and results may flow from an industrialization; replacing some human labor by the work of tools frees human time for the invention of still more tools to relieve more human labor. The drift to the cities has been another case of "To him that hath shall be given."

Only the unduly optimistic will believe that there is no limit to this sort of process and that the law of diminishing returns will never again be invoked nor the world return to a state approximating equilibrium. In fact, the tendency of most populations to saturation shows that even now and during the past century of feverish development alike of new land and of new machinery, whether of manufacture or of trade, that law has still held general regulatory sway over human increase, although in different populations at different times the sort of variation now shown by Canada has been seen for several decades on end.

Not unlikely Pearl and Reed or Yule may feel that it is pushing the formula pretty hard to carry it over to negative values of A and forced or accelerated instead of retarded populations. I admit it. I have my serious doubts about the applicability of the growth curves to human populations. Some of these doubts I have taken up in a paper (with Dr. W. J. Luyten) for the autumn meeting of the National Academy of Sciences. There is no good theoretical foundation for the curves. The simplest one $1/P = A + Be^{-nt}$, used above, is the law of autocatalytic reaction and as such was suggestive to biochemists relative to growth of organisms—children, tree shoots, etc. The analogy between an autocatalytic reaction and a living organism is not good, nor particularly bad, but the curve fits the growth of an organism tolerably well. The analogy between an organism and a society has its weak and its strong points; the curve still fits tolerably and in some cases surprisingly well.

There are analytical reasons that may be adduced to favor the curve within certain limits; this was, if I recall correctly, the point of view taken by Yule at Toronto. If it be admitted that the rate of growth of a population or organism at any time is dependent chiefly on its size at the time we have

$$\frac{dP}{dt} = f(P) = a_0 + a_1P + a_2P^2 + \ldots,$$

where $f(P)$ has been expanded into a Maclaurin series. Now as there is no growth with $P = 0$ we infer $a_0 = 0$. A first analytic approximation is

$$\frac{dP}{dt} = a_1P \text{ or } P = P_0e^{at}$$

which is the Malthusian law of geometric growth. If we retain two terms

$$\frac{dP}{dt} = a_1P + a_2P^2,$$

and with $a_2$ negative we have the autocatalytic law with a limiting population of $L = - a_1/a_2$, a growth retarded by the law of diminishing returns, by Malthusian checks of some sort. It is however analytically possible to consider the case where $a_2$ might be positive and the growth is accelerated; the solution is then of the form found above for Canada, but, as the condition is unstable, terms of higher order in P must ultimately come in to check the growth and lead to the stability of saturation.

Another way to put the analytic argument is this: The function $f(P)$, since it vanishes when $P = 0$, may be written as $f(P) = P\,\Phi(P)$. If there is to be saturation $f(P)$ and $\Phi(P)$ must also vanish when $P = L$, the limiting population. Hence we may write $f(P) = P(L - P)\Psi(P)$. Then expanding $\Psi(P)$ we have

$$\frac{dP}{dt} = P(L - P)\ (a_0 + a_1P + \ldots).$$

The first approximation is to stop with the first term $a_0$ and the autocatalytic law. A second approximation would leave the term $a_1\,P_1$; then if $a_1$ is positive and greater than $a_0/L$ there will be an initial explosive growth followed by saturation. A purely analytic argument is, however, not worth very much —it shows possibilities rather than actualities—and is here adduced merely to show that the apparently disconcerting results for Canada do fit into the same sort of analytic system as satisfactory results obtained for other populations. This is fortunate if there are as there may well be cases of temporary explosions of population growth.

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## HAPLOID MALES IN PARACOPIDOSOMOPSIS

SEVERAL years ago one of us (Patterson, 1917)[1] called attention to the fact that in certain species of parasitic Hymenoptera a majority of their polyembryonic broods are mixed, that is, contain both males and females. In a species showing polyembryonic development we should expect to find all the individuals belonging to one sex, provided the entire brood is derived from a single egg. One of the most striking cases of mixed broods is seen in Paracopidosomopsis floridanus (Copidosoma truncatellum). About 87 per cent. of the broods of this species are mixed, and a majority of such broods have less than 6 per cent. of males. In one case a single male was found in a brood of 1,550 individuals.

The rule among polyembryonic species is that the fertilized egg produces females, while the unfertilized egg develops into males. The usual explanation offered to account for the appearance of a mixed brood is that it has come from two (or more) eggs, one of which is unfertilized. In dealing with the data on the sexes of the species under consideration, the senior writer pointed out the difficulty of applying this hypothesis, and suggested that a mixed brood might arise from a fertilized egg during the early cleavage stages, by the loss of a sex chromosome in one or more blastomeres. Such a blastomere would have the diploid-minus-one number of chromosomes and might give rise to a group of males. It was further pointed out that this suggestion could be tested out by making a cytological study of the germ cells of the males appearing in mixed broods. For several years we attempted to secure the necessary material for such a study, but not until last fall (1923) did we succeed in obtaining favorable preparations.

The object of this note is to record the results of our observations. We find that the males from mixed broods possess the haploid number of chromosomes, and not the approximate diploid number. The spermatogenesis in these males is identical with that in males reared from unfertilized eggs (Patterson and Porter, 1917).[2] Our observations leave no doubt as to the fact that males in mixed broods are haploid, and this is true even in broods showing a very low percentage of males.

While the discovery of these facts settles the question of the number of chromosomes in males in mixed broods, it does not, however, definitely determine the manner of origin of mixed broods. To some it might seem to indicate that all mixed broods are to be explained on the two-egg hypothesis, but there are several facts in the development, at least of Paracopidosomopsis, that are difficult to explain on such a basis. Chief among these are (1) the appearance of asexual larvae, and (2) that while a pure male brood reared from an unfertilized egg is about as large as a pure female brood, yet if the unfertilized egg is associated with a fertilized egg in the same host it produces a few males only—sometimes but a single individual.

For the final solution of some of these problems we must look to the study of some of the more primitive polyembryonic species, such as that studied by Leibly and Hill, 1923.[3]

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[1] *Biol. Bull.*, XXXII, p. 291.

[2] *Biol. Bull.*, XXXIII, p. 38.

[3] *Jour. Agr. Research*, XXV, p. 337.

## AN ANATOMICAL SPELLING MATCH

WHEN the freshman begins his work in the anatomy laboratory he frequently fails to grasp the importance of using all possible channels for gaining the necessary anatomical information, and frequently the story of the test given Vesalius, in Paris, has been described to the students. Richardson, in his book "Disciples of Aesculapius," tells that Vesalius "was closely blindfolded. Then every bone of the body that could be distinguished by the touch was put into his hands and by the sense of touch he was able to name every bone correctly. . . ." Recently, some members of the class asked if they might not try this same test and now two classes have tried this "anatomical spelling match" and so I am passing it on for what it is worth.

The students choose two leaders and they in turn select two groups, thus dividing the class or laboratory section into two teams, as is done in the regular spelling match or in the chemical spelling match, of which so much has been heard lately. The two groups arrange themselves in two rows with their backs towards the opposing sides so that the bones may easily be placed in their hands held behind their backs. This is as satisfactory and takes much less time than blindfolding all the contestants. The one in charge passes down between the two rows and places a bone in the hands of each contestant, one after another, alternately from side to side.

We have found that only one or two men can be