common save **v**, *and if* **c** *is a third ideal, then if* **bc** *is divisible by* **a**, *the ideal* **c** *is divisible by* **a**.

Theorem II.—If **a** *and* **b** *are two integral ideals that are relatively prime and if* **a** *and* **c** *are two integral ideals that are relatively prime, then the greatest common divisor of* **a** *and* **c**, *this common divisor of* **a** *and* **c**, *this common divisor being* **v**.

Theorem III.—An integral ideal **a** *is divisible by only a finite number of other ideals.*

If an integral ideal **a** has the property that it is divisible by itself and by no other ideal save **v**, it is called a *prime* ideal.

Theorem IV.—If a product of several integral ideals is divisible by a prime ideal, one of the ideals is divisible by this prime ideal.

Theorem V.—The Fundamental Theorem. Every integral ideal that is not **v** or a prime ideal may be *factored into a product of prime ideals and this factorization is unique.*

Observe that every algebraic integer when multiplied by **v** is a principal ideal and that the above theorems are applicable to it.

The integral ideals constitute one branch of the general theory of moduls. This general theory in its incipience comprises the Kronecker modular systems and indeed many other branches of mathematics that emanate from the general realms of rationality and include the Minkowski geometry of numbers, the treatment of the moduli of periodicity of the Abelian Integrals, etc.

As a rule the text-books on the usual theory of numbers make the positive integer the starting point and the theorems regarding such integers form the foundation of the theory; it appears also that the text-books on the theory of algebraic numbers are going to start with the integral ideal. It should be emphasized that such ideals have their general setting in the general modul theory just as *number* is the more general concept of the usual positive integer.

The theory as outlined above may be made dependent upon the fundamental theorems of Dedekind as given by him in the "Begrundung der Idealtheorie. Göttingen-Nachrichten 1895."

I. *If the ideal* **c** *is divisible by the ideal* **a**, *there exists an ideal* **b** *such that* $\mathbf{c} = \mathbf{ab}$.

II. *Every ideal may be changed through multiplication by a properly chosen ideal into a principal ideal.*

III. *Every finite modul that is different from zero may through multiplication by a properly chosen modul be changed into a modul which contains the number 1 and further consists of only integers.*

IV. *If* $\alpha_1, \alpha_2, \cdots, \alpha_n$ *denote any n numbers that are not all zero of a realm* Ω *of the nth degree, it is possible to derive by rational operations n other numbers* $\beta_1, \beta_2, \cdots \beta_n$ *of* Ω *which satisfy the two conditions, first that* $\alpha_1 \beta_1 + \ldots + \alpha_n \beta_n = 1$, *and secondly, that the* n^2 *products* $\alpha_r \beta_s$ *are all integers.*

If any three of the above theorems are proved, the fourth follows as a consequence.

Observe that throughout the entire discussion of this article a fixed stock-realm R has been the realm of reference. This stock-realm was the usual realm of rational numbers. The theorems derived have been for the numbers of another realm, say Ω_1, which was deduced by adding (adjoining) to R an algebraic quantity. This algebraic quantity was in turn the root of an algebraic equation whose coefficients were rational numbers (and that is, numbers of R). It is possible to introduce a third realm Ω_2 which bears towards Ω_1, the same relation as Ω_1 had with respect to R, and so on indefinitely. Instead of the ideals that are introduced for Ω_1 other (more general) ideals exist for Ω_2 through the introduction of more general norms, discriminants, etc. It may be proved that the same rules, laws and principles exist in the more general realms as were true in Ω_1. And thus it becomes manifest that the principles of arithmetic are true universally and that is, in any algebraic realm whatever taken with respect to any arbitrary algebraic stock realm as realm of reference.

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SCIENTIFIC EVENTS

THE NANSEN POLAR EXPEDITION

THE Christiania correspondent of the London *Times* writes: The news that Dr. Nansen, after nearly 30 years spent in labor far away from the Arctic, will again return to the work of his youth is sure to attract general attention. Dr. Nansen has taken his decision. He will not only join the North Pole Expedition of the German Commander Bruns, but he will become its leader. By his famous expedition with the *Fram* in the years 1893–96, Dr. Nansen gained a reputation which, coming after his crossing of Greenland, placed him in the highest rank of Arctic explorers, and his interest in the North Polar basin has not waned.

Just as Dr. Nansen in 1893–96 had no ambition of reaching the North Pole apart from scientific exploration, so he is without this ambition on this occasion also. At the meeting of the Geographical Society he declared the flight over the Pole to be a matter of secondary importance, and in a subsequent interview he expressed the hope that Captain Amundsen will reach that goal next summer. Dr. Nansen will certainly not try to overtake Amundsen.

The projected Nansen Expedition is primarily in-

terested in obtaining scientific results. In a seaplane flying 90 to 100 miles an hour, and with a very limited accommodation, it is impossible to take topographically trustworthy photographs of regions over which the seaplane is passing, nor is it possible to sound the depths of the Arctic Ocean. But in an airship such as Commander Bruns contemplates, with a capacity of 150,000 cubic meters, supplied with four engines and with a crew of 50 men, conditions will be different. The airship will be flown at 80 miles an hour, and each engine is to be supplied with fuel for 100 hours. The whole distance, Murmansk—Alaska—Murmansk, is about 4,000 miles. The whole journey is expected to last four weeks.

Dr. Nansen is in no hurry to leave. He knows that no great things can be achieved without great preparations, and as his scientific ambition is not overshadowed by other considerations, he can afford to wait until everything is in order. At any rate, he says, we are not going to start until 1927.

HALL OF FAME FOR ENGINEERS

PLANS for an Inventors and Engineers Hall of Fame have been announced for the proposed $5,000,000 National Museum of Engineering and Industry to be erected on The Mall in Washington. Decision to establish such a hall in the central rotunda of the museum was reached unanimously by the Board of Direction of the Museum Foundation which has undertaken to raise a national endowment of $10,000,000, and among those who probably will be represented in marble or in bronze will be Charles P. Steinmetz, Alexander Graham Bell, Thomas A. Edison, Orville and Wilbur Wright, Eli Whitney, Captain John Erikson, Mergenthaler and Robert Fulton.

Thomas A. Edison, Orville Wright, Charles F. Brush, Leo H. Baekeland, Edward G. Acheson, Frank J. Sprague and Edward Weston have been elected vice-presidents of the museum movement organization.

Charles M. Schwab, president of the Bethlehem Steel Company, and Melville E. Stone, former head of The Associated Press, also have accepted posts on the Honorary Advisory Board, where they will serve with Secretary Herbert Hoover, Dr. Charles W. Eliot and General George W. Goethals.

Records of the achievements of the outstanding leaders in invention and engineering, now scattered throughout the country, are to be assembled, and all original models, so far as recoverable, are to be obtained for the museum.

Many American colleges specializing in technical education have already attempted on a limited scale to maintain museums exhibits, where members of the student body can study in graphic detail the evolution of the technical field in which they are preparing to take their places. The present movement for the National Museum of Engineering and Industry embraces, beside the central edifice in Washington, a chain of local museums of industry, situated in the various industrial sections and affiliated with the parent institution for the diffusion of technical knowledge to all parts of the country.

The Board of Direction also determined to dedicate a suitable chamber adjoining the Hall of Fame, to be designated "The Founders' Room." Herein will be honored, by suitable busts, tablets and other memorials, those public-spirited individuals, societies and corporations whose contributions in energy, time or money shall have made possible the establishment in Washington of the world's greatest industrial museum institution.

THE YALE ASTRONOMICAL STATION IN SOUTH AFRICA

DR. FRANK SCHLESINGER, accompanied by Mrs. Schlesinger, sailed from New York on December 27 for South Africa *via* England. He is taking with him the 26-inch photographic objective intended for the Yale southern telescope. After selecting the exact site, work will be begun at once on erecting the observatory and mounting the telescope, which he hopes to have in operation by May or June.

The mounting for the telescope was constructed for the most part in the Yale Observatory shop at New Haven, and was temporarily erected in New Haven. It has now been crated and will be shipped direct from New York to South Africa at the end of January. Mr. Walter O'Connell, foreman of the shop, is to accompany the mounting and to take part in its erection in South Africa.

Dr. H. L. Alden, now of the McCormick Observatory, has been appointed assistant professor of astronomy at Yale University and will be in charge of the Yale station in South Africa. He expects to sail from New York at the end of April and begin work with the telescope in June, when Dr. Schlesinger will return to this country.

Mr. C. H. Hall, Jr., formerly in charge of the Observatory of the Maryland Academy of Sciences, has been appointed assistant to Dr. Alden in the operation of the telescope. He will arrive at the station early in February.

This telescope is to be the principal instrument at what will be Yale Observatory's chief observing station. The work of the telescope will at first consist in the determination of the parallaxes of many stars to supplement similar work now being done in the northern hemisphere; the determination of the proper motions of many faint stars, and work in cooperation with other observatories in the selected areas. It is expected to send later other instruments to the station.