is strong among departments to treat themselves as little separate governments, but, whatever distribution of endeavor may be necessary for convenience or economy, government in its relation to its guardianship of scientific data should recognize its undivided responsibility.

Each nation should also acknowledge its obligation in the interest of necessary international cooperation to make readily available to other nations its assembled data and records. The mutual understanding and support of all peoples relating to any subject of research will give ultimately to each investigation and to each separate locality the largest possible measure of result. This sense of mutual interest and obligation will be of especial importance in opening opportunities throughout the world for archeological inquiries. We deprecate all suggestions of the monopolizing of such researches or their results to the prejudice of reasonable requests to prosecute investigations on fair terms. We trust that our scholars and the representatives of our museums and scientific institutions will receive a cordial welcome wherever they go throughout the world, in the realization that they are not serving selfish interests but seeking to advance the knowledge of mankind.

It is fitting that we should recognize what has already been achieved in the line of competent organization. This effort was stimulated by the great war, the conduct of which was largely based on a knowledge of science and its applications. I am informed that at the beginning of the war the Germans showed an ability "to mobilize science in a national emergency" beyond that of other nations at that time, and that since then, and in part because of that fact, the leading nations have taken definite steps towards the encouragement and support of scientific investigations. England has set up a special government Department of Scientific and Industrial Research. In the United States a National Research Council has been organized by the National Academy of Sciences. Japan and Australia have recently set up national research councils. Canada, I am told, has a similar undertaking, and in two or three European countries new scientific enterprises of similar character have been developed. Then there is the federating effort of the International Research Council which has established a number of affiliated international unions covering special fields of science, as the International Astronomical Union, an International Union of Pure and Applied Chemistry; an International Mathematical Union, an International Union of Pure and Applied Physics; an International Union of Scientific Radiotelegraphy. These organizations naturally lead to arrangements for special international scientific conferences. Thus we are at the threshold of a new era of international cooperation in the scientific field. This can not fail to add strength to the influences which make for better understandings between peoples and for a desire to adjust their differences so that they may enjoy the fruits of peace.

We should make acknowledgment to you for the benefit of the by-products of your labors. If to an increasing degree we have the security of sound public opinion, if the extravagances and diatribes of political appeals fail of their object, and if, notwithstanding the apparent confusion and welter of our life, we are able to find a steadiness of purpose and a quiet dominating intelligence, it is largely because of the multitude of our people who have been trained to a considerable extent in scientific method, who look for facts, who have cultivated the habit of inquiry and in a thousand callings face the tests of definite investigations. With scientific applications on every hand, the American people are daily winning their escape from the danger of being fooled. There are, it is true, many false prophets who are active in those areas of exertion where patient inquiry and regard for facts are not prized, but their following, while strident, is apparently not increasing.

We need your method in government; we need it in law-making and in law-administering. We need your interest in knowledge for its own sake; the self-sacrificing ardor of your leaders; your ceaseless search for truth; your distrust of phrases and catchwords; your rejection of every plausible counterfeit; your willingness to discard every disproved theory however honored by tradition, while you jealously conserve every gain of the past against madcap assault; your quiet temper, and, above all, your faith in humanity and your zeal to promote the social welfare. We need your horizon; your outlook on the world. We need the international cooperation which makes more effective the essential national endeavor and brings us nearer together as members of one human family, who in the presence of science can not remain estranged, but must find means of reconciling their several interests in the harmony of their common aspirations and for the common good.

CHARLES E. HUGHES

THE MEANING OF SCIENTIFIC RESEARCH[1]

I HAVE been assigned the task of speaking upon the meaning of scientific research. Some of you will say,

[1] Lecture delivered before the Sigma Xi Club of New York University, Tuesday, November 25, 1924.

"Why talk about that? Every one knows what scientific research is; there are no two opinions about it." I have had many discussions with my colleagues at Columbia University and in other places about this question: "Is such and such experimental work a scientific research or is it not?" The answer to it is not very obvious. That which characterizes true scientific research is the mental attitude. Two persons may be doing the same thing as far as we can see—and yet one is doing scientific research and the other is not, because one of them has a certain mental attitude and the other has not. That attitude may be the result of training, or it may be the result of innate genius.

Take, for instance, Joseph Henry, one of our best-known American scientific research men. He had only a high-school education at the Albany high school. When he graduated, he went into practical work, doing some surveying in the state of New York, which meant principally pulling chains, reading angles, etc.; nothing very scientific about it. Then he accepted a position as instructor at the Albany high school and immediately began doing experimental work all by himself, with no guidance from outside. But watch him in his work! How does he start? He starts with the problem of making a better electromagnet than any ever made—not an ordinary one. If one wishes to make a real advance in any line he must know what other people have done in that line. Henry learned everything that other people had done in constructing a fine electromagnet, and he succeeded in making a better one than anybody had ever made up to that time.

When a man builds an electromagnet, he is going of course to make and break an electrical circuit. Henry observed that he got a much longer spark when he broke the circuit—the same battery being in the circuit—than when he made the circuit. There came to his mind immediately something which had never suggested itself to anybody else's mind; namely, self-induction. He knew at once that he had made a great discovery. Then, instead of being satisfied with that discovery, he went on rapidly to study its meaning. He said to himself, "There must be, for the same reason, mutual induction," and he experimented with that until finally he proved most of the phenomena of mutual induction which we know to-day. By these experiments, his discovery of self-induction and his verification of mutual induction, of secondary sparks, etc., he also made an invention—the electromagnetic telegraph. He was the real inventor of the electromagnetic telegraph, although people do not know it, because he never made any fuss over it. He didn't care about the electromagnetic telegraph. He cared more about his scientific studies than about their application to definite, practical purposes. The invention of the telegraph did not divert him from his purely scientific work.

This is the mental attitude of the real, scientific research man, one who displays a burning desire to see more and more of the eternal truth, and to see it as soon as possible rather than to stop on the way in order to find its practical applications.

John William Draper, who taught at New York University for many years, is another brilliant illustration of this peculiar mental attitude. He was a physician, I believe, by training, and not an abstract scientist. Physicians at no time have been famous for a great knowledge of abstract science. It is not in their line. Although he was a physician, Draper was the first to study radiation, a subject which had nothing at all to do with medicine. In 1847 he published his first paper on temperature radiation, and showed how temperature changes the color of the light emitted by a hot body. In fact, he was the beginner of the theory of the black body, developed thirteen years later by Kirchhoff and by Bunsen in connection with their discovery, in 1860, of spectrum analysis.

In the same year, 1847, another physician, Herman Helmholtz, only twenty-five years of age, published a wonderful essay on the "Principle of conservation of energy," one of the most important scientific essays of the nineteenth century.

Five years before that, another physician, Robert Mayer, published his paper demonstrating that heat is a form of energy and that a certain quantity of heat corresponds to a certain definite quantity of mechanical work.

There you have two other young physicians engaged, like John Draper, in scientific research in the domain of abstract science, who could not be turned away by professional duties. Draper, for instance, did very fine photographic work. Photography in those days was a new art. Draper took it up because he saw in it something scientifically new. Do you suppose that he tried to increase his income by photographing faces? Not a bit of it. He spent his nights trying to photograph the moon and stars. That is science. The other is a commercial pursuit. He and Helmholtz and Mayer had the mental attitude of scientific research men. I will describe another remarkable case. Carnot discovered, just a hundred years ago, in 1824, one of the greatest principles in science, known to-day as Carnot's principle. He was twenty-eight years old when he did it. Now, what was his training? His father was one of the famous generals in Napoleon's army, and he saw to it that his son was educated to become an officer in Napoleon's army, so he sent him to the École Polytechnique in Paris, where he studied military science. But as soon as young Carnot could, he dropped military science and took up scientific research. He had no elaborate training in scientific research, but he had the mental attitude of the scientific research man. He was born with it.

Before I proceed further I wish to answer a question which you are entitled to ask. The question is: "Do not these examples which you have given demonstrate that it is not necessary to train a man in scientific research? There have been many men who have made great discoveries, without such training," I say, "No, they do not prove anything of the kind." These were men of great genius, and great genius does not require training. In fact, that is a distinction of a genius, a man who can do work of the highest type without training. He is born with it. He brings it into the world. But for every genius in this world there are many millions who are not geniuses, who are just ordinary mortals, and ordinary mortals have to be trained.

When, therefore, we speak of the meaning of scientific research, we must connect it with the men who are not men of genius, who are just fine, ordinary mortals—above the average, yes, perhaps considerably above the average, but, nevertheless, below the level of genius. These are the men who do the work of the world. The genius starts it; he is the pioneer. The man who develops new ideas and applies them to the happiness of mankind is the ordinary, average, scientific research man, and we need as many of them as we can possibly get. We can not have too many.

What is it we have to teach young men or young women in order to create in them the mental attitude of scientific research? I prefer to let bigger men than myself talk about that. I will only say this. The scientific research man studies nature and its various operations. Nature speaks a universal language, in which the sentences are constructed in accordance with absolute logic. She has the universal and the most correct language of all. Our human language and logic are only poor copies of the language and logic of nature, and every man and every woman must learn the language and logic of nature in order to get the attitude of a scientific research mind.

I will quote some passages from men who discussed this subject over fifty years ago. President Barnard, who gave us the first wave theory of light in America, between 1850 and 1860, and later became president of Columbia College, said this, I think at a meeting of scientific men and teachers: "If we would fit men to cultivate nature [that is, to study nature] our earliest teachings must be things and not words." He was protesting against the old method of preparing boys and girls for college. They were taught grammar—English grammar, Latin grammar, Greek grammar and sometimes a foreign-language grammar—before they were taught any science at all. Then when they entered college, they got more grammar in the various languages and did not take up science seriously until the junior year. This was too late, too late! I know this from my own experience. The earliest stimulus in science came to me before I had entered college, and the most inspirational scientific information I received when I was a little over ten years of age from my teacher in a public school—a few things, but I got hold of them and they stuck to me. They were always the foundation of my interest in science. I read about them, and kept on reading until I was about sixteen or seventeen. Then I had to take up Greek and Latin because they were required for entrance to college. I went to college, and I knew more science when I entered than when I left. That was the kind of teaching of science we had in those days, over forty years ago. It was so at Columbia College, at New York University, and at all other colleges. Hence that protest by Barnard which I quoted above. There was also a protest by Joseph Henry, who was then secretary of the Smithsonian Institution, by John William Draper, who was professor of chemistry here in New York University, and President Andrew White, of Cornell, and others. They saw that real science, the language and logic of nature, was not taught soon enough, and they started a movement in the direction of introducing a new system of scientific education and a new spirit of scientific teaching—a spirit which would produce the mental attitude of a scientific research mind.

They were not groping in the dark at all, but they thought that their cause would be better served if they had an apostle from the other side to advocate their cause before the people of the United States. In every country people are more apt to listen to a man of great reputation from three thousand miles away than to a man who is among them. So they invited Tyndall, who was a great physicist and a great lecturer. He came over here, so that he might "show the uses of experiment in the cultivation of natural knowledge, hoping that this would promote scientific education in this country," and he succeeded. He delivered a course of six lectures in ten or twelve cities like New York, Washington, Boston, New Haven, Philadelphia, etc., and he had enormous audiences who came to see his brilliant experiments (he was lecturing on light), and to hear the language and the logic of a distinguished devotee to science. Enthusiasm for scientific study was wonderfully aroused, and that is just what Joseph Henry, John William Draper, Barnard and Andrew White wanted. That was the beginning of the great movement in this country in the direction of a new method of scientific education for developing the mental attitude which characterizes a scientific research man.

When leaving, Tyndall gave a farewell address, in which he said, among other things:

> To no other country is the cultivation of science in its highest form of more importance that it is to you, the people of the United States. In no other country would it exert a more benign and elevating influence. The original investigator constitutes the fountain of knowledge. It belongs to the teacher to give this knowledge the requisite form, an honorable and often difficult task, but it is a task which receives its final sanction when the teacher himself honestly tries to add a rill to the great stream of scientific discovery. Indeed it may be doubted whether the real life of science can be fully felt and communicated by the man who has not himself been taught by direct communion with nature. For that power of science which corresponds to what the Puritan fathers would call ''experimental religion,'' you must ascend to the original investigator.

You see the plan he recommended and which was recommended by his friends who invited him over here was not only to have great scientific investigators, but that every teacher who teaches science should be given a chance to do original investigation, because without that they lacked the power of teaching. It only comes, you know, from direct communion with nature, or, as I have called it, from a direct *study of the language and logic of nature.*

People did not quite understand Tyndall's first sentence, "In no other country is the cultivation of science in its highest form of more importance than to you; in no other country would it exert a more benign and elevating influence." Why would it in this country exert a more benign and elevating influence than, say, in Russia or in Turkey or in China? Why did we need it more than any other country? That I am going to explain by quoting now from John William Draper, who made the following comment upon Tyndall's remarks: "Nowhere in the world are to be found more imposing political problems than those to be settled here," and then comes another sentence, "nowhere a greater need of scientific training." Behold, what this man said, that in a country of political problems, we need for their solution scientific training? What does this mean?

Draper did not say any more than that, but Andrew White said this, more than fifty years ago: "I will confine myself to the value in our political progress of the spirit and example of some of the scientific workers of our day and generation." (And he had undoubtedly Joseph Henry and John William Draper in mind). "What is the example of that spirit? It is an example of *zeal,* zeal in the search for truth, of *thoroughness,* of the truth sought in its wholeness, of *devotion* to duty, without which no scientific work can be accomplished, of *faith* that truth and goodness are inseparable." That was the value in our political progress of the spirit and example of some of the scientific workers of that day. That is the thing of real value which we in our political progress should imitate in the lives of these scientific men.

Then, in 1886, came Henry Rowland, of Johns Hopkins, the great American physicist. He made an address in Baltimore on the value of a scientific research laboratory, in which he eulogized the mind in which that mental attitude is found which I call the mental attitude of scientific research. He said this, "This is the mind which is destined to govern the world!" He was a bold man. Some of you may have known him. I knew him very well. He believed in what he said, and he said what he believed, and this was his bold prophecy: "This is the kind of mind [this is the kind of mental attitude] we need in the solution of all our human problems, not only the problems of science." In this he simply repeated what the others, Draper, White and Barnard, had said fourteen years before.

When Rowland said this, the status of scientific research was somewhat better than it was fourteen years before, because the movement started then had succeeded. He had organized a laboratory at Johns Hopkins University, the object of which was training in scientific research work. Then came the laboratories of Harvard, Yale and other institutions, and scientific research work began to be taught in American universities, imitating somewhat the methods of German universities. We had to imitate somebody, because we had to start somewhere before we developed a system of our own. Now, we have developed a system of our own, and I want to say a few words about that.

The scientific research movement, or, as Andrew White called it, "the movement for higher endeavor," became stronger and stronger in our universities. From the universities it was transplanted into American industries, and it is extremely strong there now. The scientific research activity in our industries is marvelous, when you compare the work done to-day with the work done no longer than twenty-five years ago. We need to-day many more research workers, not only for the proper teaching of science, which Tyndall recommended, but we need them also for another purpose. Scientific research work in the industries has increased so enormously that the industries need many men, university-trained men, to do their work. The supply of scientifically trained research men for the industrial research laboratories is smaller than the demand. The industries themselves are clamoring for better men with a better type of training, so that in this respect the universities are beginning to be led by the industries, instead of vice versa. But that will not last long. The universities will lead again, I am sure of it.

The industries are anxious that the universities should do their work of training research men as well as it possibly can be done. I heard one captain of industry remark one day that, perhaps, it would be a good thing if the industries would set aside a part of their profits, derived from the development of scientific research, for the benefit of universities, to enable them to give better and better instruction in scientific research. I also know that some of the industries are subsidizing some of the university laboratories for carrying on certain research work; not developmental work, but purely scientific research work. The best of the industries are not trying to debase the real research work of universities by giving them problems which are nothing but technical development work. The industries can do that themselves. What they would like to see the universities do is to carry on pure scientific research work, and to produce young men who have a truly scientific mental attitude. This cooperation between the scientific work in the universities and in the industries has already produced wonderful results, and it will produce more and more, and I am quite sure that some day the achievements from this cooperation will prove even to the most ordinary type of mentality that the best work can be done only by experts who have the proper training. That is the doctrine which we need in this country, and if it is adopted, not only in the industries, but in every activity of government, then the prophecy of Rowland will be fulfilled. I am sure that it will be adopted some day, because that is one of the best ways to make democracy safe for the world.

M. I. Pupin

Columbia University

THE FOUNDATION OF THE THEORY OF ALGEBRAIC NUMBERS[1]

II

We shall next see that certain modifications are to be introduced in order that the usual theorems of arithmetic hold true in the more general realms. For example, in the very simple realm that exists by adjoining $\sqrt{m}$ to the usual realm, it may be proved when m is greater than 3 and is not a perfect square that the Euclid Algorithm is not applicable, and there is no such thing as the greatest common divisor in the usual sense. By way of illustration observe that in the realm $R(\sqrt{-5})$ we have $21 = 3.7 = (4+\sqrt{-5})(4-\sqrt{-5}) = (1+2\sqrt{-5}) \times (1-2\sqrt{-5})$, where all the factors are irreducible integers. Thus it is evident that the factorization of an integer into its irreducible (or prime) factors is in these extended realms *not* a unique process as is the case in the usual realm of arithmetic and as is also true in the realms $R(i)$ and $R(\omega)$. And here is the difficulty that mathematicians at first found perplexing, a difficulty which it was necessary to overcome before the laws of arithmetic could be regarded as universal.

The problem may be recast as follows: Let ρ be the root of an algebraic equation of the nth degree whose coefficients belong to the usual (natural) realm of rationality and let ρ be adjoined to the usual realm. We thereby create an algebraic realm $R(\rho)$ of the nth degree. *Determine the arithmetic of this extended algebraic realm.*

By making use of the above example we shall anticipate the results that follow, particularly those that are connected with the *Theory of Ideals.* The reader is thus enabled to see the trend of the later theory and with this in view he is asked to accept without proof the statements given immediately below.

Write $T_1 = (3,\ 1+2\vartheta)$, $T_2 = (3,\ 1-2\vartheta)$, $T_3 = 1,\ 1+2\vartheta)$, $T_4 = (7,\ 1+2\vartheta)$ where $\vartheta = \sqrt{-5}$. It may be proved by taking the products of the ideals that

$$T_1\ T_2 = (3),\ T_1\ T_3 = (1+2\vartheta),\ T_1\ T_4 = (4-\vartheta),$$
$$T_2\ T_3 = (4+\vartheta),\ T_2\ T_4 = (1-2\vartheta),\ T_3\ T_4 = (7).$$

None of these quantities is a unit in $R(\vartheta)$ and they are all prime ideals since, if N denotes the *norm* of an algebraic quantity and that is the product of the quantity and its conjugates, so that $N(T_1) = (3,\ 1+2\vartheta)\ (3,\ 1-2\vartheta)$, then is

$$N(T_1) = N(T_2) = 3;\ N(T_3) = N(T_4) = 7.$$

Thus it is seen that the factorization of 21 into its prime ideal factors, namely, $21 = T_1\ T_2\ T_3\ T_4$ is a unique process. It is also seen that the different methods of factorization given above for the integer 21 in the realm $R(\vartheta)$ are had through the different combinations in pairs of the T's.

It thus appears that the prime ideals in this extended realm take the place of prime integers in the usual arithmetic; and one of the objects before us is to establish what is the historical origin of these prime ideals, as well as to study what they are.

Returning to the discussion of the proof that the Greater Fermat Theorem does not admit integral solutions, consider the simple case

[1] Concluding part of the address of the vice-president and chairman of Section A—Mathematics, American Association for the Advancement of Science, Washington, December 31, 1924.