has been an article of commerce in this country for some years.

In his treatment of the practical side of illumination Mr. Walsh gives primary emphasis to the problem of getting an adequate amount of light on the object or plane of interest; "the important factor in seeing is the brightness of the thing looked at and this is the product of the illumination and the reflection ratio." At the same time the almost equally important matter of keeping bright lights out of the field of vision, and the avoidance of glare in general are not lost sight of. Practically every important problem of lighting, in the home, the factory and the public institution, is treated in a manner sufficiently detailed so that the reader should have an intelligent idea of the fundamental requirements and the best practice in meeting these. The latest lighting legislation, both British and American, is extensively quoted.

The points to which exception has been taken are minor ones, and are such as are almost inevitable in any book which attempts to cover a large field. They do not prevent the book from giving, on the whole, an excellent view of the subject, and it can be cordially recommended to all who wish to obtain a good idea of the scope of modern lighting science and practice. The bibliography at the end is well chosen as a guide to further study of the subject. The book is excellently printed and attractively bound.

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SPECIAL ARTICLES

PROBABILITY-INCREASE IN SHUFFLING, AND THE ASYMMETRY OF TIME[1]

A MACROSCOPIC model to illustrate the nature of the Boltzmann H-theorem[2] has been described by P. and T. Ehrenfest. It will be recalled that the H-function is a measure of the probability P of a given state (configuration, velocity distribution) of a system of particles. It is related to the entropy S of the system through the relation $S = k \log P = -kH$. As the system approaches the steady state, the entropy S increases and the function H decreases, each approaching a limiting value. The Ehrenfest model, which operates by successive drafts of numbered tickets from urns, illustrates very effectively several characteristic properties of the H-function; in particular, its tendency to decrease continually when its value is remote from the ultimate "steady" value; the occasional lapses in which H momentarily increases, even in states remote from the steady state; the essentially discontinuous character of H, which, strictly speaking, renders the derivative (dH/dt) meaningless; and, finally, the small fluctuations of H above its minimum value when the statistically steady condition has become perceptibly established.[3]

One feature of special interest, however, the Ehrenfest model fails to exhibit, namely, the occurrence of long-continued and extended series of increases in H, such as must, according to the theorem, take place upon very rare occasions. Indeed, it seems at first sight hopeless to attempt to devise any experiment which should illustrate these exceptional high peaks in the H-curve. It seems like a contradiction of terms to speak of producing, at will, and within a closely limited period of time, an excessively rare (improbable) event. We know, indeed, that any truly representative model of an H-curve must have such high peaks at long intervals, corresponding, it may be, to billions of years or more; but how can we bring it about that such a peak shall occur during our experiment; that the particular piece of the curve under observation shall be the one containing the monstrosity?

By a simple artifice this effect can be secured. Two similar urns (boxes) are charged with a set of numbered tickets or the like. Box A receives tickets 1 to 50, which, for brevity, we may speak of as tickets a. Box B receives tickets 51 to 100, tickets b. The two boxes are thoroughly shaken to shuffle the tickets. A ticket is then drawn blindly from A, and another from B, the numbers drawn are recorded, and the tickets are returned to *opposite* urns. The urns are again thoroughly shaken and the same process is repeated as many time as may be desired. In the experiment here recorded, a series of 50 such double drafts was made.

At the termination of this first series of drafts the contents of box A are carefully noted. While this can be done from the records alone, as a matter of additional certainty, to guard against error, box A was opened and a note was made of all the tickets contained therein. They may, to distinguish them, be blackened on their back; but in any case it will now be convenient to speak of them as *black* tickets, while the remaining 50 tickets, contained in box B at the end of the first series, may be spoken of as white tickets.

The black tickets are put back in box A, and a second series of drafts precisely similar to the first is

[1] Papers from the Department of Biometry and Vital Statistics, School of Hygiene and Public Health, Johns Hopkins University, No. 91.

[2] P. and T. Ehrenfest, *Physikal Zeitschr.*, Vol. 8, 1907, p. 311; Schaefer, Einführung in die theoretische Physik, 1921, Vol. 2, p. 417.

[3] The experimental demonstration of the occurrence of these fluctuations near the steady state is one of the many remarkable developments of recent years in the physics of small dimensions. In this connection reference may be made to the work of The Svedberg (Die Existenz der Moleküle, Leipzig, 1912) and Smoluchowski (Bull. Acad. Cracovie, 1916, p. 218).

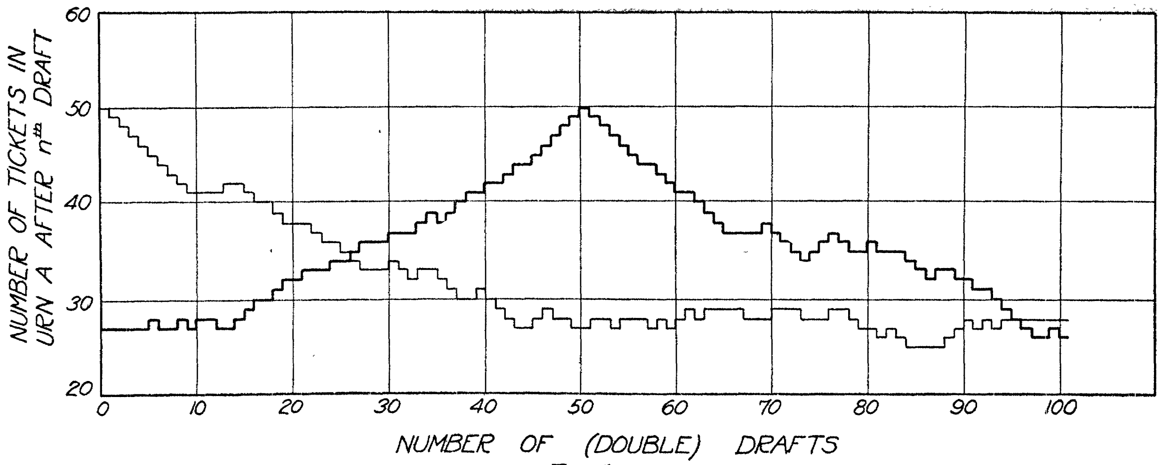

FIG. 1

now made. In the experiment here recorded the second series consisted, like the first, of 50 double drafts.

After the termination of the second series we may, by consulting the records, construct, first of all, a curve showing the tickets of kind *a* contained in box *A* at every step in the process that has been described. The lightly drawn curve shown in Fig. 1 was thus obtained. In general character it resembles the curve of the Ehrenfest model, and illustrates the same points of the *H*-theorem, no more and no less.

But if we now turn our attention to the *black* tickets (instead of tickets *a*), we see—herein lies the special interest of this method—that the method here followed enables us to trace the history of these tickets *backward* in time, from the moment (at the end of the first series) when they were all contained in box *A*. For, we know, from the record of the last draft in the first series, the numbers of all the *black* tickets; we can, therefore, go over the record of the first series of drafts and trace therein the previous history of all the black tickets, though at the time the drafts were made the identity of these black tickets was unknown to us. (We did not know, so to speak, which of the tickets were predestined to be black.)

The more heavily drawn curve of Fig. 1 shows the complete record of the history of the urn *A* as regards its contents of black tickets. The right half of this graph is, in general character, similar to the lightly drawn curve of Fig. 1. But the left half of the curve, the backwardly extended record of the history of the drafts, is the feature of special interest. It will be noted that during this part of the operation the system was *apparently* passing continually (with only occasional lapses) "from a more probable to a less probable state," until the process culminated, at the end of the first series, in the extremely "improbable" condition in which all the black tickets were in one urn.

TABLE I

NUMBER n OF TICKETS DRAWN r TIMES IN m DOUBLE DRAFTS ($2m$ INDIVIDUAL DRAFTS)

	n				
r	50 Drafts Calculated[4]	Drafts 1–50	Drafts[5] 51–100	100 Drafts Calculated[4]	Drafts 1–100
0	36.4	36	41	13.3	11
1	37.2	39	34	27.1	26
2	18.6	16	11	27.3	36
3	6.1	7	13	18.2	14
4	1.5	2	0	9.0	7
5	0.3	0	1	3.5	4
6	—	0	0	1.1	2
S[6]	28.2	27	26	25.3	28

$$n = \frac{m\,!}{(m-r)\,!\,r\,!}\left(\frac{1}{50}\right)^{r}\left(\frac{49}{50}\right)^{m-r}$$

where m = number of double drafts in the series, *i.e.*, 50 or 100.

If the tickets found in urn *A* at the end of the first 50 drafts had been blackened *before* instead of after the 50th draft, the first series of drafts would appear to an observer absolutely miraculous, urn *A* becoming filled with black tickets by a sequence that would seem positively uncanny. Yet the blackening of the tickets in no way affects the physical course of events; it only serves to draw our attention to a

4 Calculated according to the formula

5 The second series of drafts (51–100) shows evidence of imperfect shuffling (excess of tickets drawn 0 times and 3 times).

6 S denotes half the sum of the n's for even r's, *i.e.*, $\frac{n_0 + n_2 + n_4 + n_6}{2}$. It is equal to the number of those tickets present in urn *A* at the beginning of a series of drafts, which were again found in the same urn *A* also at the end of the series.

sequence of events which, without such visible marks, would escape our notice, although taking place before our very eyes.

It may be objected that the course of events here observed does not represent a *rare instance,* an *improbable case,* inasmuch as any number of cases of the kind described can be constructed at will. The answer to this objection is that the number of series of drafts thus constructible at will, though indefinitely large, is, nevertheless, a vanishingly small fraction of all the series of drafts of like character but not specially selected in the manner described, *which do not exhibit a peak.* If the reader has any doubts upon this point, let him attempt to find a peak by trying a number of series of drafts *at random* (without selection), until he encounters a high peak. But he should be warned that he will find this a time-consuming occupation. However, the experiment is unnecessary, for we can obtain an indirect estimate of the probability of success, as follows:

Suppose we attempt to name, by guessing, the 50 tickets that are going to be "black," *i.e.,* that will be found in urn *A* at the end of the 50th double draft. The chances are one in 2^{50} or one in 1,125,000,000,000,000 that we shall select all the 50 tickets correctly.

This way of looking at the matter has the advantage that it makes us independent of the physical similarity of the tickets. We may, if we please, blacken the selected tickets *before* shuffling and carrying out the series of drafts, or we may modify them in any way *before* the series, provided only that the modification is of such character as not to bias the shuffling process. Or, we may, for example, substitute balls for the tickets, and instead of blackening the selected 50 balls (those which we expect to observe as collecting in urn *A* at the end of the 50th draft), we may suppose them slightly increased or decreased in diameter, so as to render them distinct from the others, without, however, introducing any bias in favor of the selected balls collecting in urn *A* or in urn *B.* In any case our estimate remains true regarding our chance of guessing correctly, at the beginning of a series of 50 double drafts, upon which of the tickets we must fix our attention if we are to witness the remarkable spectacle of their segregation and assembling in urn *A* at the 50th draft.

It may now be objected that the series of drafts described is indeed an improbable one, but only in the sense that chances are very small that we should, without foreknowledge or knowledge *a posteriori,* select *initially* for observation a particular set of evenly distributed tickets which, at some subsequent time, are found to be all located in one urn.

In other words, it may be objected that what is improbable is not the series of drafts described, but the eventuality of fortuitously selecting for observation a certain initial condition, a certain initial set of tickets.

This objection is perfectly sound. The model is true to type. In statistical mechanics also, upon Newtonian basis, irreversibility in the course of events, that is to say, the trend from less probable to more probable states, is a matter of initial conditions. Of initial states with a large *H,* the vast majority (practically all) lead shortly to subsequent states of lesser *H*; but of initial states with a small *H* only a vanishingly small proportion lead, within reasonable time limits, to subsequent states with a large value of *H.* The model very properly displays a strictly analogous character and thus furnishes an excellent illustration of one of the difficult points in the theory of irreversible effects. In the model also, only a vanishingly small proportion of initial states in which the distribution of selected tickets is essentially even among the two urns, lead to subsequent states in which the distribution of the same tickets is very uneven (*e.g.,* all black tickets in one urn). On the contrary, a very large proportion (practically all) of the initial states in which the distribution of selected tickets is very uneven, lead shortly to subsequent states in which this distribution is essentially even.

It has been remarked that the law of evolution is the second law of thermodynamics; that an evolving system passes (in general) from less probable to more probable states.[7] The asymmetry of time has been identified with this property of evolving systems. In fact, it was for the purpose of furnishing a readily understood concrete illustration of this relation that the experiment described above was primarily designed by the writer, the original design covering only an operation of the nature of the first series of drafts described. Only when the possibility was recognized of extending the history of the tickets backward in time, by running two consecutive series, was it realized that the experiment is not competent to distinguish unequivocally between the forward and the backward direction in time. It may well be doubted whether any objective experiment can be devised that establishes, on the basis of classical mechanics or their analogue, a wholly unassailable distinction between the two directions in time. The

[7] J. Royce, SCIENCE, 1914, p. 551. It is doubtful whether a statement in such broad terms has any useful meaning. To give it such meaning it would seem necessary to specify *in what particular* a given state is more or less probable. An "improbable" state is a physically unstable state only if it is improbable with respect to a set of characteristics which, in undergoing change to a more probable set of values, furnishes an opportunity for obtaining work from the system by a macroscopic process.

There is an arbitrary element in any measure of "the probability of a given state of a system." It is only

Lagrangian equations of motion are invariant[8] for the transformation $t' = -t$. Experiments based on heat conduction may be practically quite satisfactory, but from the point of view of the kinetic theory they seem to be open to the same objection as the macroscopic model here reported; there is always the possibility, however remote, of heat transfer from the colder to the hotter of two bodies.

The "natural clock" of the Newtonian system of mechanics is a body moving through a portion of space remote from other matter,[9] and thus not acted upon by any external forces. Equal times are defined, in terms of such a body, as times occupied in the travel over equal distances. Since nothing is said, in this definition, regarding the *direction* of motion, the definition leaves the sign of t indeterminate.

The "natural clock" of the Einsteinian system is a ray of light propagated through a region of space remote from gravitating matter. The direction of this ray is fixed as *away from the source.* Here it might appear that the sign of t is determined. But to distinguish between the propagation of a light wave towards an absorbing oscillator, on the one hand, and, on the other, the propagation of a similar wave away from a radiating oscillator (source) we should have to be able to discriminate between a *rate* of loss or gain of energy *in time;* that is to say, the criterion which it is proposed to apply itself presupposes a knowledge of the direction of time. Thus, in relativistic mechanics also the sign of t is indeterterminate.

In its historical development the Newtonian system of dynamics was not based upon an objective natural clock at all, but upon our subjective judgment of time. For it is, of course, a fact, that we possess an intuitive, fairly accurate sense of equality in time intervals (rhythm), and a very deeply ingrained intuitive sense of the forward direction in time.[10] These are undoubtedly the ultimate foundations upon which the structure of dynamics was primarily reared, though the refinements of accurate clock indications were presently substituted for our approximate intuitive measures. This is, perhaps, as it should be. For the ultimate data of all observational science are psychological—sense perceptions, memory images and the like. To assume, as a starting point, an accurate mechanical clock with which our laboratory experiments in mechanics are timed, and thereupon to develop a system of mechanics which comprises in its scope the working of that very clock, seems a circular argument. The conclusions drawn, by such argument, regarding the mechanical properties of our clock (pendulum, earth, etc.) do not, of course, represent a new result; their real significance lies in the fact that they are found to lead to no conflict, thus proving that our assumptions involve no inherent contradictions, that the postulates admitted are competent to form the basis of a self-consistent system of dynamics. However, once this fact is established, we may adopt the system based on artificial time measurements, and disregard, as too coarse or imperfect, and, moreover, as unnecessary for the further devel-

with respect to certain *suitably defined* measures of probability that the entropy of the system has any significant relation to such probability. For example, it is unrelated to any "probability" that takes account of the mere identities ("personalities," so to speak) of the individual molecules of the same species.

There is here involved a question of fundamental principle. Probability is a subjective thing in so far as it depends upon the way in which we choose to classify phenomena or events. It is objective only in so far as our classification corresponds to objectively significant characters; among such we should hardly reckon, for example, the blackening of certain tickets, which would have practically no influence upon the physical course of events, but would merely serve to enable us to identify the members of a wholly arbitrary class. Indeed, we have here purposely selected a mode of marking which shall have no appreciable influence upon the draft. It is true that, as the result of this, our model illustrates primarily just those cases (such as the diffusion of a gas into itself) which are thermodynamically neutral; but indirectly, from the alternative point of view set forth above (based on the guessing of the black tickets), the model is equally representative of the typically dissipative processes accompanied by increase in entropy. For the mathematical probability of an event defined in any way is in itself independent of the thermodynamic significance or insignificance of that event, and we may, therefore, in all propriety, employ a thermodynamically insignificant event, following a certain law of probability, as a model to illustrate a thermodynamically significant event following the same or a similar law.

[8] H. Poincaré, "Thermodynamique," 1908, p. 441. Incidentally it may be noted that this holds true not only in Newtonian, but also in Einsteinian mechanics, owing to the fact that the velocity of light enters the fundamental equations of relativistic mechanics as the second power.

[9] H. Hertz, "Principien der Mechanik," 1894, p. 167; Webster, "Dynamics," 1912, p. 22.

[10] Curiously enough, our intuitive judgment of time, both as regards measure of intervals and as regards order of sequence, is vividly brought into play and strikingly exhibited in the art of music. As regards rhythm, this is obvious. But that we possess an entirely peculiar criterion of time-direction in our sense of chord resolution and of melodic progression has not, to my knowledge, been previously pointed out from this standpoint. Any one who may feel any doubt on the subject of this special sense of time-direction will find conviction in playing a phonograph record backward. Music rendered backward is an utterly meaningless jumble of noises. The same is not true of language (witness palindromes).

opment of the subject, the intuitive base from which we started. This is what the physicist, in effect, does, and his procedure finds pragmatic justification in the results.

It must be admitted that there is something unsatisfying to the mind in the conclusion that the distinction between the forward and the backward direction in time rests, physically, purely on a basis of probability. Intuition would lead us to look for a more fundamental difference. But if such there is, it escapes our present analytical scrutiny, our present analytical formulation of the laws of mechanics.[11]

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THE NORTH CAROLINA ACADEMY OF SCIENCE

The twenty-third annual meeting of the North Carolina Academy of Science was held at Trinity College, May 2 and 3. This was by far the best attended and most enthusiastic meeting the academy has ever held. The membership was reported as being 236, an increase of about ten per cent. for the year. More than half of the membership and many visitors were in attendance. The following papers were presented:

The ecological position of the eastern pine communities: B. W. Wells.

A non-mathematical interpretation of the theory of relativity: W. W. Wood.

Amoeboid behavior of the lymph cells in sea urchins: H. V. Wilson.

New dyes from amino-p-cymene: A. S. Wheeler and H. M. Taylor.

Studies in ascidian blood: W. C. George.

Extra area of hard surface due to widening roads around sharp curves: T. F. Hickerson.

Identification trials: C. S. Brimley.

Experiments on mental set: J. F. Dashiell.

Evolution of the nesting habits of birds: Z. P. Metcalf.

Present-day problems in engineering education: W. E. Wickenden.

Rainfall characteristics of North Carolina: T. Saville and J. H. Wulbern.

Opportunities for chemical industries in North Carolina: E. E. Randolph.

Local engineering problems in forestry: W. W. Ashe.

Brownian movements: F. P. Venable.

Improvements in the high school science situation: J. H. Highsmith.

Variations in the proteins of corn: H. B. Arbuckle and O. J. Thiers, Jr.

Soybean diseases: F. A. Wolfe.

New megachilid bees: T. B. Mitchell.

11 Compare A. S. Eddington, Report on Relativity Theory of Gravitation, 1920, p. XI.

Soft pork and its causes—I: J. O. Halverson and Earl Hostetler.

Methods for class demonstration of hydrogen sulfide formation by bacteria: I. V. Shunk.

The hydrogen ion concentration of the intestines in relation to the intestinal protozoan parasites: M. J. Hogue.

A botanical collecting trip up Grandfather's Mountain: P. O. Schallert.

The biological determinant vs. the environmental determinant: C. C. Taylor.

The morphology of certain phylloxera galls on the hickory: A. C. Martin.

An American vs. a tubercle bacillus: Charles Phillips.

Recent experiments on wintering honeybees: J. C. Eckert.

The effect of dry heat on certain of the cotton seed: S. G. Lehman.

Vitality of albino rats for experimental purposes: F. W. Sherwood.

Injection of the blood vessels of young chick embryos: W. R. Earle, presented by H. V. Wilson.

Water dogs (Necturus) of North Carolina: C. S. Brimley.

A neglected factor in maze learning: J. F. Dashiell.

Some results of soft pork investigation—II: J. O. Halverson and Earl Hostetler.

Structure of the heart muscle of Ascidia: W. C. George.

Homopterous head: Z. P. Metcalf.

Gossypol in relation to nutrition: W. A. Withers.

Loess of the Yellow Rivèr district, China: Collier Cobb.

A primitive gelatinous basidiomycete: W. C. Coker.

Observations on Pythium gracile, and on a fungus parasitic on it: J. N. Couch.

Stream adjustment in loessal plan of China: Collier Cobb.

A new aphanomyces parasitic on fungi: W. C. Coker and J. N. Couch.

North Carolina Section of the American Chemical Society

Latent heats of fusion of some nitrotoluenes as calculated from melting point depressions: J. M. Bell.

The action of phenylsemicarbazide on acetylacetone: A. S. Wheeler and F. P. Brooks.

Devitrification of quartz ware: F. C. Vilbrandt.

The occurrence of borneol in spruce turpentine: A. S. Wheeler and C. R. Harris.

The chemical analysis of Okra seed (Hibiscus esculentus): B. Naiman and L. M. Nixon.

New dyes derived from 2-amino-p-xylene: A. S. Wheeler and Mildred Morse.

A connection tube for gas burettes: F. C. Vilbrandt.

The action of amines on dichloro- and trichloroacetic acids: A. S. Wheeler and E. D. Jennings.

A solution of barium sulphate: O. J. Thies.

Some notes on gossypol: F. W. Sherwood.

North Carolina Physics Teachers' Association

Formation of molecules: W. E. Speas.

Scientific research in North Carolina: C. W. Edwards.

Physics laboratory manuals: W. T. Wright.