the latter journal was dated September 1910, more than 28 years after the date of the first number of volume 1.

The tendency towards forming national mathematical organizations with regular official journals is one of the noteworthy developments of the second half of the nineteenth century, and it was represented in our country by the organization of the American Mathematical Society in 1888. Within the last decade this tendency manifested itself strikingly among us by two new national mathematical organizations. The younger and larger of these is the National Council of Teachers of Mathematics, organized in 1920 and having a membership of more than 3,000. The official journal of this society is the *Mathematics Teacher,* which had been previously published by the Association of Teachers of Mathematics in the Middle States and Maryland. The older of these two mathematical organizations is the Mathematical Association of America, to which we referred above.

America has had its share of successful elementary textbook writers who secured considerable reputation on the part of the general public. In addition to these there have been hundreds who published textbooks which were designed to meet special local needs, but which often had greater faults than were possessed by the books which they replaced. Much energy has doubtless been wasted here which should have been directed towards higher mathematical attainments on the part of the authors. Substantial improvements in textbooks are, however, very important, and authors usually learn something about the subject while preparing the manuscript of a textbook.

America has also had its share of the so-called mathematical prodigies. Among these T. H. Safford (1836–1901) is well known. He became professor of astronomy in Williams College in 1876. In his eleventh year he is said to have published an almanac, computed for this city, which soon reached a sale of 24,000 copies. Mathematical prodigies, like the successful elementary textbook writers, secured considerable public notice, but most of them contributed little or nothing towards the development of our subject. Their marvelous mathematical feats are of more interest to the psychologist than to the mathematician. In Europe Ampère and Gauss are noted as prodigies and they are also noted contributors towards the advancement of our subject, but in America the mathematical prodigies have thus far contributed little to the advancement of pure mathematics.

The actual and relative mathematical advances made by Americans during the last 75 years are conspicuous, but not satisfying. We have not yet attained relatively as high a standing as we should aim to attain, or as those belonging to some of the other sections of this association—such as the astronomers and the geologists—have already attained. Not one of the 50 incorporators of the National Academy of Sciences had made important contributions to the increase of our knowledge of pure mathematics, although six of them enrolled in the section of mathematics. It is only recently (1920) that this association recognized conspicuously the advances in American mathematics by devoting an entire section to them. From 1882 to 1919 mathematics and astronomy constituted one section, and the astronomers usually commanded the major interest at our meetings. Let us hope that the letter which has been assigned to our section will represent in the future not only the fundamental character of our subject, but also the relative advances made therein. To work hard and long before receiving public recognition seems to be the lot of most of us, but the sense of growth is keen and definite in our field and this sense of growing intellectual insight and power is our main reward as regards mathematical research.

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THE PRACTICAL VALUE OF PURE SCIENCE[1]

WHEN I reflect that preceding Edison medallists have been men of the type of Charles F. Brush, who first showed the world that electricity might be used for city lighting; Alexander Graham Bell, whose invention was at the base of the whole vast system of modern communications; Frank Sprague, who was responsible for the application of electric power to railway transportation; M. I. Pupin, who made long distance telephony possible; J. J. Carty, under whose inspiration and leadership the telephone repeater and amplifier, with all that they mean to the enrichment of modern life, have been brought forth, and others of like achievement in the application of electricity to large industrial uses, I feel that there may have been a misunderstanding or a mistake in connection with this year's award. For when I look over my thirty years of scientific effort I can find no industry which has grown out of my researches, nor even any which have been very immediately benefited by them.

Since this survey certainly reveals nothing of great industrial consequence I am obliged to adopt either the mistake-theory, or, as an alternative, to assume

[1] Response to the presentation of the Edison Medal at Del Monte on the evening of October 4, when the president's presentation address was made by Dr. Frank B. Jewett from his home in New Jersey, his voice being carried over telephone lines and amplified through the magnivox so as to be very distinctly audible to the entire audience seated at dinner in the Del Monte Hotel, three thousand miles away.

that the American Institute of Electrical Engineers has this year been led to adopt a new policy—a policy of recognizing occasionally, at least, as something of vital, practical importance to the world, a type of activity which does not lead to immediate industrial advances. I am going to assume that this last hypothesis is correct, and in behalf of all workers in what is called the field of pure science, all those who are spending their lives in trying merely to ferret out nature's secrets and to better man's understanding of her laws, I wish not only to express my appreciation to the Institute for the award, but also to compliment it upon the breadth of its own vision and the service to science which it has done in recognizing before the public the value of this other field. For, in the final analysis, the thing in this world which is of most supreme importance, indeed the thing which is of most *practical* value to the race, is not, after all, useful discovery or invention, but that which lies far back of them, namely, "the way men think"—the kind of conceptions which they have about the world in which they live and their own relations to it. It is this expanding of the mind of man, this clarifying of his conceptions through the discovery of truth which is the immediate object of all studies in the field of pure science. Behind that object, however, is the conviction that human life will ultimately be enriched by every increase in man's knowledge of the way in which nature works, since obviously the first step in the beneficent control of nature is a thorough understanding of her.

To illustrate my contention that the way men think is the most important and the most practical thing in human progress, I wish to consider briefly two great epochs in history in which significant changes have been brought about in man's conception of his world and of the place he occupies in it. The first epoch began just 450 years ago; for this year happens to be the 450th anniversary of the birth of Copernicus, or, as he was known in his native Poland, Nikolaus Copernik, a man who spent his life not primarily in the pursuit of astronomy, but rather in the service of the church, for he was Canon of the Cathedral of Frauenberg. This man was more than any other responsible for changing the conceptions of mankind about what some men who call themselves practical would say had no bearing upon this life of ours at all, and yet there was not a political or social change in Europe during the two or three succeeding centuries which it did not affect.

It is not strange that through all the ages up to the time of Copernicus the earth had been the center of man's universe, nor indeed that his whole thinking had been ego-centric—that he felt that the universe had been created especially for him with every bird and beast and flower ministering to his pleasure. Nor is it strange that in spite of this self-centered conception, surrounded as he was with mysterious forces, his philosophy of life had been a supinely fatalistic one, that little idea had as yet entered his mind that he himself might have any real control over nature, or that he had any responsibility for the shaping of human destinies. In all the ancient world three blind fates sat down in dark and dank inferno and spun out the lives of men. Man himself was not a vital agent in the march of things; he was but a speck tossed hither and thither in the play of mysterious, titanic, uncontrollable forces. Even after the advent of Christianity no idea of the possibility of changing bad conditions by human effort gained access to his thinking. The best that he could do was to withdraw from the world and to cultivate his soul in a monastery, or to mortify the flesh after the manner of the pillar-saints, in the hope of reaping a reward in the next world for his piety.

Now note first the simplicity of the process by which a change in his thinking begins to come about, and then note the result. The simplicity of the process has been characteristic of the advance of science in all ages. Careful observations, such as men made in those days as well as this, had brought out difficulties in the explanations which had come down from the past. The dome of heaven rotating about the earth and carrying the fixed stars, and other transparent crystalline domes carrying the planets and the sun and rotating at different speeds, were simple enough if one is not too insistent upon the requirements of mechanical engineering, but careful observation had shown retrograde motions, at certain seasons, of the planets now known to be outside the earth's orbit, which are actually due to the fact that the earth itself is at these times speeding between these outer planets and the sun at an angular speed greater than their own, thus causing them to seem to go backward. Such phenomena imposed impossible conditions upon the crystalline domes of the ancients unless the most complicated and grotesque assumptions were made. Copernicus wrote his only book to show that all these difficulties disappeared and all explanations came out vastly more simply if the earth is assumed to be but one of a number of little planets rotating once a day upon its axis and circling once a year about the sun. But realizing that this new conception might arouse opposition because it robbed man of his central position in the scheme of things he wrote to the Pope, to whom he dedicated his book, asking him to use his influence to defend him from those who might attack his theory "because of some passage of Scripture which they had falsely distorted for their own purposes." Here is a devoutly religious man 400 years ago of sufficient vision to see, even in the dim light

of that shadowy age, that the foundations of real religion are not laid where scientific discoveries of any kind can disturb them, and who therefore keeps his mind open at all times to truth from whatever angle it tries to enter.

But now look at the results of the introduction of this new way of thinking upon a subject which had apparently no industrial or commercial bearings whatever. The shock to tradition and to established custom was too great for the unseeing of his day as it is of ours. The Inquisition came and the frightful religious wars of the next two centuries all because of the introduction of some new ideas into men's minds. Truly *the way men think* is the most practical concern in life, for all conduct flows from it!

But slowly the truth prevailed, and for four centuries the sciences of celestial mechanics and of Newtonian dynamics had greater and greater successes until it became impossible for even the most narrow visioned and unintelligent of men to doubt the fundamental correctness of Copernicus's ideas, even though they had been branded at their inception by popes and emperors and scholars, even as enlightened a one as Luther, as impious and untrue because they were "in direct contradiction to the Scriptures."

The second epoch of which I wish to speak is that in which we live; for it is quite as extraordinary as that of Copernicus in the rapidity with which new conceptions are being introduced and in the influence of these conceptions upon human life and conduct. Look at what has happened within my own life time in the field of physics, for example. When I started my graduate work in 1893 so sure were we of the physical foundations of our world, with its seventy-odd, unchangeable, indestructible elements, its well-formulated laws of matter-physics and its equally firmly established laws of ether-physics—ethereal and material phenomenon being sharply and definitely differentiable—and the principles of the conservation of energy, the conservation of mass, and the conservation of momentum acting as nature's policemen to keep the universe running eternally within the law, that it was then being frequently said, often by the ablest of physicists, that it was probable that all the great discoveries in physics had already been made and that future progress was likely to arise only by increasing the refinement of our measurements. Then came, only two years thereafter, the capital discovery of X-rays, an entirely new phenomenon, having no relation whatever to refinements of measurement. And two years later came radioactivity, which has now completely exploded the notion of the eternal character of the atom and revealed a world in which many if not all of the so-called elements are continually undergoing change, spontaneously shooting off projectiles with stupendous speeds, speeds far beyond those which it had ever been thought possible that matter in any form could attain. And then three years later came the beginnings of the quantum theory, which has shown us unmistakably (so it appears) that in the domain in which electrons live even Newton's laws no longer hold. And then a few years later, through Einstein's insight or speculations, as you prefer, mass and energy became interconvertible terms, and now we are all agreed that our former sharp distinctions between material, electrical and ethereal phenomenon have got to be abandoned. And, most important of all, from the amazing progress of physics has come more and more to the fore the idea that man has himself the ultimate ability to control for his own ends many of the changes going on in this changing world. How could it be otherwise when within a hundred years the very greatest of modern industries, that represented by the American Institute of Electrical Engineers, has been created through man's gaining year by year a larger and larger control of what used to be the most mysterious and apparently the most uncontrollable phenomenon of nature, the thunderbolts of Jove.

And geology, too, is telling us the same kind of a story. It is but fifty years since the death of Lyell, a man who, more perhaps than any other, first taught us to read the story of the rocks, yet now so well have we learned the lesson that to use a single illustration we count the exact number of years since the last ice age by the annual deposits on the shores of the Baltic Sea in just the same way as we obtain the age of a tree by the counting of its rings. And through all this expanding knowledge has come the definite evidence that man himself has been here perhaps a hundred thousand years, and yet that his extraordinary development, not physiologically, perhaps, but socially, has been exceedingly recent, much of it within a generation or two, since the developments in physics have given him control of the giant forces formerly wielded only by the Titans.

And the developments in biology taking place under our very eyes tell the same story. Bacteriology has already banished some of man's most dreaded scourges. In the fields of agriculture, horticulture and animal husbandry, man now builds plants and fruits, and even animals almost to suit his taste. If he hasn't a drought-resisting wheat, a seedless orange or a grape he goes to work to make one. If he does not find an ox adapted to resist the Texas winds like the buffalo and to put on flesh like a Durham, he creates one. As a result, then, of the modern pursuit of pure science in all its branches—the following of that inner urge simply to know, to explore, to understand, two new ideas completely foreign to the ancient world and to many races of the modern world as well have come into our western civilization. The one is

the idea of the possibility of progress, of continuous development; and the other the idea of man's ability to control and in the Providence of God (and I say it in all reverence) to determine to a large extent his own destiny, the idea of his own responsibility for the kind of an external world in which he lives. And if you wish to see the practical result of this changing of "the way men think," look at the difference between our own civilization and the static civilizations of Asia, where Nirvana is the goal of human life and a large fraction of the population reaches it quickly through starvation. Why is it that "fifty years of Europe is better than a cycle of Cathay"? Is it not simply because in certain sections of the world, primarily those inhabited by the Nordic race, a certain set of ideas have got a start in men's minds, the ideas of progress and of responsibility.

And these ideas have come about, I think, because in a few sections of the earth men have been led to follow simply the urge to know. First, to know this earth geographically, to explore it clear to the north pole and to the south pole, even when they knew there was not the remotest prospect of growing wheat or potatoes there. But now the days of geographical exploration are gone, and yet it is the same urge which leads on the descendants of these voyagers into the unknown—the astronomer to explore the heavens, however useless that may be, the physicist to study the properties of matter and radiant energy whether he sees any immediate use for his results or not, the biologist to delve as far as he can into the secrets of life and of organic growth.

On behalf of all those who are working in the field of pure science, all those pioneers who are pushing out beyond the present frontiers of human knowledge —where a few years hence the engineer and the other builders of a future more perfect civilization than our own will follow them—on behalf of all those who are struggling on in this field, which does not often meet with large public appreciation, I extend my heartfelt thanks to the American Institute of Electrical Engineers for helping to educate the public up to its values by recognizing it with an occasional Edison award.

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GUSTAF ENESTRÖM

THE recent death of Gustaf Eneström, of Stockholm, removes a most distinguished investigator from the field of the history of mathematics. For eighteen years editor of the *Bibliotheca mathematica,* a journal founded by him and devoted to the history of mathematics, he wielded a most powerful influence for greater thoroughness in historical research.

Eneström was born at Nora, in Sweden, on September 5, 1852. Trained mainly in mathematics and languages, he became connected in 1875 with the library of the University of Upsala and in 1879 with the royal library of Stockholm. Through his work as librarian he acquired extraordinary ability in matters of bibliography. Before 1899 he had comparatively little time for historical investigation. Later he came to enjoy greater leisure for research and he also acquired possession of large collections of mathematical books which afforded facilities for consulting original sources, such as perhaps few historians of mathematics have enjoyed.

Trained as a bibliographer, he possessed to a degree probably never before equalled, the art of bringing all the resources of a library to bear upon a particular problem. He had the patience for working out the details of an inquiry with extreme precision. As a result, his conclusions have been found to be almost invariably correct. He is the author of numerous short papers on mathematical history, but much of his work consisted in correcting errors of others, particularly of Moritz Cantor, the author of the well-known "Vorlesungen über Geschichte der Mathematik." Unlike the performances of certain other modern critics of historical works, Eneström's findings almost without exception constituted the final word on the subject.

Cantor and Eneström were wholly different minds. Cantor possessed the faculty of portraying in bold relief the history of his science extending over long periods of time. Unaided, he prepared three massive volumes carrying the history of mathematics from the earliest historic times to the year 1759. In this general survey some of the minute details received inaccurate statement. Eneström, on the other hand, never wrote a general history, nor even the history of a particular period. He devoted fifteen years to the microscopic examination of the ponderous volumes of Cantor. His notes are a store-house of information which no historian of mathematics can afford to ignore. Eneström's influence upon the younger generation of writers has been great. The historical notes in the French "Encyclopédie des sciences mathématiques" carry the marks of his thoroughness.

At the outbreak of the great war, the printing of the "Bibliotheca mathematica" was permanently suspended by Teubner in Leipzig. It was a great disappointment to Eneström that after the war no one in America seemed willing to finance the journal as an American publication.

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