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## AMERICAN MATHEMATICS DURING THREE QUARTERS OF A CENTURY[1]

BETWEEN the time of the discovery of America and the middle of the nineteenth century, Europe as a whole had had Euler and Lagrange; England had had Wallis and Newton; France had had Descartes, Fermat, Pascal, Monge and Galois; Italy had had Galileo and Ruffini; Switzerland had had the famous family of Bernoullis; Germany had had Leibnitz and Lambert; Norway had had Abel; but America had had no one who could reasonably be classed with these men as regards mathematical contributions. Nathaniel Bowditch, 1773–1838, is probably most worthy of consideration in this connection, but he fails to measure up to such high standards.

When the first meeting of the American Association for the Advancement of Science was held in 1848, various mathematicians of Europe who were then living had already made contributions which unquestionably outranked the best mathematical contributions that had been made in America up to that time. In support of this we need only remind ourselves of the discoveries relating to the convergence of Taylor's series and the founding of group theory by A. L. Cauchy, the fundamental contributions to number theory and to several other fields by C. F. Gauss, the interpretation of ordinary complex numbers by couples of real numbers and the discovery of quaternions by W. R. Hamilton, the Ausdehnungslehre of H. Grassmann, the projective methods of generating geometric figures due to J. Steiner, etc.

During the year of the first meeting of this association there occurred the death of a very original Bohemian mathematical philosopher whose work failed to receive sufficient recognition until quite recently, *viz.*, B. Bolzano, who had an Italian father and was professor of the philosophy of religion in the University of Prague. It has recently been announced that Bolzano gave the first known example of a continuous function which has no derivative at any of its points, and that as early as 1830 he had developed a method for constructing continuous functions having no derivatives. This is of great historic interest in view of the fact that the later work of Weierstrass and others along the same line attracted unusual attention

[1] Address of vice-president and chairman of Section A—Mathematics—American Association for the Advancement of Science. Cincinnati, Ohio, December, 1923.

when it was made public after 1860. Ignorance of the earlier work due to Bolzano doubtless enhanced the reputation of Weierstrass and others at that time.

In 1848 Europe had half a dozen regular mathematical periodicals, while America had none. The few American mathematical periodicals which had been started earlier had had an ephemeral existence, and this was also the fate of the *Mathematical Monthly* started at Cambridge, Massachusetts, by J. D. Runkle, ten years later. In fact, our first really permanent mathematical periodical was the *American Journal of Mathematics,* of which the first volume appeared 30 years after the first meeting of this association. Four years earlier, the *Analyst* began to appear at Des Moines, Iowa, under the editorship of H. E. Hendricks. It was followed in 1884 by the *Annals of Mathematics,* which has since then rendered very valuable service to the interests of mathematics in our country and is now being published by the Princeton University Press.

The preceding remarks may serve to show that when this association was started America had contributed very little to the advancement of mathematical knowledge, while Europe had a considerable number of mathematical investigators who had already made contributions of the greatest value. This great lead on the part of European mathematicians throws light on the later mathematical developments in our own country. Twenty years after this association was started, a well and favorably known mathematical review, entitled *Jahrbuch über die Fortschritte der Mathematik,* began to appear. In the list of abbreviations for the journals reviewed in this yearbook, no American publication is noted in the first four volumes, although as many as 78 foreign journals appeared already in the first volume and more in later volumes. In the fifth volume, relating to the publications of 1873, the *Transactions of the Connecticut Academy of Arts and Sciences* were noted, and in the later volumes the number of American journals thus noted gradually increased, so that in Volume 45, for 1914 and 1915, there are 13 American journals in a total list of 180. A ratio of about 7¼ per cent. is evidently still too small for our country and does not give due credit to American mathematical contribution a decade ago.

Mathematicians took a relatively insignificant part in the early development of the American Association for the Advancement of Science. The similar organization of Great Britain, France and Italy make a better showing as regards our subject. The most prominent mathematical figure in the early history of our association was Benjamin Peirce, who was elected as the sixth president, and presided at the second meeting held in this state, in 1853,—the first such meeting having been held two years earlier in the city in which we are privileged to meet to-day. The most significant mathematical contribution of Benjamin Peirce was, however, made much later and related to systems of complex numbers obeying the associate law of multiplication; *i.e.,* associative algebras. A memoir on this subject containing the earliest classification of systems of complex numbers was read by him in 1870, before the National Academy of Sciences, and a small number of copies in lithographed form were then distributed. It was published posthumously in 1881, in Volume 4 of the *American Journal of Mathematics.*

The publications on pure mathematics which appeared in the early volumes of the *Proceedings* of this association were insignificant as regards important advances. To illustrate still further the backwardness of American mathematics at that time, we may recall a few facts relating to the history of substitution groups and the Galois theory of equations. During the year in which this association held its first meeting, J. A. Serret taught the former subject in Paris and ten years later R. Dedekind taught the latter in the University of Göttingen. About the same time E. Mathieu and C. Jordan wrote doctors' theses on the theory of substitutions in the University of Paris, and in the early fifties of last century E. Betti wrote various expository articles on substitutions and the Galois theory of equations for the early volumes of the Italian mathematical journal known as the *Annali.* In Great Britain, A. Cayley and W. R. Hamilton were working along the same line at about this time.

We therefore find that during the first decade of the life of this association the mathematicians of France, England, Germany and Italy took an active part in developing and expounding the comparatively new theory of substitutions and the Galois theory of equations which was based thereon. Not only did the American mathematicians take no active part in this development then, but they postponed such activity for about a quarter of a century longer, when J. J. Sylvester gave the first course along this line in Johns Hopkins University during the first half year of 1882–1883. It is well known that American mathematicians soon thereafter began to take a significant part in the development of this field. This activity was inaugurated largely by O. Bolza, F. N. Cole, H. Maschke and E. H. Moore. The success with which later American mathematicians met in this field is partly reflected by the fact that two names of Americans appeared among the eleven which were cited in 1909 in the *Encyclopédie des Sciences Mathématiques* as those who had especially aided in developing this theory. The gradual relative increase of emphasis on the group concept in algebraic work may be illustrated by the fact that in the first 27 volumes of the Jahrbuch (1868–1897) this subject was reviewed under the general heading "Elimination," "Substitution," etc. In Volume 28 the term "Group" appeared for the first time in the heading. From Volume 29 to Volume 45 the heading began with "Substitution and group theory," but was concluded with "Determinants, elimination and symmetric functions." In Volume 46, covering the three years 1916–1918, the heading was changed to "Group theory" alone. The chapter headings of this volume exhibit also increasing emphasis on this concept in other fields. What is perhaps of more general interest is the fact that while articles on the Galois theory of equations are not found here those relating to the abstract theories of fields, moduli and systems of hypercomplex numbers are classed under the heading of "Group theory" in this volume, so that the first significant American research paper in pure mathematics, *viz.*, the memoir of Benjamin Peirce, to which we referred above, would now be classed in this review as belonging to the general domain of group theory. Hence, the fact that American mathematicians were so slow in entering this field in its substitution group form is the more significant.

The history of the theory of determinants presents a somewhat similar picture as regards late participation on the part of Americans. In 1855 the "Mathematical Dictionary," by Davies and Peck, was copyrighted. The word determinant is not defined therein, although the subject of determinants had then been developed in Europe during several decades and two textbooks on this very useful subject had appeared there. It should, however, be added that a few of the leading American mathematicians began to take notice of this subject about this time, as may be seen from the fact that Benjamin Peirce developed its elements in his "System of Analytic Mechanics," 1855, and that J. E. Oliver published the first article of "A treatise on Determinants" in the closing volume of the *Mathematical Monthly*, 1860. Americans made few contributions towards the development of this subject until much later, and the first American textbook thereon was published by P. Hanus as late as 1886. This was just a quarter of a century after the appearance in England of a small textbook by Spottiswoode on the same subject.

If a well-informed mathematician of the present day could be transported with all his attainments and powers to a world like ours was in the middle of the nineteenth century he would find it easy to attain great eminence in a short time by exhibiting the solution of a system of $m$ linear equations in $n$ unknowns by means of matrices and their ranks, by explaining the applications of the modern theory of integral equations or of the theory of aggregates, by outlining the fundamental concepts of Klein's Erlangen Program, and in many other ways. The great mathematical world progress during the three quarters of a century just closed is, however, not confined to the new theories which have arisen within this period. The additions to the older theories are equally substantial.

During the year in which this association held its first meeting, B. A. Gould, who later became a noted astronomer and president of this association (1868), secured the Ph.D. degree at the University of Göttingen, having studied with the noted mathematician, C. F. Gauss. This calls to mind two important facts relating to the American mathematical situation in the middle of the nineteenth century, *viz.*, many of the ablest mathematicians were also astronomers and made their reputation in the field of astronomy, and some students realized that the European universities offered much greater mathematical opportunities than those of our own country. The latter realization became more pronounced in the early eighties of the last century, when a large number of American mathematical students began to go to Europe, especially to the German universities, for advanced study. As soon as a considerable number of young American mathematical students had an opportunity to become practically as fully acquainted with the fertile fields for research as the young men of Europe, a few Americans began to exhibit as deep an interest in research as their European colleagues. The great mathematical lead which Europe had maintained for centuries seems to be largely due to the fact that its young men enjoyed better advantages than those of our country. At any rate, when Americans in sufficiently large numbers enjoyed similar advantages, they were soon able to effect a marked change in their own country.

In America, productive mathematical research has always been practically confined to the colleges and universities. The most noteworthy exception is furnished by the work of G. W. Hill. We have had no kings who were anxious to attract to their courts the leading mathematicians of the world, nor have we had academies whose income was sufficient to support leading scientific investigators. Among the great national capitals of the world, Washington is conspicuous for not yet having produced or maintained any eminent investigators in pure mathematics. In this respect the comparison with Paris, Berlin, London, Rome, Stockholm, Christiania, etc., is humiliating. If we turn to the catalogues of our colleges for the year when this association held its first meeting we find that Harvard under the leadership of Peirce had then made great recent progress, but even here the comparison with present conditions is very striking.

The catalogue for 1848–49 states that candidates for the freshman class were examined by the mathematical department in the following books: Davies's and Hill's arithmetics; Euler's algebra, or Davies's first lesson in algebra to extraction of the square root; and "An introduction to geometry and the science of form, prepared from the most approved Prussian textbooks," to "VII of proportions." In the catalogue of Yale for the same years we find that no examination in geometry was required for admission, but that there was an examination in arithmetic and in Day's algebra to quadratic equations. The catalogue of Princeton for these years does also not impose any examination in geometry. In fact, the catalogue for the two preceding years did not even impose an examination in algebra, but the catalogue for the years in question states that an examination is held in arithmetic and in the elements of algebra through simple equations.

What is perhaps of more importance from the standpoint of mathematical research than the limited mathematical preparation on the part of the students is the small number of those giving instruction in our subject. At Harvard we find that Benjamin Peirce was professor of astronomy and mathematics, and that Joseph Lovering was professor of mathematics and natural philosophy. Besides these two professorships, which were only partly devoted to pure mathematics, we find here only one tutor in mathematics. At Yale we find one professor of mathematics and one tutor in the same subject, while at Princeton we find that Stephen Alexander, who about ten years later was elected president of this association, was professor of mathematics and astronomy, while J. T. Duffield was adjunct professor of mathematics.

It should also be noted that mathematicians in these early days allowed themselves to be drawn into too many activities which did not promote their advance in their chosen profession. For instance, Benjamin Peirce was not only professor of astronomy and mathematics, but he did much work on the Nautical Almanac, whose office was located at Cambridge from its inception in 1849 until its removal to Washington in 1866. It is, however, not true that he was in charge of this almanac for some years, as is stated on page 338 of the second edition of Cajori's "History of Mathematics." The men in charge during the life time of Benjamin Peirce were, in order: C. H. Davis (1849–1856), Joseph Winlock (1856–1859), C. H. Davis (1859–1861), Joseph Winlock (1861–1865), J. H. Coffin (1865–1877), and Simon Newcomb (1877–1894). Peirce was consulting astronomer from 1849 to 1867.

He wrote a considerable number of elementary mathematical textbooks for the Harvard students and was also superintendent of the U. S. Coast Survey from 1867 to 1874. From 1852 to 1867 he was in charge of the longitude determination of this survey, etc. In view of these varied and absorbing interests, it is not surprising that he failed to keep in close touch with some of the results of the eminent European mathematical investigators of his days. Such close touch on a large scale was not maintained by American mathematicians until about the middle of the period under consideration.

The wide difference between the mathematical situation of those days and of our own time is also illustrated by the biography of J. H. Van Amringe, the first president of the American Mathematical Society. Even before he graduated from Columbia College in 1860, with the A.B. degree, he was tendered an instructorship in no fewer than five widely different departments, *viz.*, Greek, Latin, history, chemistry and mathematics. He chose mathematics and taught this subject there for almost half a century, serving also as dean of the college for a number of years, but never becoming a mathematician in the modern sense of this term. A dozen years after this association was founded, and for about two decades longer, we find that a graduate with the A.B. degree was regarded as sufficiently well trained to assume a regular instructorship even in some of our leading institutions.

The list of presidents of this associatioon includes the names of a considerable number of the most eminent scientists of our country. We find therein several who are known in the history of mathematics, but not until 1922 do we find one who was noted principally on account of his achievements in pure mathematics. In fact, there was only one predecessor who held the simple title of professor of mathematics at the time he served as president of this association, *viz.*, H. A. Newton, who was president in 1885 while he occupied the chair of professor of mathematics in Yale University. His reputation was, however, based principally on his work on meteors and not on contributions towards the advancement of mathematical knowledge. He never attained eminence in this field.

Benjamin Peirce was professor of astronomy and mathematics in Harvard, when he served as president of this association in 1853, and was then widely known on account of his researches relating to the perturbations of Uranus and Neptune. Joseph Lovering, who presided over this association twenty years later, was then professor of mathematics and natural philosophy in the same institution and was known chiefly for his contributions to the latter subject. Alexis Caswell, who presided in 1857, was professor of mathematics and astronomy in Brown University and was noted principally as an astronomer and educator. His scientific attainments were meager. He later became president of Brown. R. S. Woodward, who presided in 1900, was then professor of mechanics and mathematical physics in Columbia University, and was noted for work in applied mathematics.

The list of presidents of the British Association is naturally more favorable as regards pure mathematics. Four years before our association was started the British association had as president George Peacock, who is known chiefly for his work in formal mathematics and for historical articles on our subject. He was one of the founders of the Analytic Society, which had for its object the introduction into England of the calculus notation used on the continent of Europe, but he was never very eminent as a mathematician. In 1878 William Spottiswoode was president of the British association. He was a more eminent mathematician and was also known chiefly for his work in pure mathematics, having published the first book on the theory of determinants in 1851, and having assisted in the development of various mathematical subjects. Five years later, Arthur Cayley, the most eminent of the pure mathematicians of Great Britain, served as president of the British association. J. J. Sylvester, the second in eminence among these mathematicians, was president of Section A (mathematics and physics) in 1869, but he never served as president of the entire association.

During the first third of the period under consideration, American mathematicians were comparatively isolated and they developed like the isolated trees, with many relatively large branches—not growing very tall, but having the breadth and the grandeur associated with the tree that stands alone. Later, when mathematical journals became more general, many of them developed like the trees of the forest, with much greater height, but with much less imposing branches and breadth. They became members of the common mathematical forest, and had to grow tall to receive sunshine. In particular, the pure mathematician had either to grow tall or to stifle in the shade of his European colleagues working along similar lines. Comparatively few such forest trees have developed into giants among us, but these few have tended to remove the stigma formerly associated in our land with being only a mathematician. It is beginning to be recognized that to be a giant in the forest of mathematics it is necessary to develop upward rather than to develop imposing branches.

If we may carry this figure a little further, it may be noted that the fruit trees, as a rule, do best when they are somewhat isolated. These are typified by those who are interested mainly in teaching. The teacher who desires to serve most efficiently has usually found it necessary to direct his development along the line of the isolated tree. The same is true to a large extent as regards those working in applied mathematics. The dense mathematical forest is not the place for these. They need more space for breath. Until recently the American Association for the Advancement of Science could find no tree in the mathematical forest which was both American and of sufficient magnitude to bear its highest office with dignity. We all rejoice that this association recently acknowledged the existence of one such in the person of E. H. Moore, a native of the state in which we are meeting.

While we are proud of the fact that the common mathematical forest of the world contains already a considerable number of American trees whose size is attracting favorable comment everywhere, and that the number of smaller American trees in this forest which are growing upward rapidly and give great promise of becoming some of the most imposing trees therein is rapidly increasing, we can not afford to neglect paying due respect to the fruit trees and to the other isolated trees with their magnificent proportions and symmetry. The forest tree is most valuable for certain types of service, but for other types the fruit trees are more valuable. The former are naturally attracting most of our attention at present because they are such recent products of our history and they have a more permanent value. They are now especially the centers of interest of the American Mathematical Society, while the Mathematical Association of America and the National Council of Teachers of Mathematics center their interests in the latter.

It was noted above that E. H. Moore was one of the inaugurators of the successful study of group theory in our land. His most eminent student in this field is L. E. Dickson. It has also been noted that Americans entered this field long after various Europeans had begun to cultivate it with marked success. This observation might give an erroneous impression unless we add that new phases of this subject presented themselves later and some Americans aided in the development of these from the beginning. In particular, the subject of finite continuous transformation groups was opened by Sophus Lie in about 1874, only two years before Johns Hopkins University opened its doors to students, and hence this subject was practically new when a considerable number of Americans began to take an active part in mathematical research. The foundation of the theory of infinite continuous groups was laid about ten years later (1883). The geometry of infinite discontinuous groups was opened by Camille Jordan, Sophus Lie and Felix Klein, at a slightly earlier date. A little later, about 1882, Henri Poincaré illustrated in a striking manner by his theory of Fuchsian functions the services which certain discontinuous infinite groups are able to render in the theory of functions. About 1896 Georg Frobenius opened up a practically new and extensive field in the theory of linear groups of finite order by his study of group characteristics and related subjects. A number of smaller new fields in group theory, such as those of the group of isomorphisms (1893) and the commutator sub-group

(1896) were opened from time to time. Into these newer fields of group theory certain Americans entered either from the beginning or soon thereafter. There is, therefore, a striking contrast between their entrance into these newer fields and their entrance into the fields of the elements of finite substitution groups and abstract groups, as noted above. Fashions in group theory, as well as in other mathematical subjects, have changed fairly rapidly during the last three quarters of a century, so that the stylish intellectual dresser in this subject could easily be distinguished from those who were more or less old fashioned.

While styles have changed within group theory itself, it seems that these changes have not been sufficiently rapid or sufficiently radical to satisfy all those who dress in accord with the most recent approved intellectual fashion. Some of these abandoned, either temporarily or permanently, the ranks of the group theory force and joined with success others, in particular the postulationalists, when these came to render conspicuous and much needed service to our science. Oscar Bolza, on the other hand, left the ranks of the workers in group theory to join the force which had created a new and wholesome interest in the older subject of calculus of variations. American mathematical contributions have been greatly enriched by the disciples of Bolza, such as G. A. Bliss, in this great field. Changes in main interests have doubtless been very wholesome when they were effected with such success as has been the case in several instances in the history of American mathematics.

We have already mentioned three fields in which American mathematicians have rendered conspicuous service during the last forty years, *viz.*, group theory, postulates and the calculus of variations. It would be very unfortunate if we should convey the impression that American mathematical contributions were practically limited to these fields. In at least two other fields Americans have secured international prizes during the period under consideration, and in others the recognition has been equally definite. The reasons why we have paid more attention to the first of the fields mentioned above are that it comes first historically from the standpoint of considerable researches, and that we are able to speak about it with more certainty than about some of the other fields. The question of relative importance is a difficult one and has not been raised here. We all delight in every serious mathematical contribution, and may be pardoned if we delight especially in American contributions of fundamental importance regardless of the particular line of work to which they relate.

The greatest mathematical monuments which America has raised during the period under consideration are its series of periodicals. In particular, the *American Journal of Mathematics,* the *Annals of Mathematics,* the *Bulletin of the American Mathematical Society,* the *American Mathematical Monthly,* the *Transactions of the American Mathematical Society,* and the *Mathematics Teacher.* Compared with the older European series, several of which extend beyond 100 volumes apiece, these American series are as yet quite limited, since none of them has reached its 50th volume. They represent, however, a very substantial mathematical contribution made by Americans collectively within half a century. On the other hand, the mathematical advances found in American textbooks and treatises have not yet been extensive and are practically confined to works which appeared during the last two decades. In particular, our country has not produced a calculus which compares favorably with Jordan's "Cours d' Analyse" as regards extent and originality, nor has it produced an algebra which appears creditable from this point of view when compared with Weber's "Lehrbuch der Algebra." In function theory the works of Osgood and Pierpont, and in geometry the works of Veblen and Eisenhart, are conspicuous examples of recent advances in American textbooks as regards the point of view in question.

During the period under consideration several brief series of mathematical periodicals have also arisen in our land. The most advanced of these was the *Mathematical Review* which was expected to appear bimonthly under the editorship of W. E. Story, of Clark University. This university lately discontinued its mathematical department on account of lack of funds. During the second half of the period under consideration Clark University exerted a strong influence on advanced mathematical work in our country, as may be seen from the work of those who secured their doctor degrees at this institution. Hence the mathematical public regrets very much that this source of mathematical activity had to be discontinued. It is, however, a consolation to be able to note that since the time when this *Review* was started and when mathematical activity at Clark was at its peak many new centers of such activity have been created in our midst. The first number of this *Review* appeared in July, 1896; the second number appeared about nine months later, while a third number appeared in 1899. No additional numbers have appeared. Two journals of the older type, *viz.,* the *Mathematical Visitor* and the *Mathematical Magazine,* were started by Artemas Martin in 1877 and 1882, respectively. These journals were devoted mainly to solutions of problems, and the successive numbers appeared too irregularly and at too long intervals to be very useful. In particular, the second part of number 12 of volume 2 of the latter journal was dated September 1910, more than 28 years after the date of the first number of volume 1.

The tendency towards forming national mathematical organizations with regular official journals is one of the noteworthy developments of the second half of the nineteenth century, and it was represented in our country by the organization of the American Mathematical Society in 1888. Within the last decade this tendency manifested itself strikingly among us by two new national mathematical organizations. The younger and larger of these is the National Council of Teachers of Mathematics, organized in 1920 and having a membership of more than 3,000. The official journal of this society is the *Mathematics Teacher,* which had been previously published by the Association of Teachers of Mathematics in the Middle States and Maryland. The older of these two mathematical organizations is the Mathematical Association of America, to which we referred above.

America has had its share of successful elementary textbook writers who secured considerable reputation on the part of the general public. In addition to these there have been hundreds who published textbooks which were designed to meet special local needs, but which often had greater faults than were possessed by the books which they replaced. Much energy has doubtless been wasted here which should have been directed towards higher mathematical attainments on the part of the authors. Substantial improvements in textbooks are, however, very important, and authors usually learn something about the subject while preparing the manuscript of a textbook.

America has also had its share of the so-called mathematical prodigies. Among these T. H. Safford (1836–1901) is well known. He became professor of astronomy in Williams College in 1876. In his eleventh year he is said to have published an almanac, computed for this city, which soon reached a sale of 24,000 copies. Mathematical prodigies, like the successful elementary textbook writers, secured considerable public notice, but most of them contributed little or nothing towards the development of our subject. Their marvelous mathematical feats are of more interest to the psychologist than to the mathematician. In Europe Ampère and Gauss are noted as prodigies and they are also noted contributors towards the advancement of our subject, but in America the mathematical prodigies have thus far contributed little to the advancement of pure mathematics.

The actual and relative mathematical advances made by Americans during the last 75 years are conspicuous, but not satisfying. We have not yet attained relatively as high a standing as we should aim to attain, or as those belonging to some of the other sections of this association—such as the astronomers and the geologists—have already attained. Not one of the 50 incorporators of the National Academy of Sciences had made important contributions to the increase of our knowledge of pure mathematics, although six of them enrolled in the section of mathematics. It is only recently (1920) that this association recognized conspicuously the advances in American mathematics by devoting an entire section to them. From 1882 to 1919 mathematics and astronomy constituted one section, and the astronomers usually commanded the major interest at our meetings. Let us hope that the letter which has been assigned to our section will represent in the future not only the fundamental character of our subject, but also the relative advances made therein. To work hard and long before receiving public recognition seems to be the lot of most of us, but the sense of growth is keen and definite in our field and this sense of growing intellectual insight and power is our main reward as regards mathematical research.

G. A. MILLER

UNIVERSITY OF ILLINOIS

## THE PRACTICAL VALUE OF PURE SCIENCE[1]

WHEN I reflect that preceding Edison medallists have been men of the type of Charles F. Brush, who first showed the world that electricity might be used for city lighting; Alexander Graham Bell, whose invention was at the base of the whole vast system of modern communications; Frank Sprague, who was responsible for the application of electric power to railway transportation; M. I. Pupin, who made long distance telephony possible; J. J. Carty, under whose inspiration and leadership the telephone repeater and amplifier, with all that they mean to the enrichment of modern life, have been brought forth, and others of like achievement in the application of electricity to large industrial uses, I feel that there may have been a misunderstanding or a mistake in connection with this year's award. For when I look over my thirty years of scientific effort I can find no industry which has grown out of my researches, nor even any which have been very immediately benefited by them.

Since this survey certainly reveals nothing of great industrial consequence I am obliged to adopt either the mistake-theory, or, as an alternative, to assume

[1] Response to the presentation of the Edison Medal at Del Monte on the evening of October 4, when the president's presentation address was made by Dr. Frank B. Jewett from his home in New Jersey, his voice being carried over telephone lines and amplified through the magnivox so as to be very distinctly audible to the entire audience seated at dinner in the Del Monte Hotel, three thousand miles away.