SCIENTIFIC BOOKS

Relativity and Modern Physics. By G. D. BIRKHOFF, Cambridge, Harvard University Press, 1923. xi + 283 pp.

WHEN one reads the opening sentence of the preface, "Although great interest has been aroused by Einstein's theory of relativity, there has not been available an approach to the subject which treats it adequately but with a minimum of technical requirements," he is apt to question the statement in view of the many books which had appeared before the statement was written, but then he concludes that the author is going to give us an account of how he thought himself into the theory. And he finds that the treatment is by no means a mere rearrangement of the material in other books. In many respects there is an originality in the point of view and treatment which makes it a book worthy of very careful study.

More than half of the book deals with special relativity, and in the greater part of this the phenomena of a space-time continuum of two dimensions are considered, although the generalizations to the space-time continuum of four dimensions are adequately presented. This specialization of the problem to two dimensions makes graphical representation easy, and consequently is very helpful. However, we question the advisability of carrying it to such an extent. For, after all, the student must, sooner or later, learn to think in four dimensions.

After a brief review of mathematical physics from the classical point of view, the next three chapters are devoted to a postulational treatment of space-time continua of two dimensions. After defining the apparent time t and apparent distance x of a particle B relative to a particle A by means of light-flashes from A to B and back in times t_1 and t_2, as measured at A, he introduces the graphs in terms of cartesian coordinates t_1 t_2 and x, t. Then comes the assumption (p. 24), "If in the space-time under consideration any naturally moving particle is assumed to be at rest and the velocity of light is taken to be constant (in particular, unity), then all other naturally moving particles will appear to move with constant velocities less than that of light." One asks "What is a naturally moving particle; is this its definition?" We should prefer a statement somewhat as follows: Any straight-line on the chart of an observer A making an angle ϑ with the t-axis would be interpreted by A as the world-line (previously defined) of a particle whose velocity relative to A is tan ϑ; we call it a naturally moving particle. The assumption tan $\vartheta < 1$, which follows later from another assumption, is evidently introduced here to avoid imaginaries when the equations connecting the coordinates of two charts are obtained (p. 31).

In these equations there appears a function λ whose determination depends upon further assumptions concerning the nature of the space-time continuum, and chapters III and IV are given over largely to this question. Instead of starting with Einstein's fundamental postulate: "Natural phenomena run their course according to the same general laws with respect to coordinate systems moving relative to one another with constant velocity," which is essentially equivalent to Birkhoff's definition of an isotropic space-time (pp. 34, 49), the latter considers also what he calls "aeolotropic" space-time, from the definition of which it follows that space and time have an absolute significance. Both isotropic and aeolotropic space-times are isometric in the sense that "all portions of the space-time continuum are the same in so far as the metric relations which obtain in them are concerned" (p. 33). I believe that if the author had developed the isotropic case first and then considered the aeolotropic case, there would have been a gain in clarity. Moreover, repetitions could have been avoided, and the space thus saved devoted to added details which would help the reader. Particular attention should be called to the manner in which the expressions for the proper time are developed both in the two-dimensional and four-dimensional continua (pp. 31, 156); it emphasizes the fact that methods adopted in the former case can not be used necessarily in the latter.

After deriving in Chapter V the well-known formula for the composition of velocities in isotropic space and interpreting its significance, the author obtains by very careful physical analysis the equations of motion of a particle under the action of a force and points out the significance of the apparent mass, $m/\sqrt{1-v^2}$. Following the consideration of collision of particles, the dynamical equations of a system of particles are derived. These are used for the determination of the statistical mass of a system of particles in steady state. An interesting application is made in determining the expression for the pressure of light, when it is assumed that a beam of light is approximately a system of particles of slight mass moving with a velocity nearly unity. Another important application appears at the close of Chapter VII in the derivation of the differential equations of a perfect fluid in one-dimensional hydro-dynamics. The reader gets in this investigation and in the applications to a particle, measuring rod and clock, a valuable insight into the ideas and methods of special relativity. In the first part of this chapter the same equations are derived from a set of postulates. They serve to characterize the form of the equations, but I prefer the derivation previously referred to, as it emphasizes the physical character of the problem. In Chapter IX the same topics for four-dimensional space-time are developed in essentially the same manner.

The development of tensor calculus is contained in Chapters VI and VIII. Although much of the treatment of the former deals with two dimensions and linear transformations, the formulas are general and the reader is so informed. Whether or not it was advisable to make these restrictions in presenting this subject depends upon the reader. After the introduction in Chapter VIII of the idea of geodesic coordnates at a point, that is, coordinates for which the first derivatives of the components- g_{ij} of the metric tensor are zero at a point, there is a full and clear presentation of the particular type of geodesic coordinates introduced by Riemann in terms of which the equations of the geodesics through the particular point are linear. The author makes frequent use of these Riemannian coordinates throughout the remainder of the book. In this chapter he uses them to determine the Riemann curvature tensor, to define and explain covariant differentiation and to obtain the four fundamental identities connecting the first covariant derivative of the contracted tensor R_{ij} and the first derivative of the curvature invariant R. There is no denying the fact that the use of geodesic coordinates adds greatly to the simplification of processes involving covariant differentiation, but it would have been helpful had the general expression for covariant differentiation of a mixed tensor been written down. Also, it is to be regretted that the reader must look elsewhere for an interpretation of the geometrical significance of the Riemann tensor, either from the Riemann point of view, or in connection with the parallel displacement of a vector about a closed circuit. A further interesting and important application is made in Chapter XIII, namely, the determination of all space-time continua characterized by tensor equations which are linear and homogeneous in the differential coefficients of the second order and do not contain any higher derivatives. There are three essential types: (a) the Einstein spaces $R_{ij} = 0$; (b) the spaces for which $R = 0$ and $R_{ij} \pm 0$; (c) the spaces which can be mapped conformably upon euclidean space of four dimensions.

In the preface special attention is called to the postulational treatment of electromagnetism which appears in Chapter XII. The first postulate concerning the character of the force acting on a test charge yields a skew-symmetric tensor whose components are identified with the electric and magnetic intensities of the field. The second deals with the state of the field; the third requires the equations to hold for a state of equilibrium and the fourth that the derivatives of the intensities and the density of electricity enter linearly. The problem is to find the equations which satisfy these requirements and in addition are tensor equations with respect to the space-time continuum of special relativity. By very interesting mathematical analysis Maxwell's equations are derived. Here is a good evidence of what Einstein's requirement that the equations of physics be in tensor form means for the development of mathematical physics.

The reader is introduced to gravitation first in two dimensional space-time (Chapter IX). After a brief presentation of the Newtonian theory, Einstein's principle of equivalence is clearly defined (p. 141) and is used as the point of departure. On page 144 it is assumed "that the intrinsic equations $\int ds =$ an extremum, and $ds = 0$, respectively determine, as heretofore, the paths of naturally moving particles and light waves." To agree with the Einstein theory this must mean that a naturally moving particle is one in free space. It will be noticed that the author, like Einstein, does not assume that the paths of light are geodesics. However, in Chapter XVI, which deals with planetary motion, the deflection of light rays and the displacement of the spectral lines, as derived from the Schwarzschild form for ds^2, he takes the equation of a path of light as a limiting case of the motion of a particle. In obtaining the equations of hydrodynamics and electrodynamics for general relativity Einstein makes use of the explicit hypothesis, "With an appropriate choice of a local reference system special relativity holds for every infinitesimal four-dimensional domain or volume-element of the world." Birkhoff states its equivalent in terms of isotropic space-time (p. 145) and then proceeds to use it in connection with the equations

$$\frac{\delta T^{i\alpha}}{\delta x^{\alpha}} = 0,$$

where

$$T^{i\alpha} = \rho \frac{dx^i}{ds}\frac{dx^{\alpha}}{ds},$$

which are the equations of hydrodynamics in tensor form as shown in Chapter VII. He says that in general relativity these equations "must have the extended form

$$\frac{\delta T^{i\alpha}}{\delta \xi^{\alpha}} = 0\text{''},$$

where ξ^{α} are Riemannian coordinates. This is clearly an additional assumption; it means that the local coordinate system referred to is geodesic. Einstein[1] recognizes it as an assumption. In general coordinates it means that covariant derivatives are to replace ordinary derivatives. These remarks apply

[1] "The Meaning of Relativity," p. 91.

also to the generalization of the equations of electrodynamics of special relativity. Since gravitation is necessarily a four-dimensional space-time problem, this chapter merely serves to break the ground, but in Chapter XIV the reader finds the full treatment of the problem. The author's determination of tensor equations (previously mentioned) plays in well with his development of the subject, which is essentially a combination of Einstein's treatments of gravitation in his 1916 paper and in "The Meaning of Relativity," although the author makes no reference to the latter. Chapter XV is given over to a study of the solar field. In deriving the Schwarzschild form it is shown that the usual assumption that the field is independent of the time is unnecessary; this is an important result.

There are no footnotes, but at the end of the book there is a bibliography arranged according to the chapters in which the special subjects are treated. The book is well printed, and the only criticism as to form which I have to make deals with the page headings. It seems useless to put the title of the book at the head of alternative pages, when, had this space been used for section headings, it would have served a good purpose. Also an indication of chapter and section on each pair of pages would have made cross-reference easy.

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SPECIAL ARTICLES

SOME EFFECTS OF INSULIN AND GLUCOKININ ON MAIZE SEEDLINGS

IN experiments with a genetic type of maize, which is chlorotic and apparently unable to use its carbohydrates properly, tests were made to determine the action of insulin and glucokinin on the seedlings while still dependent upon the endosperm for food. The plants were started in sand, and when the first leaves began to unroll the seedlings were transferred to individual test tubes containing 20 cubic centimeters of distilled water and known quantities of either insulin or glucokinin. Each plant was supported at the top of the tube by absorbent cotton, and the roots were protected from the light. All of the fluid in the tube was poured off and fresh solution added every 48 hours, without removing the plant from the tube or disturbing its root system. Neither nutrient solution nor iron were given the plants during the experiment. The insulin and glucokinin were prepared by Collip's methods[1, 2] from fresh beef pancreas and onion tops, respectively.

[1] *Trans. Royal Soc.*, Canada, XVI, 1922.

[2] *Jour. Biol. Chem.*, LVII, 65–78, 1923.

As insulin and glucokinin produced in general the same response in seedlings, although different dilutions of the two substances were required, it is possible to group the results. In solutions carrying from 1 per cent. to 0.005 per cent. of glucokinin (or corresponding dilutions of insulin) growth was retarded, more or less, in direct proportion to the amount of glucokinin or insulin present. This retardation was particularly evident in the higher concentrations, and was more striking in root growth than in top growth. In the stronger solutions of both insulin and glucokinin the formation of secondary roots was practiaclly stopped. The growth of the primary roots and of such secondary roots as had appeared was behind that of the untreated controls growing in distilled water. The delayed growth of the tops was less evident during the first few days of insulin or glucokinin treatment, but became progressively more apparent as the experiment proceeded.

Seedlings grown in solutions of less than 0.005 per cent. glucokinin (or corresponding dilutions of insulin) showed some evidence of beneficial effects, as measured by the amount and character of root growth and the amount of top growth when compared with untreated controls. The retardation of growth by strong solutions and the beneficial effects of very dilute solutions were noted both in the series of chlorotic seedlings and in series of normal green seedlings treated with insulin or glucokinin.

A third reaction, an increase in the development of the chloroplastid pigments, was found in chlorotic plants growing in insulin and glucokinin solutions. Even when grown in strong solutions which were unfavorable to root development, chlorotic plants produced enough of the green pigments to appear distinctly green when compared with the untreated controls. A pair of chlorotic plants chosen at random, the one from an untreated series and the other from a series receiving so much insulin that the root growth was reduced to about one third normal, may serve as examples of the relative amounts of chloroplastid pigments developed. The untreated plant assayed less than 1 per cent. green pigments (chlorophylls), and the plant grown in insulin 28.7 per cent. in terms of the green pigments present in an untreated green seedling. The yellow pigments (xanthophyl and carotin) in the treated chlorotic seedling assayed almost 200 per cent. as compared with the untreated chlorotic seedling. Large numbers of seedlings of the strain of chlorotic maize used in these experiments have been grown under both field and greenhouse conditions, in connection with genetic studies, but no plant was ever found which had developed an appreciable green color.

In view of the fact that insulin and glucokinin may not be absorbed readily by the roots of plants