author says on page 101 "The operation of finding the square root of a number is distinctly Greek."

In closing, the reviewer desires to record one more surprise to himself when he read on page 77 that Diophantus "was searching in general for classes of numbers instead of particular numbers, and it is the class, as such, that is primarily sought in an indeterminate equation." It is well known that Diophantus usually gave only one solution even when the equation under consideration admitted an infinite number of solutions. Mathematical historians usually direct special attention to the fact that the Greeks were satisfied with one solution even in their geometric constructions. The reviewer never saw any evidence in support of the statement that Diophantus searched in general for classes of numbers in the solution of indeterminate equations.

G. A. MILLER

UNIVERSITY OF ILLINOIS

SPECIAL ARTICLES

POSITIVE ION CURRENTS FROM THE POSITIVE COLUMN OF MERCURY ARCS

A NEGATIVELY charged auxiliary electrode in the path of a mercury arc (as in a mercury rectifier) takes a current which is practically *independent of the impressed voltage* even if several hundred volts be employed. This current, which is usually a few milliamperes per cm², might conceivably be due either to emission of electrons from the electrode (as for example by photo-electric effect) or to positive ions taken up by the negative electrode. By placing in the ionized gas a negatively charged grid completely enclosing a positively charged electrode, it is found that the current to the positive electrode may remain nearly zero although the positive current of many milliamperes flows to the grid. This proves that the currents are due almost wholly to positive ions taken up by the negative electrode, since electrons from the grid would pass to the positive electrode.

Why are these positive ion currents so nearly independent of the voltage? The explanation seems comparatively simple and is in excellent accord with experiment.

Electrons are repelled from the negative electrode while positive ions are drawn towards it. Around each negative electrode there is thus a *sheath* of definite thickness containing only positive ions and neutral atoms. The thickness of this sheath can be calculated from the space charge equations used for pure electron discharges. Since mercury ions are 200×1848 times heavier than electrons, the currents carried with equal voltage will be $\sqrt{200 \times 1848}$ or 608 times smaller.

Thus X the thickness (in cm.) of the sheath in the case of a plane electrode receiving positive mercury ions with a current density i/A, (amperes per cm²) can be calculated from the equation[1]

$$\frac{i}{A} = \frac{2.33 \times 10^{-6}}{608} \frac{V^{3/2}}{X^2}$$

where V is the potential of the electrode with respect to the surrounding gas. With a current density of ten milliamperes per cm² the thickness of the sheath is thus only 0.02 cm with 100 volts on the electrode; and 0.0035 cm with 10 volts.

Electrons are reflected from the outside surface of the sheath while all *positive* ions which reach the sheath are attracted to the electrode. A change in the negative voltage of the electrode from 10 to 100 volts thus only changes the sheath thickness from 0.0035 up to 0.02 cm and since this displacement of the edge of the sheath is small compared to the free path of the electrons or ions, and the dimensions of the tube, it follows directly that no change occurs in the positive ion current reaching the electrode. The electrode is in fact perfectly screened from the discharge by the positive ion sheath, and its potential can not influence the phenomena occurring in the arc, nor the current flowing to the electrode.

With cylindrical electrodes of diameters comparable with the thickness of the sheath, the variation of the sheath diameter with the voltage causes the effective collecting area for the ions to change so that the currents are not strictly independent of the voltage. This conclusion affords a crucial test of the correctness of the theory, especially since electron emission would follow entirely different laws. The positive ion current flowing to the electrode should be proportional to the area of the outside of the sheath, or in other words to its diameter. This can be calculated by means of the space charge equation for concentric cylinders. For positive mercury ions this becomes

$$\frac{i}{L} = \frac{14.69 \times 10^{-6}}{608} \frac{V^{3/2}}{r\beta^2}$$

where L is the length and r is the radius of the cylindrical electrode and β is a function of a/r where a is the radius of the outside of the sheath. The method of calculating this function has been given[2] and a table of its value as a function of a/r will appear in a forthcoming number of the *Physical Review*.

The experimental data have confirmed the theory by showing that a small diameter of the collecting electrodes and low intensities of ionization cause an increased variation of current with voltage, both of these factors tending to make the sheath diameter large compared to the electrode diameter.

The following typical experimental data were ob-

[1] Langmuir, *Phys. Rev.*, 2, 450 (1913).

[2] Langmuir, *l.c.*

tained with a glow discharge in mercury vapor using a hot tungsten cathode emitting 30 milliamperes. The conditions were such as to give a rather weak ionization, but the results are quite comparable with those observed in the portions of a mercury arc tube where similar ionization occurs.

ELECTRODE RADIUS 0.0635 CM. LENGTH 6.2 CM.

Volts	I obs. m.a.	β	$\frac{a}{r}$	I cal. m.a.
40	0.31	1.40	2.58	0.308
60	0.36	1.77	3.06	0.366
80	0.40	2.05	3.43	0.410
100	0.46	2.26	3.72	0.443
140	0.52	2.74	4.39	0.524

The last column gives currents calculated on the assumption that they are proportional to the values of a/r in the 4th column as they should be by the theory. It is seen that the agreement with the currents in the 2nd column is within the probable experimental error.

The positive ion currents flowing into the sheath in this case correspond to a current density of 49 microamperes per cm². With the more intense ionization in a five-ampere mercury arc rectifier, positive ion current densities of about 30–60 milliamperes per cm² are obtained and these currents, because of the small thickness of the sheath, are much more nearly independent of the voltage even if the electrodes are of small diameter.

This theory, for reasons which can not well be stated in a brief note, leads to the following conception of the positive column of the mercury arc (and of glow discharges in general).

The glass walls of the discharge tube become negatively charged and repel (or reflect) all but a very minute fraction of the electrons that move towards the walls. Since electrons in general make elastic collisions with mercury atoms, the motion of the electrons is in random directions, almost as many electrons moving against the potential gradient in the arc, as with it. Thus an apparent current density of one ampere per cm² in an arc is to be regarded as consisting of an electron current of perhaps 20 amperes per cm² in one direction and a similar current of 19 amperes per cm² in the opposite direction. The positive ion currents, because of the large mass of the ions, are many hundreds of times smaller (10 to 60 milliamperes per cm²). The positive ions, in general, lose a great part of their energy in each collision with the atoms, and thus behave as though all collisions were inelastic.

Few, if any, of the ions move against the potential gradient, but by collisions with atoms they are thrown against the glass walls before having travelled more than one or two free paths, these paths usually being comparable with the diameter of the discharge tube. Enough electrons pass to the walls to neutralize the charges of the positive ions and in doing so liberate in the form of heat, an energy corresponding to the ionizing potential 10.4 volts. Thus a positive ion current density of 10 milliamperes per cm² generates a heat of over 0.1 watt per cm². This accounts for the influences of the size of the tube on the voltage drop in the arc.

A more complete discussion of these conclusions will be published in the November number of the G. E. Review.

IRVING LANGMUIR
RESEARCH LABORATORY, GENERAL ELECTRIC COMPANY, SCHENECTADY, N. Y.

LATE FERTILIZATION AND SEX-RATIO IN TROUT

THE experiments of R. Hertwig[1] and his pupils have shown conclusively that delayed fertilization in frogs will produce a preponderance of males, in some cultures 99–100 per cent. males being produced. It occurred to me that the trout would afford favorable material for seeing whether this phenomenon could be reproduced in other species. Accordingly, in 1921 and 1922, by the courtesy of the Midlands Fisheries Ltd., and in especial of their manager at Nailsworth, Glos., Mr. Stevens, I carried out some experiments on the matter.

I since discovered that Dr. Mrsic, in Professor Hertwig's laboratory at Munich, had also been working at the subject and had obtained more definite results than I. He had kindly sent me the proofs of his article due to appear shortly in the *Arch. Ent. Mech.*,[2] and allowed me to mention his results.

My own results were as follows:

	Total	Per cent. ♂♂ P.E.
Control '21	119	47.90 ± 3.09
Control '22	274	48.17 ± 2.04
Late-fertilized '21 (some 4, some 7 days late)	332	54.52 ± 1.83
Early-fertilized '22 (7 days early)	319	50.78 ± 1.89

Dr. Mrsic's were as under (I have calculated the probable errors for his figures, as these do not appear in his paper):

Control (I)	90	47.8 ± 3.55
4 days—7 days late (II–III)	199	39.7 ± 2.34
13 days late (IV)	77	50.6 ± 3.84
21 days late (V)	57	66.7 ± 4.21

[1] Hertwig, R., '05–'07, Verh. Dtsch. Zool. Ges.: '12, Biol. Zentr. 32; '21, Sitzb. Bayr. Ak. Wiss. 1921; Kuschakewitsch, '10, Festschr. f. R. Hertwig, 1910.

[2] Since published while the above was passing through the press.