hard technical argument, discussed some of the difficult problems in stratigraphy and metamorphosis presented by local formations. The zoologists, amongst whom the presence of students and the younger generation of workers was notable, were also occupied chiefly with strictly technical matters, in which the conjunction of laboratory workers, museum systematists, and those who deal with living animals at sea or on land was very advantageous. In engineering also there was a useful collaboration of the "practical" and the theoretical sides, of the laboratory and the workshop. The physiologists made several concessions to publicity well justified by the contemporary importance of such subjects as diabetes and cancer, but they also had a valuable discussion with the chemists and physicists on the extremely important recent advances in knowledge of the physics of living membranes described by the chemical president in his address. By general agreement the proceedings in the section of botany were of unusual scientific value, although they were of a kind for the most part difficult for laymen.

But the meeting of 1923 owes its success above all to its achievements in physical science. On the borderland of chemistry and physics theories are pressing on each other concerning the material stuff of the universe. A single instance may serve to explain the general trend of the new knowledge. Although for long it has been suspected that the elements were built up of common units differing in number and arrangement, the fractional quantities assigned to them by the most careful observation seemed to forbid the existence of any simple relationship. Professor Soddy and his fellow-workers have now shown that the atomic weights are a mere statistical average, representing the proportions in which substances not hitherto suspected to have separate existences are found mingled in nature. The elements themselves are simple multiples of a common unit. And so in various ways older complexities are being resolved in what are at once higher and simpler unities. Chemical, physical, electrical and magnetic properties are all being reconciled as expressions or presentations of more fundamental properties of more elementary constituents of matter. Nature is turning out to be articulated, built of unit pieces, and these in their mass, size and movements are comparable with the phenomena of light, at present the ultimate and most nearly absolute standard of the universe. The vital interest of the proceedings at the Liverpool meeting of the British Association lay less in the announcement of completed results to the public than in the actual shaping of knowledge in an assemblage of leading physicists and chemists from almost every country in the world under the honored presidency of Sir Ernest Rutherford.—*The London Times.*

SCIENTIFIC BOOKS

Mathematics. By David Eugene Smith. Marshall Jones Company, Boston, 1923, pp. x + 175.

This interesting little volume belongs to a series bearing the general title, "Our debt to Greece and Rome," edited by George Depue Hadzsits, University of Pennsylvania, and David Moore Robinson, Johns Hopkins University. An announcement appearing at the end of the volume gives 50 titles of the series together with the names of the authors in most cases. The present volume contains a brief introduction by T. L. Heath, who is well known on account of his extensive contributions to the history of Greek mathematics. Its four main divisions bear the following headings: Preliminary survey, the contributions in details, influence of the contributions, and conclusion.

The volume gives a very appreciative popular account of the mathematical contributions by the Greeks and the Romans, and brings out a number of historical facts which are not usually found in a history of mathematics. Hence, it will doubtless be read with profit by many mathematicians as well as by others to whom its popular style and very meager use of technical mathematics should appeal strongly. Mathematics has been called a Greek science, not only by those who find it difficult but also by those who are in position to understand its nature and who are familiar with the fundamental contributions of the Greeks along this line. It should, however, not be assumed that the Greeks developed the greater part of the mathematics of our times. They merely made a good start along certain important lines.

The reader who is mainly interested in actual facts relating to the contributions by the Greeks and Romans might sometimes wish that our author had not made such free use of the hyperbole. For instance, on page 90 we read: "In the first place we owe to the ancients our technical vocabulary, not merely that of mathematics in general and of notation in particular, but that of all the sciences"; while on page 160 we find the following sentence, "It is quite possible that our indebtedness in matters of notation and symbols is not great, and this should be frankly admitted." On page 114 we are told that Fermat was "the greatest genius of modern times in the theory of numbers," and on page 120 it is stated that "with respect to our indebtedness to Euclid, our modern text-books in mathematics are modeled primarily upon his works." This statement may profitably be compared with those relating to the modern tendency towards arithmetizing mathematics.

A question of a more serious nature may be raised as regards the mathematical contributions of the Romans. Our author emphasizes the fact that the Romans contributed practically nothing towards the advancement of pure mathematics, but he seems to give them too much credit as regards applied mathematics when he speaks on page 12 and elsewhere of the Romans and the Greeks as complements of each other. In particular, the works of Archimedes and of Heron stand out more prominently in applied mathematics than those of any Roman authors, and the Greek work along the line of mathematical astronomy seems to be more important than that of the Romans along the line of land surveying. Our author makes it clear, however, that the Romans were not gifted as mathematicians and this is the main point in question in this connection.

In view of the fact that the author of the present volume is so widely and favorably known it is likely that many readers thereof will be inclined to place unusual confidence in the accuracy of the statements made therein. It seems, therefore, desirable to note here a few modifications and corrections which might otherwise appear uncalled for in such a brief review. Beginning with one of the most important cases we note that on page 137 there appears the following sentence: "It was Eratosthenes the mathematician who found the circumference of the earth to a degree of approximation not equaled by Ptolemy the astronomer, and, indeed, not equaled until modern times." On page 131 it is stated that this result was approximately 25,000 miles. By consulting volume 6 of the well-known *Encyklopädie der Mathematischen Wissenschaften* one finds on page 223 thereof that Posidonius, who was born 141 years later than Eratosthenes, had already obtained a somewhat more nearly accurate value for the circumference of the earth than the one due to Eratosthenes, and that the Arabians who made measurements by order of the caliph Almamun obtained in 827 a still closer approximation. Moreover, according to this authority, none of these results is very close to the truth, since even the best of them misses the actual value by more than 10 per cent.

On page 42 it is stated that Euclid used the term "even-times even numbers" for numbers of the form 2^n. The inaccuracy of this assertion can easily be established by consulting the well-known work entitled "The Thirteen Books of Euclid," by T. L. Heath. On page 282 of volume 2 thereof appears a discussion of Euclid's use of this particular term, and on page 419 of the same volume we find Euclid's proof of a theorem relating to numbers which are both even-times even and even-times odd. This proves definitely that Euclid used the term in question for a much larger class of numbers than those which are of the form 2^n.

On page 66 our author refers to the three different types of algebra noted by Nesselmann in 1842 and frequently quoted in the histories of mathematics; *viz.*, the rhetorical, the syncopated and the symbolic. He adds that "the first is, generally speaking, pre-Grecian, but extends through the classical period as well; the second is late Greek and medieval; the third is modern." On page 4 of volume 2 of Tropfke's *Geschichte der Elementar-Mathematik,* 1921, it is stated that the first of these three types of algebras is found among the Greeks up to the first century after Christ. This is in substantial agreement with the statement noted above, but Tropfke adds that the East Arabs, the Persians, the West Arabs up to the thirteenth century, the medieval mathematicians, such as Leonardo of Pisa, Jordanus Nemorarius and their pupils up to Regiomontanus (1436–1476) also employed this type of algebra. The Arabs avoided symbols to such an extent that they even used words in place of number symbols.

One of the striking features of the volume under review is the fact that the Greek contributions to algebra are given such a prominent position in comparison with those of the Arabs and the Hindus. In fact, on page 129 it is stated that the Arabs "added not a single proposition of importance, nor did they make any progress towards the solution of the cubic or biquadratic equation or towards the approximation of the roots of numerical equations of higher degree. They were translators, popularizers, and text-book writers, but they were not creative algebraists. As to the Hindus, they added nothing worthy of note to the stock of algebraic knowledge except in the way of a symbolism which no later writers adopted, and in the way of numerous interesting problems." These views are especially interesting, since the Hindus and the Arabs have frequently been incorrectly credited with the founding of algebra. The reviewer is, however, inclined to believe that the above quotation does not give enough credit to the Hindus and the Arabs as regards contributions towards the development of algebra.

On page 61 it is stated that the first text-book on conics is due to Apollonius of Perga. This is surprising in view of the fact that it is very well known that Euclid wrote four books on conics, which have been lost, and that Aristaeus wrote five books on the same subject, probably at a somewhat earlier date. It is also surprising to note that on page 37 our author speaks of the choice of the base of the sexagesimal system of numerical notation and of the division of the circle into 360 degrees as if we knew definitely the motives leading to these choices. Most mathematical historians seem to regard these as very difficult unsettled historical questions and various theories have been advanced from time to time to account for these particular choices. In view of the fact that the ancient Egyptians and Babylonians extracted the square root it is difficult to see why our author says on page 101 "The operation of finding the square root of a number is distinctly Greek."

In closing, the reviewer desires to record one more surprise to himself when he read on page 77 that Diophantus "was searching in general for classes of numbers instead of particular numbers, and it is the class, as such, that is primarily sought in an indeterminate equation." It is well known that Diophantus usually gave only one solution even when the equation under consideration admitted an infinite number of solutions. Mathematical historians usually direct special attention to the fact that the Greeks were satisfied with one solution even in their geometric constructions. The reviewer never saw any evidence in support of the statement that Diophantus searched in general for classes of numbers in the solution of indeterminate equations.

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SPECIAL ARTICLES

POSITIVE ION CURRENTS FROM THE POSITIVE COLUMN OF MERCURY ARCS

A NEGATIVELY charged auxiliary electrode in the path of a mercury arc (as in a mercury rectifier) takes a current which is practically *independent of the impressed voltage* even if several hundred volts be employed. This current, which is usually a few milliamperes per cm^2., might conceivably be due either to emission of electrons from the electrode (as for example by photo-electric effect) or to positive ions taken up by the negative electrode. By placing in the ionized gas a negatively charged grid completely enclosing a positively charged electrode, it is found that the current to the positive electrode may remain nearly zero although the positive current of many milliamperes flows to the grid. This proves that the currents are due almost wholly to positive ions taken up by the negative electrode, since electrons from the grid would pass to the positive electrode.

Why are these positive ion currents so nearly independent of the voltage? The explanation seems comparatively simple and is in excellent accord with experiment.

Electrons are repelled from the negative electrode while positive ions are drawn towards it. Around each negative electrode there is thus a *sheath* of definite thickness containing only positive ions and neutral atoms. The thickness of this sheath can be calculated from the space charge equations used for pure electron discharges. Since mercury ions are 200 $\times$ 1848 times heavier than electrons, the currents carried with equal voltage will be $\sqrt{200 \times 1848}$ or 608 times smaller.

Thus X the thickness (in cm.) of the sheath in the case of a plane electrode receiving positive mercury ions with a current density i/A, (amperes per cm^2) can be calculated from the equation[1]

$$\frac{i}{A} = \frac{2.33 \times 10^{-6}}{608} \frac{V^{3/2}}{X^2}$$

where V is the potential of the electrode with respect to the surrounding gas. With a current density of ten milliamperes per cm^2 the thickness of the sheath is thus only 0.02 cm with 100 volts on the electrode; and 0.0035 cm with 10 volts.

Electrons are reflected from the outside surface of the sheath while all *positive* ions which reach the sheath are attracted to the electrode. A change in the negative voltage of the electrode from 10 to 100 volts thus only changes the sheath thickness from 0.0035 up to 0.02 cm and since this displacement of the edge of the sheath is small compared to the free path of the electrons or ions, and the dimensions of the tube, it follows directly that no change occurs in the positive ion current reaching the electrode. The electrode is in fact perfectly screened from the discharge by the positive ion sheath, and its potential can not influence the phenomena occurring in the arc, nor the current flowing to the electrode.

With cylindrical electrodes of diameters comparable with the thickness of the sheath, the variation of the sheath diameter with the voltage causes the effective collecting area for the ions to change so that the currents are not strictly independent of the voltage. This conclusion affords a crucial test of the correctness of the theory, especially since electron emission would follow entirely different laws. The positive ion current flowing to the electrode should be proportional to the area of the outside of the sheath, or in other words to its diameter. This can be calculated by means of the space charge equation for concentric cylinders. For positive mercury ions this becomes

$$\frac{i}{L} = \frac{14.69 \times 10^{-6}}{608} \frac{V^{3/2}}{r\beta^2}$$

where L is the length and r is the radius of the cylindrical electrode and β is a function of a/r where a is the radius of the outside of the sheath. The method of calculating this function has been given[2] and a table of its value as a function of a/r will appear in a forthcoming number of the *Physical Review*.

The experimental data have confirmed the theory by showing that a small diameter of the collecting electrodes and low intensities of ionization cause an increased variation of current with voltage, both of these factors tending to make the sheath diameter large compared to the electrode diameter.

The following typical experimental data were ob-

[1] Langmuir, *Phys. Rev.*, 2, 450 (1913).

[2] Langmuir, *l.c.*